CHAPTER 2 POLYNOMIALS

EXERCISE 2.3 PAGE:35

1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let $p(x) = x^3+x^2+x+1$

The zero of x+1 is -1. [x+1 = 0 means x = -1]

$p(-1) = (-1)^3+(-1)^2+(-1)+1$

$= -1+1-1+1$

$= 0$

∴ By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4+x^3+x^2+x+1$

Solution:

Let $p(x)= x^4+x^3+x^2+x+1$

The zero of x+1 is -1. [x+1= 0 means x = -1]

$p(-1) = (-1)^4+(-1)^3+(-1)^2+(-1)+1$

$= 1-1+1-1+1$

$= 1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4+3x^3+3x^2+x+1$

Solution:

Let $p(x)= x^4+3x^3+3x^2+x+1$

The zero of x+1 is -1.

$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$

$=1-3+3-1+1$

$=1 \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^4+3x^3+3x^2+x+1$

(iv) $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

Solution:

Let $p(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2} = -1-1+2+\sqrt{2}+\sqrt{2}$

$= 2\sqrt{2} \neq 0$

∴ By factor theorem, x+1 is not a factor of $x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3+x^2-2x-1$, $g(x) = x+1$

Solution:

$p(x) = 2x^3+x^2-2x-1$, $g(x) = x+1$

$g(x) = 0$

$\Rightarrow x+1 = 0$

$\Rightarrow x = -1$

∴ Zero of g(x) is -1.

Now,

$p(-1) = 2(-1)^3+(-1)^2-2(-1)-1$

$= -2+1+2-1$

$= 0$

∴ By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x+2$

Solution:

$p(x) = x^3+3x^2+3x+1$, $g(x) = x+2$

$g(x) = 0$

⇒ $x+2 = 0$

⇒ $x = -2$

∴ Zero of g(x) is -2.

Now,

$p(-2) = (-2)^3+3(-2)^2+3(-2)+1$

$= -8+12-6+1$

$= -1 \neq 0$

∴ By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x-3$

Solution:

$p(x) = x^3-4x^2+x+6$, $g(x) = x - 3$

$g(x) = 0$

⇒ $x-3 = 0$

⇒ $x = 3$

∴ Zero of g(x) is 3.

Now,

$p(3) = (3)^3-4(3)^2+(3)+6$

$= 27-36+3+6$

= 0

∴ By factor theorem, g(x) is a factor of p(x).

3. Find the value of k, if x–1 is a factor of p(x) in each of the following cases:

(i) p(x) = x^2+x+k

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

⇒ $(1)^2+(1)+k = 0$

⇒ 1+1+k = 0

⇒ 2+k = 0

⇒ k = -2

(ii) p(x) = $2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

⇒ $2(1)^2+k(1)+\sqrt{2} = 0$

⇒ $2+k+\sqrt{2} = 0$

⇒ $k = -(2+\sqrt{2})$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

Solution:

If x-1 is a factor of p(x), then p(1)=0

By Factor Theorem

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k = \sqrt{2} - 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution:

If x-1 is a factor of p(x), then p(1) = 0

By Factor Theorem

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$\Rightarrow k - 3 + k = 0$

$\Rightarrow 2k - 3 = 0$

$\Rightarrow k = 3/2$

4. Factorise:

(i) $12x^2 - 7x + 1$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -7 and product =1×12 = 12

We get -3 and -4 as the numbers [-3+-4=-7 and -3×-4 = 12]

$12x^2-7x+1 = 12x^2-4x-3x+1$

$= 4x(3x-1)-1(3x-1)$

$= (4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = 7 and product = 2×3 = 6

We get 6 and 1 as the numbers [6+1 = 7 and 6×1 = 6]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$= 2x\ (x+3)+1(x+3)$

$= (2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = 5 and product = 6×-6 = -36

We get -4 and 9 as the numbers [-4+9 = 5 and -4×9 = -36]

$6x^2+5x-6 = 6x^2+9x-4x-6$

$= 3x(2x+3)-2(2x+3)$

$= (2x+3)(3x-2)$

(iv) $3x^2-x-4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum = -1 and product = 3×-4 = -12

We get -4 and 3 as the numbers [-4+3 = -1 and -4×3 = -12]

$3x^2-x-4 = 3x^2-4x+3x-4$

$= x(3x-4)+1(3x-4)$

$= (3x-4)(x+1)$

5. Factorise:

(i) x^3-2x^2-x+2

Solution:

Let $p(x) = x^3-2x^2-x+2$

Factors of 2 are ±1 and ± 2

Now,

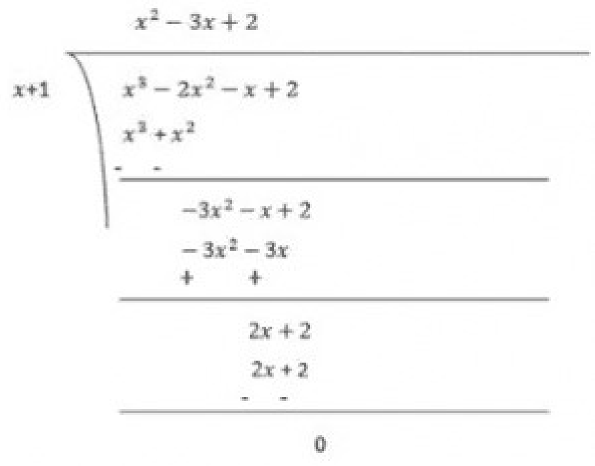
$p(x) = x^3 - 2x^2 - x + 2$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 2 + 1 + 2$

