Exercise 3.1 Page: 42

Question1. In the matrix A = $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$ **, write:**

(i) The order of the matrix.

(ii) The number of elements.

(iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution

(i) There are 3 horizontal lines (rows) and 4 vertical lines (columns) in the given matrix A.

Therefore, Order of the matrix is 3 x 4.

(ii) The number of elements in the matrix A is 3 x 4 = 12.

(iii) a_{13} Element in first row and third column = 19

a_{21} Element in second row and first column = 35

a_{33} Element in third row and third column = -5

a_{24} Element in second row and fourth column = 12

a_{23} Element in second row and third column = 5/2

Question2. If a matrix has 24 elements, what are possible orders it can order? What, if it has 13 elements?

Solution

We know that if a matrix is of the order m x n, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)

Hence, the possible orders of a matrix having 24 elements are:

1 × 24, 24 × 1, 2 × 12, 12 × 2, 3 × 8, 8 × 3, 4 × 6, and 6 × 4

(1, 13) and (13, 1) are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are 1 × 13 and 13 × 1.

Question3. If a matrix has 18 elements, what are the possible orders it can have? What if has 5 elements?

Solution
We know that if a matrix is of the order m × n, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6,), and (6, 3)

Hence, the possible orders of a matrix having 18 elements are:

1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6, and 6 × 3

(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1 × 5 and 5 × 1.

Question4. Construct a 2 x 2 matrix A = [a$_{ij}$] whose elements are given by:

(i) $a_{ij} = \dfrac{(i+j)^2}{2}$

(ii) $a_{ij} = i/j$

(iii) $a_{ij} = \dfrac{(1+2j)^2}{2}$

Solution

In general, a 2 × 2 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{ij} = \dfrac{(i+j)^2}{2}; i, j = 1, 2$

$\therefore a_{11} = \dfrac{(1+1)^2}{2} = \dfrac{4}{2} = 2 \quad a_{12} = \dfrac{(1+2)^2}{2} = \dfrac{9}{2}$

$a_{21} = \dfrac{(2+1)^2}{2} = \dfrac{9}{2} \quad a_{22} = \dfrac{(2+2)^2}{2} = \dfrac{16}{2} = 8$

Therefore, the required matrix is $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

(i)

In general, a 2 × 2 matrix is given by A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{ij} = \dfrac{i}{j}, i, j = 1, 2$

$\therefore a_{11} = \dfrac{1}{1} = 1 \quad a_{12} = \dfrac{1}{2}$

$a_{21} = \dfrac{2}{1} = 2 \quad a_{22} = \dfrac{2}{2} = 1$

Therefore, the required matrix is A = $\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$

(ii)

In general, a 2 × 2 matrix is given by A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$a_{ij} = \dfrac{(i+2j)^2}{2}, i, j = 1, 2$

$\therefore a_{11} = \dfrac{(1+2)^2}{2} = \dfrac{3^2}{2} = \dfrac{9}{2} \quad a_{12} = \dfrac{(1+4)^2}{2} = \dfrac{5^2}{2} = \dfrac{25}{2}$

$a_{21} = \dfrac{(2+2)^2}{2} = \dfrac{4^2}{2} = 8 \quad a_{22} = \dfrac{(2+4)^2}{2} = \dfrac{6^2}{2} = 18$

Therefore, the required matrix is $A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$

(iii)

Question 5. Construct a 3 x 4 matrix, whose elements are given by:

(i) $a_{ij} = \dfrac{1}{2}|-3i+j|$

(ii) $a_{ij} = 2i - j$

In general, a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$a_{(ij)} = 1/2\,|-3i+j|$, i = 1,2,3 and j = 1,2,3,4

$\therefore a_{11} = \dfrac{1}{2}|-3\times1+1| = \dfrac{1}{2}|-3+1| = \dfrac{1}{2}|-2| = \dfrac{2}{2} = 1$

$a_{21} = \dfrac{1}{2}|-3\times2+1| = \dfrac{1}{2}|-6+1| = \dfrac{1}{2}|-5| = \dfrac{5}{2}$

$a_{31} = \dfrac{1}{2}|-3\times3+1| = \dfrac{1}{2}|-9+1| = \dfrac{1}{2}|-8| = \dfrac{8}{2} = 4$

$a_{12} = \dfrac{1}{2}|-3\times1+2| = \dfrac{1}{2}|-3+2| = \dfrac{1}{2}|-1| = \dfrac{1}{2}$

$a_{22} = \dfrac{1}{2}|-3\times2+2| = \dfrac{1}{2}|-6+2| = \dfrac{1}{2}|-4| = \dfrac{4}{2} = 2$

$a_{32} = \dfrac{1}{2}|-3\times3+2| = \dfrac{1}{2}|-9+2| = \dfrac{1}{2}|-7| = \dfrac{7}{2}$

$a_{13} = \dfrac{1}{2}|-3\times1+3| = \dfrac{1}{2}|-3+3| = 0$

$a_{23} = \dfrac{1}{2}|-3\times2+3| = \dfrac{1}{2}|-6+3| = \dfrac{1}{2}|-3| = \dfrac{3}{2}$

$a_{33} = \dfrac{1}{2}|-3\times3+3| = \dfrac{1}{2}|-9+3| = \dfrac{1}{2}|-6| = \dfrac{6}{2} = 3$

$a_{14} = \dfrac{1}{2}|-3\times1+4| = \dfrac{1}{2}|-3+4| = \dfrac{1}{2}|1| = \dfrac{1}{2}$

$a_{24} = \dfrac{1}{2}|-3\times2+4| = \dfrac{1}{2}\left|-6+4\right| = \dfrac{1}{2}\left|-2\right| = \dfrac{2}{2} = 1$

$a_{34} = \dfrac{1}{2}|-3\times3+4| = \dfrac{1}{2}|-9+4| = \dfrac{1}{2}|-5| = \dfrac{5}{2}$

Therefore, the required matrix is A = $\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$

Solution : (i)

In general, a 3 × 4 matrix is given by $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$

