CHAPTER 8 MECHANICAL PROPERTIES OF SOLIDS

EXERCISES **PAGE:177**

Question 1. A steel wire of length 4.7 m and cross-sectional area 3.0 x 10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0 x 10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Answer:

For steel

$$l_1 = 4.7\text{m}, \quad A_1 = 3.0 \times 10^{-5} \text{ m}^2$$

If F newton is the stretching force and Δl metre the extension in each case, then

$$Y_1 = \frac{Fl_1}{A_1 \Delta l}$$

$$\Rightarrow \quad Y_1 = \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta l} \quad ...(i)$$

For copper

$$l_2 = 3.5\text{m}, \quad A_2 = 4.0 \times 10^{-5} \text{ m}^2$$

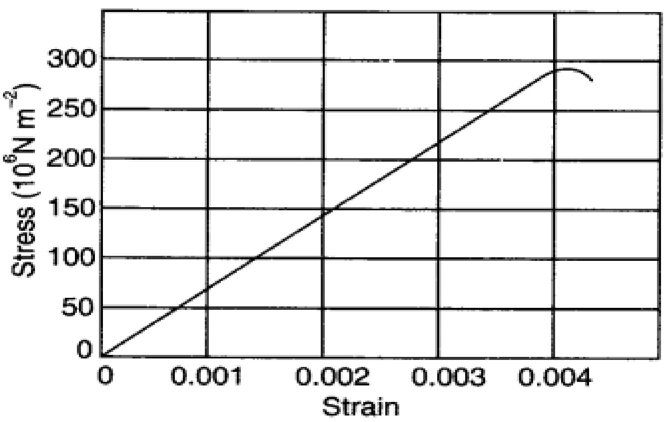
Now, $$Y_2 = \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta l} \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{Y_1}{Y_2} = \frac{4.7}{3.0 \times 10^{-5}} \times \frac{4.0 \times 10^{-5}}{3.5} = \frac{4.7 \times 4.0}{3.0 \times 3.5} = 1.79.$$

Question 2. Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?

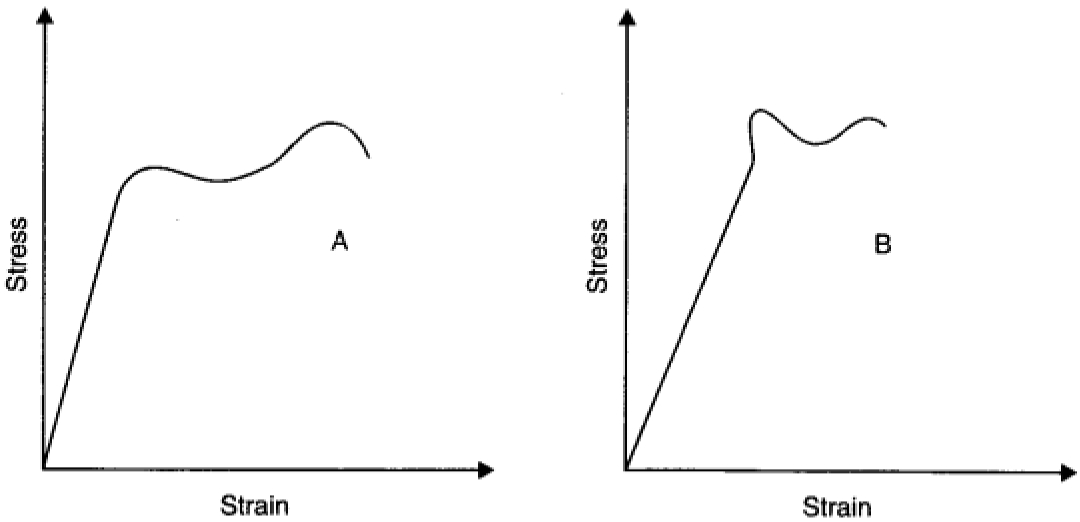
Answer:



(a) Young's modulus of the material (Y) is given by

Y =Stress/Strain
$=150 \times 10^6/0.002$
$150 \times 10^6/2 \times 10\text{-}3$
$=75 \times 10^9 \ Nm^{-2}$
$=75 \times 10^{10} \ Nm^{-2}$
(a)Yield strength of a material is defined as the maximum stress it can sustain. From graph, the approximate yield strength of the given material
$= 300 \times 10^6 \ Nm^{-2}$
$= 3 \times 10^8 \ Nm^{-2}$.

Question 3. The stress-strain graphs for materials A and B are shown in figure.