$= 0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2 - 3x + 2) = (x+1)(x^2 - x - 2x + 2)$

$= (x+1)(x(x-1) - 2(x-1))$

$= (x+1)(x-1)(x-2)$

(ii) $x^3 - 3x^2 - 9x - 5$

Solution:

Let $p(x) = x^3 - 3x^2 - 9x - 5$

Factors of 5 are ±1 and ±5

By the trial method, we find that

p(5) = 0

So, (x-5) is factor of p(x)

Now,

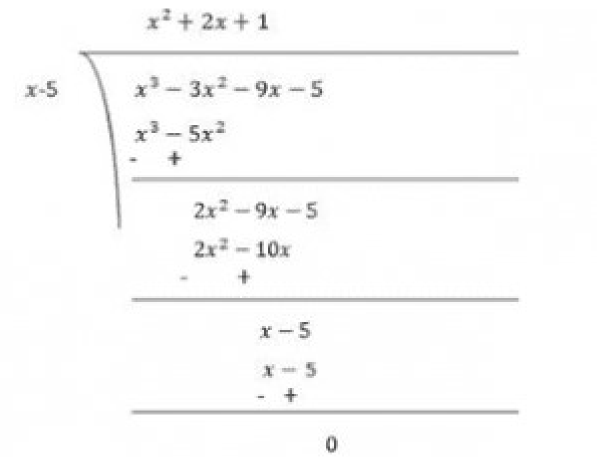
$p(x) = x^3 - 3x^2 - 9x - 5$

$p(5) = (5)^3 - 3(5)^2 - 9(5) - 5$

= 125-75-45-5

= 0

Therefore, (x-5) is the factor of p(x)



Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2+2x+1) = (x-5)(x^2+x+x+1)$

= (x-5)(x(x+1)+1(x+1))

= (x-5)(x+1)(x+1)

(iii) $x^3+13x^2+32x+20$

Solution:

Let $p(x) = x^3+13x^2+32x+20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By the trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

Now,

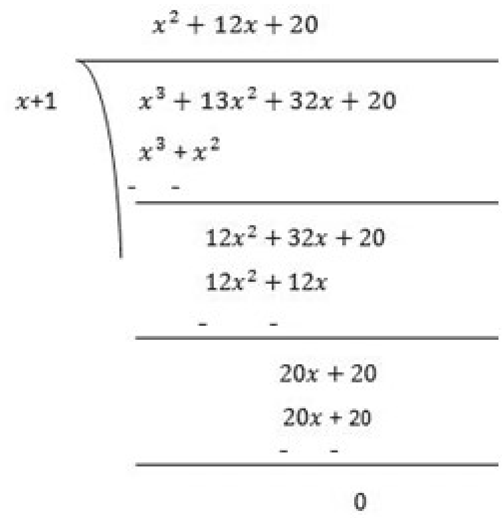
$p(x)= x^3+13x^2+32x+20$

$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$

$= -1+13-32+20$

$= 0$

Therefore, $(x+1)$ is the factor of $p(x)$



Now, Dividend = Divisor × Quotient +Remainder

$(x+1)(x^2+12x+20) = (x+1)(x^2+2x+10x+20)$

$= (x+1)x(x+2)+10(x+2)$

$= (x+1)(x+2)(x+10)$

(iv) $2y^3+y^2-2y-1$

Solution:

Let $p(y) = 2y^3+y^2-2y-1$

Factors $= 2\times(-1)= -2$ are ±1 and ±2

By the trial method, we find that

$p(1) = 0$

So, $(y-1)$ is factor of $p(y)$

Now,

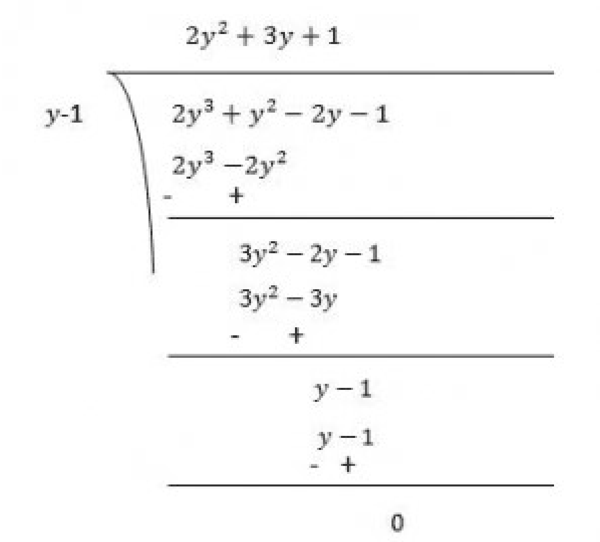
$p(y) = 2y^3+y^2-2y-1$

$p(1) = 2(1)^3+(1)^2-2(1)-1$

$= 2+1-2$

$= 0$

Therefore, (y-1) is the factor of p(y)



Now, Dividend = Divisor × Quotient + Remainder

$(y-1)(2y^2+3y+1) = (y-1)(2y^2+2y+y+1)$

$= (y-1)(2y(y+1)+1(y+1))$

$= (y-1)(2y+1)(y+1)$