$a_{ij} = 2i - j, i = 1, 2, 3$ and $j = 1, 2, 3, 4$

$\therefore a_{11} = 2 \times 1 - 1 = 2 - 1 = 1$

$a_{21} = 2 \times 2 - 1 = 4 - 1 = 3$

$a_{31} = 2 \times 3 - 1 = 6 - 1 = 5$

$a_{12} = 2 \times 1 - 2 == 2 - 2 = 0$

$a_{22} = 2 \times 2 - 2 = 4 - 2 = 2$

$a_{32} = 2 \times 3 - 2 = 6 - 2 = 4$

$a_{13} = 2 \times 1 - 3 = 2 - 3 = -1$

$a_{23} = 2 \times 2 - 3 = 4 - 3 = 1$

$a_{33} = 2 \times 3 - 3 = 6 - 3 = 3$

$a_{14} = 2 \times 1 - 4 = 2 - 4 = -2$

$a_{24} = 2 \times 2 - 4 = 4 - 4 = 0$

$a_{34} = 2 \times 3 - 4 = 6 - 4 = 2$

Therefore, the required matrix is $A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$

(ii)

Question6. Find the values of x,y and z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

(ii)

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Solution

(i) Given:

$$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

x = 1, y = 4 = z = 3

(ii)

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

x + y = 6, xy = 8 and 5 + z = 5

Now, 5 + z = 5 ⇒ z = 0

We know that:

$(x - y)^2 = (x + y)^2 - 4xy$

⇒ $(x - y)^2 = 36 - 32 = 4$

⇒x - y = ± 2

Now, when x - y = 2 and x + y = 6 , we get x = 4, y = 2

When x - y = -2 and x + y = 6, we get x = 2, y = 4

$\therefore$ x = 4, y = 2, and z = 0, or x = 2, y = 4 and z = 0

$$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

x + y + z = 9 ... (1)

x + z = 5 ... (2)

y + z = 7 ... (3)

From (1) and (2), we have:

y + 5 = 9

$\Rightarrow$ y = 4

Then, from (3), we have:

4 + z = 7

$\Rightarrow$ z = 3

$\therefore$ x + z = 5

$\Rightarrow$ x = 2

$\therefore$ x = 2, y = 4, and z = 3

(iii)

Question7. Find the values of a,b, c and d from the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3x+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

Solution

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3x+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

a − b = −1 ... (1)

2a − b = 0 … (2)

2a + c = 5 … (3)

3c + d = 13 … (4)

From (2), we have:

b = 2a

Then, from (1), we have:

a − 2a = −1

⇒ a = 1

⇒ b = 2

Now, from (3), we have:

2 ×1 + c = 5

⇒ c = 3

From (4) we have:

3 ×3 + d = 13

⇒ 9 + d = 13 ⇒ d = 4

∴a = 1, b = 2, c = 3, and d = 4

Question 8. A = [a_{ij}]$_{m \times n}$ is a square matrix if:

(A) m < n (B) m > n (C) m = n (D) None of these

Solution
It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns. A = [a_{ij}]$_{m \times n}$

option (C) is correct.

Question9. Which of the given values of x and y make the following pairs of matrices equal:

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(A) $x = \dfrac{-1}{3}, y = 7$

(B) Not possible to find

(C) $y = 7, x = \dfrac{-2}{3}$

(D) $x = \dfrac{-1}{3}, y = \dfrac{-2}{3}$

Solution :

It is given that

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

Equating the corresponding elements, we get:

$$3x+7 = 0 \Rightarrow x = -\dfrac{7}{3}$$

$$5 = y-2 \Rightarrow y = 7$$

$$y+1 = 8 \Rightarrow y = 7$$

$$2 - 3x = 4 \Rightarrow x = -\dfrac{2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x , which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Therefore, option (B) is correct.

Question10. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is:

(A) 27

(B) 18

(C) 81

(D) 512

Solution
The correct answer is D.The given matrix of the order 3 × 3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$

Exercise 3.2 Page: 58

Question1. Let A = $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$ **. Find each of the following:**

(i) A + B

(ii) A − B

(iii) 3A − C

(iv) AB

(v) BA

Solution

(i) $A + B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(ii) $A - B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2-1 & 4-3 \\ 3-(-2) & 2-5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

$$3A - C = 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \times 2 & 3 \times 4 \\ 3 \times 3 & 3 \times 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

(iii)

$$AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{bmatrix}$$

$$= \begin{bmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

(iv)

$$BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

(v)

Question2. Compute the following:

(i)
$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

(ii)
$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

(iii)
$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

(iv)
$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

Solution

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a+a & b+b \\ -b+b & a+a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(ii)

$$\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

(iii)

$$\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \quad [\because \sin^2 x + \cos^2 x = 1]$$

(iv)

Question3. Compute the indicated products:

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

(i)

(ii)
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2, 3, 4]$$

(iii)
$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

(iv)
$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

(v)
$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$$

(vi)
$$\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$$

Solution

(i)
$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a(a) + b(b) & a(-b) + b(a) \\ -b(a) + a(b) & -b(-b) + a(a) \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

(ii)
$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2, 3, 4]$$

$$= \begin{bmatrix} 1(2) & 1(3) & 1(4) \\ 2(3) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{bmatrix}$$

(iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

$$= \begin{bmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{bmatrix}$$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

$$= \begin{bmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{bmatrix}$$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2-1 & 0+2 & 2+1 \\ 3-2 & 0+4 & 3+2 \\ -1-1 & 0+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{bmatrix}$$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 6-1+9 & -9-0+3 \\ -2+0+6 & 3+0+2 \end{bmatrix} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

Question4. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ then compute (A + B) and (B – C). Also, verify that A + (B – C) = (A + B) – C.

Solution

$$A + B = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3 & 2-1 & -3+2 \\ 5+4 & 0+2 & 2+5 \\ 1+2 & -1+0 & 1+3 \end{bmatrix} = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$B - C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 & -1-1 & 2-2 \\ 4-0 & 2-3 & 5-2 \\ 2-1 & 0-(-2) & 3-3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$A + (B - C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(-1) & 2+(-2) & -3+0 \\ 5+4 & 0+(-1) & 2+3 \\ 1+1 & -1+2 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

$$(A + B) - C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 1-1 & -1-2 \\ 9-0 & 2-3 & 7-2 \\ 3-1 & -1-(-2) & 4-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

L.H.S. = R.H.S. Proved.

Question5. If
$$A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$
then compute 3A – 5B.