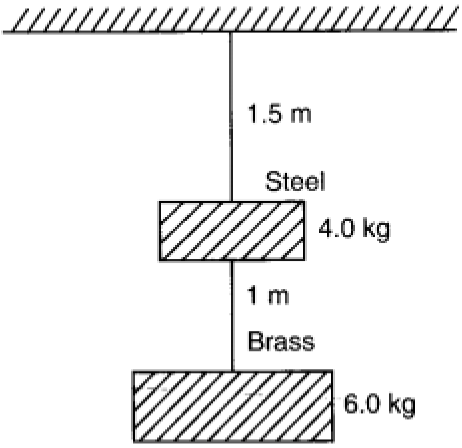


The graphs are drawn to the same scale.
(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?
Answer: (a) From the two graphs we note that for a given strain, stress for A is more than that of B. Hence Young's modulus =(Stress /Strain) is greater for A than that of B.
(b) Strength of a material is determined by the amount of stress required to cause fracture. This stress corresponds to the point of fracture. The stress corresponding to the point of fracture in A is more than for B. So, material A is stronger than material B.

Question 4. Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.
Answer: (a) False. The-Young's modulus is defined as the ratio of stress to the strain within elastic limit. For a given stretching force elongation is more in rubber and quite less in steel. Hence, rubber is less elastic than steel.

(b) True. Stretching of a coil is determined by its shear modulus. When equal and opposite forces are applied at opposite ends of a coil, the distance, as well as shape of helicals of the coil change and it, involves shear modulus.

Question 5. Two wires of diameter 0.25 cm, one made of steel and other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m.Young's modulus of steel is 2.0 x 10[11] Pa. Compute the elongations of steel and brass wires. (1 Pa = 1 N m[2]).



Answer:

For steel wire; total force on steel wire;

$$F_1 = 4 + 6 = 10 \text{ kg} f = 10 \times 9.8 \ N;$$

$$l_1 = 1.5 \text{ m}, \quad \Delta l_1 = ?; \quad 2r_1 = 0.25$$

cm

or $$r_1 = \left(\frac{0.25}{2}\right) \text{cm} = 0.125 \times 10^{-2} \text{ m}$$

$$Y_1 = 2.0 \times 10^{11} \text{ Pa}$$

For brass wire, $$F_2 = 6.0 \text{ kg} f = 6 \times 9.8 \text{ N};$$

$$2r_2 = 0.25 \text{ cm}$$

or $$r_2 = \left(\frac{0.25}{2}\right) = 0.125 \times 10^{-2} \text{ m};$$

$$Y_2 = 0.91 \times 10^{11} \text{ Pa}, \quad l_2 = 1.0 \text{ m}, \quad \Delta l_2 = ?$$

Since, $$Y_1 = \frac{F_1 \times l_1}{A_1 \times \Delta l_1} = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1} \Rightarrow \Delta l_1 = \frac{F_1 \times l_1}{\pi r_1^2 \times \Delta l_1}$$

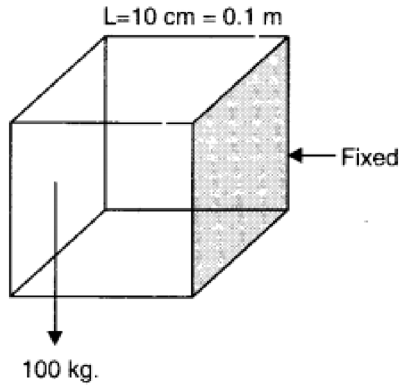
or $$\Delta l_1 = \frac{(10 \times 9.8) \times 1.5 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times 2 \times 10^{11}} = 1.49 \times 10^{-4} \text{ m.}$$

And $$\Delta l_2 = \frac{F_2 \times l_2}{\pi r_2^2 \times Y_2} = \frac{(6 \times 9.8) \times 1 \times 7}{22 \times (0.125 \times 10^{-2})^2 \times (0.91 \times 10^{11})} = 1.3 \times 10^{-4} \text{ m.}$$

Question 6. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Answer: Here, side of cube, L = 10 cm =10/100= 0.1 m

∴ Area of each face, $A = (0.1)^2 = 0.01\ m^2$



Tangential force acting on the face,

$$F = 100\ kg = 100 \times 9.8 = 980\ N$$

$$\text{Shear modulus, } \eta = 25\ GPa = 25 \times 10^9\ Nm^{-2}$$

Since shear modulus is given as:

$$\eta = \frac{\text{Tangential stress}}{\text{Shearing strain}}$$

$$\therefore \text{Shearing strain} = \frac{\text{Tangential stress}}{\text{Shear modulus}}$$

$$= \frac{F}{A\eta} = \frac{980}{0.01 \times 25 \times 10^9} = 3.92 \times 10^{-6}$$

$$\text{Now, } \frac{\text{Lateral Strain}}{\text{Side of cube}} = \text{Shearing strain}$$

$$\therefore \text{Lateral Strain} = \text{Shearing strain} \times \text{Side of the cube}$$

$$= 3.92 \times 10^{-6} \times 0.1$$

$$= 3.92 \times 10^{-7}\ m \approx 4 \times 10^{-7}\ m.$$

Question 7. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. Young's modulus, $Y = 2.0 \times 10^{11}$ Pa.