Solution

$$3A - 5B = 3 \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5 \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Question6. Simplify: $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Solution

$\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}+\sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\sin\theta\cos\theta & \cos^2\theta\end{bmatrix}+\begin{bmatrix}\sin^2\theta & -\sin\theta\cos\theta\\ \sin\theta\cos\theta & \sin^2\theta\end{bmatrix}$

$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & \cos\theta\sin\theta-\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\sin\theta\cos\theta & \cos^2\theta+\sin^2\theta\end{bmatrix}$

$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $(\because \cos^2\theta+\sin^2\theta=1)$

Question7. Find X and Y, if:

(i) $X+Y=\begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$

(ii) $2X+3Y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ and $3X+2Y=\begin{bmatrix}2 & -1\\ -1 & 5\end{bmatrix}$

Solution

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad ....(1)$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad ......(2)$$

Adding equations (1) and (2), we get:

$$2X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 7+3 & 0+0 \\ 2+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

Now, $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 7-5 & 0-0 \\ 2-1 & 5-4 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

(i)

$$2X + 3Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \quad ...(3)$$

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \quad ....(4)$$

Multiplying equation (3) with (2), we get:

$$2(2X + 3Y) = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 4X + 6Y = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} \quad ....(5)$$

Multiplying equation (4) with (3), we get:

$$3(3X + 2Y) = 3 \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$\Rightarrow 9X + 6Y = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} \quad ....(6)$$

(ii)

From (5) and (6), we have:

$$(4X+6Y)-(9X+6Y)=\begin{bmatrix}4 & 6 \\ 8 & 0\end{bmatrix}-\begin{bmatrix}6 & -6 \\ -3 & 15\end{bmatrix}$$

$$\Rightarrow -5X=\begin{bmatrix}4-6 & 6-(-6) \\ 8-(-3) & 0-15\end{bmatrix}=\begin{bmatrix}-2 & 12 \\ 11 & -15\end{bmatrix}$$

$$\therefore X=-\frac{1}{5}\begin{bmatrix}-2 & 12 \\ 11 & -15\end{bmatrix}=\begin{bmatrix}\dfrac{2}{5} & -\dfrac{12}{5} \\ -\dfrac{11}{5} & 3\end{bmatrix}$$

Now, $2X+3Y=\begin{bmatrix}2 & 3 \\ 4 & 0\end{bmatrix}$

$$\Rightarrow 2\begin{bmatrix}\dfrac{2}{5} & -\dfrac{12}{5} \\ -\dfrac{11}{5} & 3\end{bmatrix}+3Y=\begin{bmatrix}2 & 3 \\ 4 & 0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\dfrac{4}{5} & -\dfrac{24}{5} \\ -\dfrac{22}{5} & 6\end{bmatrix}+3Y=\begin{bmatrix}2 & 3 \\ 4 & 0\end{bmatrix}$$

$$\Rightarrow 3Y=\begin{bmatrix}2 & 3 \\ 4 & 0\end{bmatrix}-\begin{bmatrix}\dfrac{4}{5} & -\dfrac{24}{5} \\ -\dfrac{22}{5} & 6\end{bmatrix}$$

$$\Rightarrow 3Y=\begin{bmatrix}2-\dfrac{4}{5} & 3+\dfrac{24}{5} \\ 4+\dfrac{22}{5} & 0-6\end{bmatrix}=\begin{bmatrix}\dfrac{6}{5} & \dfrac{39}{5} \\ \dfrac{42}{5} & -6\end{bmatrix}$$

$$\therefore Y=\frac{1}{3}\begin{bmatrix}\dfrac{6}{5} & \dfrac{39}{5} \\ \dfrac{42}{5} & -6\end{bmatrix}=\begin{bmatrix}\dfrac{2}{5} & \dfrac{13}{5} \\ \dfrac{14}{5} & -2\end{bmatrix}$$

Question8. Find X if Y = $Y=\begin{bmatrix}3 & 2 \\ 1 & 4\end{bmatrix}$ and $2X+Y=\begin{bmatrix}1 & 0 \\ -3 & 2\end{bmatrix}$

Solution

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Question9. Find x and y if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Solution

$$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we have:

$2 + y = 5$

$\Rightarrow y = 3$

$2x + 2 = 8$

$\Rightarrow x = 3$

$\therefore x = 3$ and $y = 3$

Question10. Solve the equation for x,y,z and t if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Solution

$$2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$2x+3 = 9$
$\Rightarrow 2x = 6$
$\Rightarrow x = 3$

$2y = 12$
$\Rightarrow y = 6$

$2z-3 = 15$
$\Rightarrow 2z = 18$
$\Rightarrow z = 9$

$2t+6 = 18$
$\Rightarrow 2t = 12$
$\Rightarrow t = 6$

$\therefore x = 3, y = 6, z = 9,$ and $t = 6$

Question11. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, **find the values of x and y**

Solution :

$$x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

2x − y = 10 and 3x + y = 5

Adding these two equations, we have:

5x = 15

⇒ x = 3

Now, 3x + y = 5

⇒ y = 5 − 3x

⇒ y = 5 − 9 = −4

∴x = 3 and y = −4

Question12. Given: $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2W \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ Z+W & 3 \end{bmatrix}$ **find the values of x ,y,z and w**

Solution

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$3x = x+4$

$\Rightarrow 2x = 4$

$\Rightarrow x = 2$

$3y = 6+x+y$

$\Rightarrow 2y = 6+x = 6+2 = 8$

$\Rightarrow y = 4$

$3w = 2w+3$

$\Rightarrow w = 3$

$3z = -1+z+w$

$\Rightarrow 2z = -1+w = -1+3 = 2$

$\Rightarrow z = 1$

$\therefore x = 2,\ y = 4,\ z = 1,$ and $w = 3$

Question13. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **show that** $F(x)\,F(y) = F(x+y)$

Solution

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}, F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$F(x)F(y)$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$= F(x+y)$

$\therefore F(x)F(y) = F(x+y)$

Question14. Show that:

(i)
$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(ii)
$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$=\begin{bmatrix} 5(2)-1(3) & 5(1)-1(4) \\ 6(2)+7(3) & 6(1)+7(4) \end{bmatrix}$$

$$=\begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}=\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$=\begin{bmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{bmatrix}$$

$$=\begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}=\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\neq\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

(i)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(ii)

Question15. Find A² – 5A + 6I if A = $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

We have $A^2 = A \times A$

$$A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+3(-1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$\therefore A^2 - 5A + 6I$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5+6 & -1+0 & -3+0 \\ -1+0 & -7+6 & -10+0 \\ -5+0 & 4+0 & -2+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

Solution:

Question16. If A = $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ prove that $A^3 - 6A^2 + 7A + 2I = 0$.

Solution
L.H.S. = $A^3 - 6A^2 + 7A + 2I$

$$A^2 = AA = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now $A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$$

$\therefore A^3 - 6A^2 + 7A + 2I = O$

= R.H.S.   Proved.

Question17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $l = Matric\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **find k so that A² = kA − 2I**

Solution

$$A^2 = A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3)+(-2)(4) & 3(-2)+(-2)(-2) \\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k - 2 = 1$

$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

Thus, the value of k is 1.

Question18. If $A = \begin{bmatrix} 0 & -\frac{\tan\alpha}{2} \\ \frac{\tan\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Solution

On the L.H.S.

$I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 1 \end{bmatrix} \qquad \ldots(1)$$

On the R.H.S.

$$(I-A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 0 \end{bmatrix} \right) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\dfrac{\alpha}{2} \\ -\tan\dfrac{\alpha}{2} & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\dfrac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\dfrac{\alpha}{2} \\ -\cos\alpha\tan\dfrac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\dfrac{\alpha}{2} + \cos\alpha \end{bmatrix} \qquad \ldots(2)$$

$$= \begin{bmatrix} 1 - 2\sin^2\dfrac{\alpha}{2} + 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}\tan\dfrac{\alpha}{2} & -2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} + \left(2\cos^2\dfrac{\alpha}{2} - 1\right)\tan\dfrac{\alpha}{2} \\ -\left(2\cos^2\dfrac{\alpha}{2} - 1\right)\tan\dfrac{\alpha}{2} + 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} & 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}\tan\dfrac{\alpha}{2} + 1 - 2\sin^2\dfrac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2\sin^2\dfrac{\alpha}{2} + 2\sin^2\dfrac{\alpha}{2} & -2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} + 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} - \tan\dfrac{\alpha}{2} \\ -2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} + \tan\dfrac{\alpha}{2} + 2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2} & 2\sin^2\dfrac{\alpha}{2} + 1 - 2\sin^2\dfrac{\alpha}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 1 \end{bmatrix}$$

Thus, from (1) and (2), we get L.H.S. = R.H.S.

Question19. A trust fund has ` 30,000 that must be invested in two different types of bond. The first bond pays 5% interest per year and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ` 30,000 in two types of bonds, if the trust fund must obtain an annual interest of (a) ` 1800, (b) ` 2000.

Solution
(a) It is given that Rs.30,000 must be invested into two types of bonds with 5% and 7% interest rates.

Let Rs. x be invested in bonds of the first type. Thus, Rs. (30,000 − x) will be invested in the other type.

Hence, the amount invested in each type of the bonds can be represented in matrix form with each column corresponding to a different type of bond as:

X = [x 30,000 − x]

Annual interest obtained is Rs. 1800. We know, Interest = PTR/100

Here, the time is one year and thus T = 1.

Hence, the interest obtained after one year can be expressed in matrix representation as-

$$[x\ 30,000 - x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [1800]$$

$$\Rightarrow \left[x \times \frac{5}{100} + (30000 - x) \times \frac{7}{100} \right] = [1800]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

⇒ 5x + 210000 − 7x = 180000

⇒ − 2x = − 30000

∴ x = 15000

Amount invested in the first bond = x = Rs. 15000

⇒ Amount invested second bond = Rs. (30000 − x) = Rs. (30000 − 15000) = Rs. 15000

∴ The trust has to invest Rs. 15000 each in both the bonds in order to obtain an annual interest of Rs. 1800.

(b) It is given that Rs.30,000 must be invested into two types of bonds with 5% and 7% interest rates.

Let Rs. x be invested in bonds of the first type. Thus, Rs. (30,000 − x) will be invested in the other type.

Hence, the amount invested in each type of the bonds can be represented in matrix form with each column corresponding to a different type of bond as:

X = [x 30,000 − x]

Annual interest obtained is Rs. 2000.

Hence, the interest obtained after one year can be expressed in matrix representation as-

$$[x \ 30000 - x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [2000]$$

$$\Rightarrow \left[x \times \frac{5}{100} + (30000 - x) \times \frac{7}{100} \right] = [2000]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 2000$$

⇒ 5x + 210000 − 7x = 200000

⇒ −2x = − 10000

∴ x = 5000

Amount invested in the first bond = x = Rs. 5000

⇒ Amount invested second bond = Rs. (30000 − x) = Rs. (30000 − 5000) = Rs. 25000

∴ The trust has to invest Rs.5000 in the first bond and Rs.25000 in the second bond in order to obtain an annual interest of Rs.2000.

Question20. The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ` 80, ` 60 and ` 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution

The bookshop has 10 dozen chemistry books, 8 dozen physics books, and 10 dozen economics books.

The selling prices of a chemistry book, a physics book, and an economics book are respectively given as Rs 80, Rs 60, and Rs 40.

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$12\begin{bmatrix} 10 & 8 & 10 \end{bmatrix}\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$$= 12\begin{bmatrix} 10\times80+8\times60+10\times40 \end{bmatrix}$$
$$= 12(800+480+400)$$
$$= 12(1680)$$
$$= 20160$$

Thus, the bookshop will receive Rs 20160 from the sale of all these books.

Question21. Assume X, Y, Z, W and P are matrices of order 2 x n,3 x k,2 x p, n x 3 and p x k repectively. The restriction on n,k and p so that PY + WY will be define are:

A. k = 3, p = n

B. k is arbitrary, p = 2

C. p is arbitrary, k = 3

D. k = 2, p = 3

Solution

Matrices P and Y are of the orders p × k and 3 × k respectively.

Therefore, matrix PY will be defined if k = 3. Consequently, PY will be of the order p × k.

Matrices W and Y are of the orders n × 3 and 3 × k respectively.

Since the number of columns in W is equal to the number of rows in Y, matrix WY is well-defined and is of the order n × k.

Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order p × k and WY is of the order n × k. Therefore, we must have p = n.

Thus, k = 3 and p = n are the restrictions on n, k, and p so that PY + WY will be defined.