Answer:

Here total mass to be supported, M = 50,000 kg

∴ Total weight of the structure to be supported = Mg

$$= 50,000 \times 9.8 \text{ N}$$

Since this weight is to be su ported by 4 columns,

∴ Compressional force on each column (F) is given by

$$F = \frac{Mg}{4} = \frac{50,000 \times 9.8}{4} \text{ N}$$

Inner radius of a column, r_1 = 30 cm = 0.3 m

Outer radius of a column, r_2 = 60 cm = 0.6 m.

∴ Area of cross-section of each column is given by

$$A = \pi(r_2^2 - r_1^2)$$

$$= \pi[(0.6)^2 - (0.3)^2] = 0.27\pi \text{ m}^2$$

Young's modulus, $Y = 2 \times 10^{11}$ Pa

Compressional strain of each column =?

$$\therefore \quad Y = \frac{\text{Compressional force / area}}{\text{Compressional Strain}}$$

$$= \frac{F/A}{\text{Compressional Strain}}$$

or Compressional strain of each column

$$= \frac{F}{AY} = \frac{50,000 \times 9.8 \times 7}{4 \times 0.27 \times 22 \times 2 \times 10^{11}}$$

$$= 0.722 \times 10^{-6}$$

∴ Compressional strain of all columns is given by

$$= 0.722 \times 10^{-6} \times 4 = 2.88 \times 10^{-6}$$

$$= 2.88 \times 10^{-6} \; .$$

Question 8. A piece of copper having a rectangular cross-section of 15.2 mm x 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain? Shear modulus of elasticity of copper is 42 x 10^9 N/m^2.

Answer:

Here, $A = 15.2 \times 19.2 \times 10^{-6}\ m^2$; $F = 44500$ N; $\eta = 42 \times 10^9\ Nm^{-2}$

$$\text{Strain} = \frac{\text{Stress}}{\text{modulus of elasticity}} = \frac{F/A}{\eta}$$

$$= \frac{F}{A\eta} = \frac{44500}{(15.2 \times 19.2 \times 10^{-6}) \times 42 \times 10^9} = 3.65 \times 10^{-3}.$$

Question 9. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 Nm^{-2} what is the maximum load the cable can support ?

Answer:

Maximum load = Maximum stress × Cross-sectional area

$$= 10^8\ Nm^{-2} \times \frac{22}{7} \times (1.5 \times 10^{-2}\ m)^2$$

$$= 7.07 \times 10^4\ N.$$

Question 10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Answer: Since each wire is to have same tension therefore, each wire has same extension. Moreover, each wire has the same initial length.So, strain is same for each wire.

Now, $$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/(\pi D^2/4)}{\text{Strain}}$$

or $$Y \propto \frac{1}{D^2} \Rightarrow D \propto \frac{1}{\sqrt{Y}}$$

$$\frac{D_{copper}}{D_{iron}} = \sqrt{\frac{Y_{iron}}{Y_{copper}}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.314$$

Question 11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1 m, is whirled in a vertical circle with an angular velocity of 2 rero./s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the zvire when the mass is at the lowest point of its path. Ysteel = 2 x 10^{11} Nm^{-2}.

Ans. Here, m = 14.5 kg; l = r = 1 m; v = 2 rps; A = 0.065 x 10^{-4} m^2 Total pulling force on mass, when it is at the lowest position of the vertical circle is F = mg + mr w^2 = mg +

mr 4,π2 v2

$$= 14.5 \times 9.8 + 14.5 \times 1 \times 4 \times (22/7)^2 \times 2^2$$
$$= 142.1 + 2291.6 = 2433.9 \text{ N}$$

$$Y = \frac{F}{A} \times \frac{l}{\Delta l}$$

or $$\Delta l = \frac{Fl}{AY} = \frac{2433.7 \times 1}{(0.065 \times 10^{-4}) \times (2 \times 10^{11})} = 1.87 \times 10^{-3} \text{ m} = 1.87 \text{ m.}$$

Question 12.Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 x 10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.
Answer:

Here $P = 100$ atmosphere
$= 100 \times 1.013 \times 10^5$ Pa $\quad (\because 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa})$

Initial volume, $V_1 = 100 \text{ litre} = 100 \times 10^{-3} \text{ m}^3$

Final volume, $V_2 = 100.5 \text{ litre} = 100.5 \times 10^{-3} \text{ m}^3$

∴ Change in volume $= \Delta V = V_2 - V_1$
$= (100.5 - 100) \times 10^{-3} \text{ m}^3$
$= 0.5 \times 10^{-3} \text{ m}^3$

Using formula of bulk modulus,

$$B = \frac{P}{\frac{\Delta V}{V}} = \frac{PV}{\Delta V}$$

$$= \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}}$$

$$B = 2.026 \times 10^9 \text{ Pa}$$

Also we know that the bulk modulus of air = 1.0×10^5 Pa

Now, $$\frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1.0 \times 10^5}$$
$$= 2.026 \times 10^4$$
$$= 20260$$

The ratio is too large. This is due to the fact that the strain for air is much larger than for water at the same temperature. In other words, the intermolecular distances in case of liquids are very small as compared to the corresponding distances in the case of gases. Hence there are larger interatomic forces in liquids than in gases.

Question 13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 x 103kg m^{-3}?

Answer:

Compressibility of water,

$$k = \frac{1}{B} = 45.8 \times 10^{-11}\ \text{Pa}^{-1}$$

Change in pressure,

$$\Delta p = 80\ \text{atm} - 1\ \text{atm}$$
$$= 79\ \text{atm} = 79 \times 1.013 \times 10^5\ \text{Pa}$$

Density of water at the surface,

$$\rho = 1.03 \times 10^3\ \text{kg m}^{-3}$$

As $$B = \frac{\Delta p . V}{\Delta V} \quad \text{or} \quad \frac{\Delta V}{V} = \frac{\Delta p}{B} = \Delta p \times \frac{1}{B} = \Delta p \times k$$

or $$\frac{\Delta V}{V} = 79 \times 1.013 \times 10^5 \times 45.8 \times 10^{-11} = 3.665 \times 10^{-5}$$

Now $$\frac{\Delta V}{V} = \frac{(M/\rho)-(M/\rho')}{(M/\rho)} = 1 - \frac{\rho}{\rho'}$$

or $$\frac{\rho}{\rho'} = 1 - \frac{\Delta V}{V}$$

or $$\rho' = \frac{\rho}{1-(\Delta V/V)}$$

or $$\rho' = \frac{1.03 \times 10^3}{1 - 3.665 \times 10^{-3}} = \frac{1.03 \times 10^3}{0.996}$$
$$= 1.034 \times 10^3\ \text{kg/m}^3.$$

Question 14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Answer:

Here, $P = 10\ \text{atm} = 10 \times 1.013 \times 10^5\ \text{Pa}$; $k = 37 \times 10^9\ \text{Nm}^{-2}$

$$\text{Volumetric strain} = \frac{\Delta V}{V} = \frac{P}{K} = \frac{10 \times 1.013 \times 10^5}{37 \times 10^9} = 2.74 \times 10^{-5}$$

$$\therefore \text{Fractional change in volume} = \frac{\Delta V}{V} = 2.74 \times 10^{-5}.$$

Question 15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7.0 x 10^6 Pa.

Answer: Here a side of copper cube a = 10 cm, hence volume V = a^3 = $10^{-3}\ m^3$, hydraulic pressure applied p = 7.0 x 10^6 Pa and from table we find that bulk modulus of copper B = 140 G Pa = 140 x 10^9 Pa.

Using the relation $B = -\frac{P}{\frac{\Delta V}{V}}$, we have decrease in

$$\text{volume } \Delta V = \frac{PV}{B}$$

$$\therefore \quad \Delta V = \frac{7.0 \times 10^6 \times 10^{-3}}{140 \times 10^9} = 5 \times 10^{-8} \text{ m}^3 = 5 \times 10^{-2} \text{ cm}^3.$$

Question 16. How much should be pressure the a litre of water be changed to compress it by 0.10 %? Bulk modulus of elasticity of water = 2.2 x 10^9 Nm^{-2}.

Answer:

Here, $V = 1 \text{ litre} = 10^{-3} \text{ m}^3; \quad \Delta V/V = 0.10/100 = 10^{-3}$

$$K = \frac{pV}{\Delta V}$$

or $p = K\frac{\Delta V}{V} = (2.2 \times 10^9) \times 10^{-3} = 2.2 \times 10^6 \text{ Pa}.$