Therefore, option (A) is correct.

Question22. Assume X, Y, Z, W and P are matrices of order 2 x n,3 x k,2 x p, n x 3 and p x k repectively. If n = p then order of matrix 7X – 5Z is:

(A) p × 2

(B) 2 × n

(C) n × 3

(D)p × n

Solution :

Matrix X is of the order 2 × n.

Therefore, matrix 7X is also of the same order.

Matrix Z is of the order 2 × p, i.e., 2 × n   [Since n = p]

Therefore, matrix 5Z is also of the same order.

Now, both the matrices 7X and 5Z are of the order 2 × n.

Thus, matrix 7X − 5Z is well-defined and is of the order 2 × n.

The correct answer is B.

Exercise 3.3 Page: 66

Question1. Find the transpose of each of the following matrices:

(i)
$$\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$$

(ii)
$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

(iii)
$$\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$$

Solution

(i) Let A =
$$\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$$

Transpose of A = A' or $A^T = [5 \quad 1/2 \quad -1]$

(ii)
$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

Transpose of A = A' or $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii)
$$\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$$

Transpose of A = A' or $A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

Question2. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ **then verify that:**

(i) (A + B)' = A' + B'

(ii) (A − B)' = A' − B'

Solution

We have:

$$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}, \; B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

(i)

$$A + B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$$

Hence, we have verified that $(A + B)' = A' + B'$

(ii)

$$A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$$

Hence, we have verified that $(A - B)' = A' - B'$.

Question3. If $A\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} \end{bmatrix}$ **then verify that:**

(i) (A + B)' = A' + B'

(ii) (A − B)' = A' − B'

Solution

(i) It is known that $A = (A')'$

Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$$

$$\therefore (A + B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

$$A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$$

Thus, we have verified that $(A + B)' = A' + B'$.

(ii)

$$A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$$

$$\therefore (A - B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

$$A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Thus, we have verified that $(A - B)' = A' - B'$.

Question4. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ then find (A + 2B)'.

Solution

We know that $A = (A')'$

$$\therefore A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$\therefore A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$\therefore (A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$

Question5. For the matrices A and B, verify that (AB)' = B'A', where:

(i)
$$A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = [-1, 21]$$

(ii)
$$A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = [1, 5, 7]$$

Solution

(i)

$$AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$, $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

(ii)

$$AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$$

$$\therefore (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now, $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$, $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

$$\therefore B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Hence, we have verified that $(AB)' = B'A'$.

Question6. (i) If A = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ **then verify that A'A = I.**

(ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$ then verify that A'A = I.

Solution

(i)

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} (\cos\alpha)(\cos\alpha)+(-\sin\alpha)(-\sin\alpha) & (\cos\alpha)(\sin\alpha)+(-\sin\alpha)(\cos\alpha) \\ (\sin\alpha)(\cos\alpha)+(\cos\alpha)(-\sin\alpha) & (\sin\alpha)(\sin\alpha)+(\cos\alpha)(\cos\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha+\sin^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \sin^2\alpha+\cos^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, we have verified that $A'A = I$.

(ii)

$$A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$\begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}\begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$= \begin{bmatrix} (\sin\alpha)(\sin\alpha)+(-\cos\alpha)(-\cos\alpha) & (\sin\alpha)(\cos\alpha)+(-\cos\alpha)(\sin\alpha) \\ (\cos\alpha)(\sin\alpha)+(\sin\alpha)(-\cos\alpha) & (\cos\alpha)(\cos\alpha)+(\sin\alpha)(\sin\alpha) \end{bmatrix}$$

$$= \begin{bmatrix} \sin^2\alpha+\cos^2\alpha & \sin\alpha\cos\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Hence, we have verified that $A'A = I$.

Question7. (i) Show that the matrix A = $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ **is a symmetric matrix.**

(ii) Show that the matrix A = $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ **is a skew symmetric matrix.**

Solution

(i) Given: A = $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

Changing rows of matrix A as the columns of new matrix A' = $\begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ = A

∴ A' = A

Therefore, by definitions of symmetric matrix, A is a symmetric matrix.

(ii) Given: A = $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

A' = $\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ = $-\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ = − A

∴ A' = − A

Therefore, by definition matrix A is a skew-symmetric matrix

Question8. For a matrix A = $\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ **verify that:**

(i) (A + A') is a symmetric matrix.

(ii) (A – A') is a skew symmetric matrix.

Solution

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

(i) $A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\therefore (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

Hence, $(A + A')$ is a symmetric matrix.

(ii) $A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$(A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A - A')$

Hence, $(A - A')$ is a skew-symmetric matrix.

Question9. Find 1/2 (A + A') and 1/2(A – A') when A = $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Solution

The given matrix is $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

$\therefore A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$

$\therefore \frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Question10. Express the following matrices as the sum of a symmetric and skew symmetric matrix:

(i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

$$\text{(iv)} \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$$

Solution

(i)

Let $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$, then $A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$

Let $P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

Let $Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P + Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

(ii)

Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, then $A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$

Let $P = \dfrac{1}{2}(A + A') = \dfrac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P$

Thus, $P = \dfrac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Let $Q = \dfrac{1}{2}(A - A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$

Thus, $Q = \dfrac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$P + Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$

Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$, then $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Now, $A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

Let $P = \dfrac{1}{2}(A + A') = \dfrac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \dfrac{1}{2} & -\dfrac{5}{2} \\ \dfrac{1}{2} & -2 & -2 \\ -\dfrac{5}{2} & -2 & 2 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 3 & \dfrac{1}{2} & -\dfrac{5}{2} \\ \dfrac{1}{2} & -2 & -2 \\ -\dfrac{5}{2} & -2 & 2 \end{bmatrix} = P$

Thus, $P = \dfrac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$

Let $Q = \dfrac{1}{2}(A - A') = \dfrac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{5}{2} & \dfrac{3}{2} \\ -\dfrac{5}{2} & 0 & 3 \\ -\dfrac{3}{2} & -3 & 0 \end{bmatrix}$

(iii)

Now, $Q' = \begin{bmatrix} 0 & -\dfrac{5}{2} & -\dfrac{3}{2} \\ \dfrac{5}{2} & 0 & -3 \\ \dfrac{3}{2} & 3 & 0 \end{bmatrix} = -Q$

Thus, $Q = \dfrac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$$P + Q = \begin{bmatrix} 3 & \dfrac{1}{2} & -\dfrac{5}{2} \\ \dfrac{1}{2} & -2 & -2 \\ -\dfrac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \dfrac{5}{2} & \dfrac{3}{2} \\ -\dfrac{5}{2} & 0 & 3 \\ -\dfrac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = A$$

Let $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$, then $A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

Now $A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

Let $P = \dfrac{1}{2}(A + A') = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

Now, $P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P$

Thus, $P = \dfrac{1}{2}(A + A')$ is a symmetric matrix.

Now, $A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Let $Q = \dfrac{1}{2}(A - A') = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Now, $Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -Q$

Thus, $Q = \dfrac{1}{2}(A - A')$ is a skew-symmetric matrix.

Representing A as the sum of P and Q:

$P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A$

(iv)

Choose the correct answer in Exercises 11 and 12.

Question11. If A and B are symmetric matrices of same order, AB − BA is a:

(A) Skew-symmetric matrix

(B) Symmetric matrix

(C) Zero matrix

(D) Identity matrix

Solution
Given: A and B are symmetric matrices ∴ A = A' and B = B'

Now, (AB – BA)' = (AB)' – (BA)' ∴ (AB – BA)' = B'A' – A'B' [Reversal law]

∴ (AB – BA)' = BA – AB [From eq. (i)] ∴ (AB – BA)' = – (AB – BA)

∴ (AB – BA) is a skew matrix.

Therefore, option (A) is correct.

Question12. If A = $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ **, then A + A' = I, if the value of α is:**

(A) π/6

(B) π/3

(C) π

(D) 3π/2

Solution

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Now, $A + A' = I$

$$\therefore \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements of the two matrices, we have:

$2\cos\alpha = 1$

$$\Rightarrow \cos\alpha = \frac{1}{2} = \cos\frac{\pi}{3}$$

$$\therefore \alpha = \frac{\pi}{3}$$

Therefore, option (B) is correct.

Exercise 3.4 Page: 69

Using elementary transformation, find the inverse of each of the matrices, if it exists in Exercises 1 to 6.

Question 1. $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ $(R_2 \to R_2 - 2R_1)$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{2}{5} & \dfrac{1}{5} \end{bmatrix} A$ $\left(R_2 \to \dfrac{1}{5} R_2\right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{3}{5} & \dfrac{1}{5} \\ -\dfrac{2}{5} & \dfrac{1}{5} \end{bmatrix} A$ $(R_1 \to R_1 + R_2)$

$\therefore A^{-1} = \begin{bmatrix} \dfrac{3}{5} & \dfrac{1}{5} \\ -\dfrac{2}{5} & \dfrac{1}{5} \end{bmatrix}$

Question 2. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ $\qquad (R_1 \rightarrow R_1 - R_2)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$ $\qquad (R_2 \rightarrow R_2 - R_1)$

$\therefore A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

Question3. $\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ $\qquad (R_2 \rightarrow R_2 - 2R_1)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$ $\qquad (R_1 \rightarrow R_1 - 3R_2)$

$\therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$

Question4. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Question5. $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

Solution

Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & \dfrac{1}{2} \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \rightarrow \dfrac{1}{2} R_1 \right)$

$\Rightarrow \begin{bmatrix} 1 & \dfrac{1}{2} \\ 0 & \dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 \\ -\dfrac{7}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \rightarrow R_2 - 7R_1 \right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -\dfrac{7}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \rightarrow R_1 - R_2 \right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} A \qquad \left(R_2 \rightarrow 2R_2 \right)$

$\therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

Let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & \dfrac{3}{2} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$ $\qquad \left(R_1 \to \dfrac{1}{2} R_1 \right)$

$\Rightarrow \begin{bmatrix} 1 & \dfrac{3}{2} \\ 0 & -\dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 \\ -\dfrac{5}{2} & 1 \end{bmatrix} A$ $\qquad \left(R_2 \to R_2 - 5R_1 \right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -\dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -\dfrac{5}{2} & 1 \end{bmatrix} A$ $\qquad \left(R_1 \to R_1 + 3R_2 \right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$ $\qquad \left(R_2 \to -2R_1 \right)$

$\therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

Question6. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Solution :

Let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & \dfrac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \dfrac{1}{2} R_1 \right)$

$\Rightarrow \begin{bmatrix} 1 & \dfrac{5}{2} \\ 0 & \dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 \\ -\dfrac{1}{2} & 1 \end{bmatrix} A \qquad \left(R_2 \to R_2 - R_1 \right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -\dfrac{1}{2} & 1 \end{bmatrix} A \qquad \left(R_1 \to R_2 - 5R_2 \right)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A \qquad \left(R_2 \to 2R_2 \right)$

$\therefore A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

Using elementary transformation, find the inverse of each of the matrices, if it exists in Exercises 7 to 14.

Question7. $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Solution :

Let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

We know that $A = AI$

$\therefore \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ $\qquad (C_1 \rightarrow C_1 - 2C_2)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ $\qquad (C_2 \rightarrow C_2 - C_1)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ $\qquad (C_1 \rightarrow C_1 - C_2)$

$\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

Question8. $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Solution :

Let $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$ $\qquad (R_1 \rightarrow R_1 - R_2)$

$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$ $\qquad (R_2 \rightarrow R_2 - 3R_1)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$ $\qquad (R_1 \rightarrow R_1 - R_2)$

$\therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

Question9. $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

Solution :

Let A = $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

We know that $A = IA$

$\therefore \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \rightarrow R_1 - R_2)$

$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 - 2R_1)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A \qquad (R_1 \rightarrow R_1 - 3R_2)$

$\therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

Question10. $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Solution :

Let A = $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

We know that $A = AI$

$$\therefore \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \qquad (C_1 \rightarrow C_1 + 2C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \qquad (C_2 \rightarrow C_2 + C_1)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \dfrac{1}{2} \\ 2 & \dfrac{3}{2} \end{bmatrix} \qquad \left(C_2 \rightarrow \dfrac{1}{2}C_2\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \dfrac{1}{2} \\ 2 & \dfrac{3}{2} \end{bmatrix}$$

Question11. $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Solution   :

Let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

We know that $A = AI$

$$\therefore \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \qquad (C_2 \rightarrow C_2 + 3C_1)$$

$$\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -2 & 3 \\ -1 & 1 \end{bmatrix} \qquad (C_1 \rightarrow C_1 - C_2)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \qquad \left(C_1 \rightarrow \frac{1}{2}C_1 \right)$$

$$\therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

Question 12. $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Solution :

Let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A \qquad \left(R_1 \rightarrow \frac{1}{6}R_1 \right)$$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 + 2R_1)$$

Now, in the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Therefore, A^{-1} does not exist.

Question13. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Solution :

Let A = $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

We know that A = IA

$\therefore \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A \qquad (R_1 \rightarrow R_1 + R_2)$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 + R_1)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A \qquad (R_1 \rightarrow R_1 + R_2)$

$\therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

Question14. $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Solution :

Let A = $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \dfrac{1}{2} R_2$, we have:

$$\begin{bmatrix} 0 & 0 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

Now, in the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Therefore, A^{-1} does not exist.

Using elementary transformation, find the inverse of each of the matrices, if it exists in Exercises 15 to 17.

Question15. $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Solution   :

Let A = $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \left(R_2 \rightarrow \dfrac{1}{5}\right)$$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{bmatrix} = \begin{bmatrix} \dfrac{4}{5} & \dfrac{1}{5} & -1 \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ -\dfrac{2}{5} & \dfrac{2}{5} & 1 \end{bmatrix} A \qquad (R_1 \rightarrow R_1 + R_2 \text{ and } R_3 \rightarrow R_3 + 2R_2)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \dfrac{4}{5} & \dfrac{1}{5} & -1 \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ 2 & 1 & -2 \end{bmatrix} A \qquad (R_3 \rightarrow R_3 + 3R_1)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{4}{5} & \dfrac{1}{5} & -1 \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ \dfrac{2}{5} & \dfrac{1}{5} & -\dfrac{2}{5} \end{bmatrix} A \qquad \left(R_3 \rightarrow \dfrac{1}{5}R_3\right)$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{2}{5} & 0 & -\dfrac{3}{5} \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ \dfrac{2}{5} & \dfrac{1}{5} & -\dfrac{2}{5} \end{bmatrix} A \qquad (R_1 \to R_1 - R_3)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\dfrac{2}{5} & 0 & \dfrac{3}{5} \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ \dfrac{2}{5} & \dfrac{1}{5} & -\dfrac{2}{5} \end{bmatrix} A \qquad (R_1 \to (-1)R_1)$$

$$\therefore A^{-1} = \begin{bmatrix} -\dfrac{2}{5} & 0 & \dfrac{3}{5} \\ -\dfrac{1}{5} & \dfrac{1}{5} & 0 \\ \dfrac{2}{5} & \dfrac{1}{5} & -\dfrac{2}{5} \end{bmatrix}$$

Question16.
$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

Solution

Let A = $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$,

We know that $A = IA$

$$\therefore \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 + 3R_1$ and $R_3 \to R_3 - 2R_1$, we have:

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 + 3R_3$ and $R_2 \to R_2 + 8R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

Applying $R_3 \to \dfrac{1}{25} R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -\dfrac{3}{5} & \dfrac{1}{25} & \dfrac{9}{25} \end{bmatrix} A$$

Applying $R_1 \to R_1 - 10R_3$, and $R_2 \to R_2 - 21R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\dfrac{2}{5} & -\dfrac{3}{5} \\ -\dfrac{2}{5} & \dfrac{4}{25} & \dfrac{11}{25} \\ -\dfrac{3}{5} & \dfrac{1}{25} & \dfrac{9}{25} \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & -\dfrac{2}{5} & -\dfrac{3}{5} \\ -\dfrac{2}{5} & \dfrac{4}{25} & \dfrac{11}{25} \\ -\dfrac{3}{5} & \dfrac{1}{25} & \dfrac{9}{25} \end{bmatrix}$$

Question17. $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Solution

Let A = $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$,

We know that $A = IA$

$$\therefore \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to \dfrac{1}{2} R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\dfrac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 5R_1$, we have:

$$\begin{bmatrix} 1 & 0 & -\dfrac{1}{2} \\ 0 & 1 & \dfrac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 & 0 \\ -\dfrac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 - R_2$, we have:

$$\begin{bmatrix} 1 & 0 & -\dfrac{1}{2} \\ 0 & 1 & \dfrac{5}{2} \\ 0 & 0 & \dfrac{1}{2} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 & 0 \\ -\dfrac{5}{2} & 1 & 0 \\ \dfrac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$, we have:

$$\begin{bmatrix} 1 & 0 & -\dfrac{1}{2} \\ 0 & 1 & \dfrac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} & 0 & 0 \\ -\dfrac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \dfrac{1}{2}R_3$, and $R_2 \rightarrow R_2 - \dfrac{5}{2}R_3$, we have:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Question18. Matrices A and B will be inverse of each other only if:

(A) AB = BA

(B) AB = BA = 0

(C) AB = 0, BA = I

(D) AB = BA = I

Solution
By definition of inverse of square matrix,

Option (A) is correct.

Miscellaneous Exercise **Page: 72**

Question1. If A and B are symmetric matrices, prove that AB – BA is a skew symmetric matrix.

Solution
A and B are symmetric matrices.

A' = A and B' = B ……….(i)

Now, (AB – BA)' = (AB)' – (BA)'   ∴ (AB – BA)' = B'A' – A'B' [Reversal law]

∴ (AB – BA)' = BA – AB   [Using eq. (i)]

∴ (AB – BA)' = – (AB – BA)

Therefore, (AB – BA) is a skew symmetric.

Question2. Show that the matrix B'AB is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution
(B'AB)' = [B'(AB]' = (AB)' (B')'   [∴ (CD)' = D'C']

∴ (B'AB)' = B'A'B   ……….(i)

Case I: A is a symmetric matrix, then   A' = A

From eq. (i) (B'AB)' = B'AB

B'AB is a symmetric matrix.

Case II: A is a skew symmetric matrix. A' = – A

Putting A' = – A in eq. (i), (B'AB)' = B'(– A)B = – B'AB

B'AB is a skew symmetric matrix.

Question3. Find the values of x,y,z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfies the equation A'A = I.**

Solution

Given: $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$$\therefore A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Now, $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-xz+zx & 2yz-yz-yz & z^2+z^2+z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing the corresponding elements, we have:

$$2x^2 = 1 \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

$$6y^2 = 1 \Rightarrow y = \pm\frac{1}{\sqrt{6}}$$

$$3z^2 = 1 \Rightarrow z = \pm\frac{1}{\sqrt{3}}$$

Question4. For what value of x $[1,2,1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix}$ = O?

Solution

$$[1 \quad 2 \quad 1]\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [1+4+1 \quad 2+0+0 \quad 0+2+2]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6 \quad 2 \quad 4]\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6(0)+2(2)+4(x)] = O$$

$$\Rightarrow [4+4x] = [0]$$

$\therefore 4 + 4x = 0$

$\Rightarrow x = -1$

Thus, the required value of x is -1.

Question5. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I = 0$.

Solution

Given: A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O = \text{R.H.S.}$$

$$\therefore A^2 - 5A + 7I = O$$

Question6. Find x if
$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

Solution

$$[x \quad -5 \quad -1] \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x+0-2 \quad 0-10+0 \quad 2x-5-3] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x-2 \quad -10 \quad 2x-8] \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x(x-2)-40+2x-8] = O$$
$$\Rightarrow [x^2-2x-40+2x-8] = [0]$$
$$\Rightarrow [x^2-48] = [0]$$

$$\therefore x^2 - 48 = 0$$
$$\Rightarrow x^2 = 48$$
$$\Rightarrow x = \pm 4\sqrt{3}$$

Question7. A manufacturer produces three products, x,y,z which he sells in two markets. Annual sales are indicated below:

	Market	Products
I. 10,000	2,000	18,000
II. 6,000	20,000	8,000

(a) If unit sales prices of x,y and z are ` 2.50, ` 1.50 and ` 1.00 respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are ` 2.00, ` 1.00 and 50 paise respectively. Find the gross profit.

Solution :

(a) The unit sale prices of x, y, and z are respectively given as Rs 2.50, Rs 1.50, and Rs 1.00.

Consequently, the total revenue in market I can be represented in the form of a matrix as:

$$[10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

= 10000 x 2.50 + 2000 x 1.50 + 18000 x 1.00

= 25000 + 3000 + 18000

= 46000

The total revenue in market II can be represented in the form of a matrix as:

$$[6000 \quad 20000 \quad 8000] \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

= 6000 x 2.50 + 20000 x 1.50 + 8000 x 1.00

= 15000 + 30000 + 8000

= 53000

Therefore, the total revenue in market I isRs 46000 and the same in market II isRs 53000.

(b) The unit cost prices of x, y, and z are respectively given as Rs 2.00, Rs 1.00, and 50 paise.

Consequently, the total cost prices of all the products in market I can be represented in the form of a matrix as:

$$[10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

= 10000 x 2.00 + 2000 x 1.00 + 18000 x 0.50

= 20000 + 2000 + 9000

= 31000

Since the total revenue in market I isRs 46000, the gross profit in this marketis (Rs 46000 − Rs 31000) Rs 15000.

$$\begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$$

= 6000 x 2.00 + 20000 x 1.00 + 8000 x 0.50

= 12000 + 20000 + 4000

= 36000

The total cost prices of all the products in market II can be represented in the form of a matrix as:

Since the total revenue in market II isRs 53000, the gross profit in this market is (Rs 53000 − Rs 36000) Rs 17000.

Question8. Find the matrix X so that X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Solution

Given: X $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$ ……….(i)

The matrix given on the R.H.S. of the equation is a 2 × 3 matrix and the one given on the L.H.S. of the equation is a 2 × 3 matrix. Therefore, X has to be a 2 × 2 matrix.

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

Therefore, we have:

$\begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Equating the corresponding elements of the two matrices, we have:

$a + 4c = -7, \quad 2a + 5c = -8, \quad 3a + 6c = -9$
$b + 4d = 2, \quad 2b + 5d = 4, \quad 3b + 6d = 6$

Now, $a + 4c = -7 \Rightarrow a = -7 - 4c$

$\therefore 2a + 5c = -8 \Rightarrow -14 - 8c + 5c = -8$
$\Rightarrow -3c = 6$
$\Rightarrow c = -2$

$\therefore a = -7 - 4(-2) = -7 + 8 = 1$

Now, $b + 4d = 2 \Rightarrow b = 2 - 4d$

$\therefore 2b + 5d = 4 \Rightarrow 4 - 8d + 5d = 4$
$\Rightarrow -3d = 0$
$\Rightarrow d = 0$

$\therefore b = 2 - 4(0) = 2$

Thus, a = 1, b = 2, c = −2, d = 0

Hence, the required matrix X is $\begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

Question9. If A = $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ **is such that A² = I, then:**

(A) $1 + \alpha^2 + \beta\gamma = 0$

(B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$

(D) $1 + \alpha^2 - \beta\gamma = 0$

Solution

Given: $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and A² = I

$$\therefore A^2 = A \cdot A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix}$$

Now, $A^2 = I \Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

On comparing the corresponding elements, we have:

$\alpha^2 + \beta\gamma = 1$

$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

Equating corresponding entries, we have Therefore, option (C) is correct.

Question10. If the matrix A is both symmetric and skew symmetric, then:

(A) A is a diagonal matrix

(B) A is a zero matrix

(C) A is a square matrix

(D) None of these

Solution
Since, A is symmetric, therefore, A' = A ……..(i)

And A is skew-symmetric, therefore, A' = – A

A = – A [From eq. (i)]

A + A = 0 ⇒2A = 0 ⇒A = 0

Therefore, A is zero matrix.

Therefore, option (B) is correct.

Question15. If A is a square matrix such that A^2 = A, then $(I + A)^3$ – 7A is equal to:

(A) A

(B) I – A

(C) I

(D) 3A

Solution
Given: A^2 = A   …..(i)

Multiplying both sides by A, $A^3 = A^2$ = A [From eq. (i)] ……(ii)

Also given $(I + A)^3 – 7A = I^3 + A^3 + 3I^2A + 3IA^2 – 7A$

Putting A^2 = A [from eq. (i)] and A^3 = A [from eq. (ii)],

= I + A + 3IA + 3IA – 7A = I + A + 3A + 3A – 7A   [IA = A]

= I + 7A – 7A = I

Therefore, option (C) is correct.