CHAPTER 1 UNITS AND MEASUREMENT

EXERCISES PAGE:10

Q1. Fill in the blanks
(a) The volume of a cube of side 1 cm is equal to…………m^3.
(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to ……..$(mm)^2$.
(c) A vehicle moving with a speed of 18 km h^{-1} covers ………. m in 1 s.
(d) The relative density of lead is 11.3. Its density is …….. g cm^{-3} or ………. kg m^{-3}.

Answer: (a) Volume of cube, V = $(1\ cm)^3$ = $(10^{-2}\ m)^3$ 10^{-6} m^3.
Hence, answer is 10^{-6}
(b) Surface area = 2πrh + $2\pi r^2$ = 2πr (h + r)
= 2 x 22/7 x 2 x 10 (10 x 10 + 2 x 10) mm^2 = 1.5 x 10^4 mm^2 Hence, answer is 1.5 x 10^4.
(c) Speed of vehicle = 18 km/h = 18 x 1000/3600 m/s
= 5 m/s ; so the vehicle covers 5 m in 1 s. = 11.3
(d) Density= 11.3 g cm^{-3}
=11.3 x 10^3 kg m^{-3} [1 kg =10^3 g,1m=10^2 cm]
=11.3 x 10^3 kg m^{-4}

Question 2. Fill in the blanks by suitable conversion of units
(a) 1 kg m2 s^{-2} = …. g cm^2 s^{-2}
(b) 1 m =………… ly
(c) 3.0 m s^{-2} = …. km h^{-2}
(d) G = 6.67 x 10^{-11} N m^2 $(kg)^{-2}$ = …. $(cm)^3$ s^{-2} g^{-1}.

Answer:

(a) $1 \text{ kg m}^2 \text{ s}^{-2} = \frac{1 \text{ kg m}^2}{s^2} = \frac{1 \times 1000 \times (10^2)^2}{s^2} \text{ g cm}^2$

$= 10^7 \text{ g cm}^2 \text{ s}^{-2}$

(b) $1 \text{ m} = \frac{1}{9.46 \times 10^5} \text{ ly} \approx \frac{1}{10^{16}} \text{ ly} = 10^{-16} \text{ ly}$

(c) $3 \text{ ms}^{-2} = \frac{3 \times 10^{-3} \text{ km}}{\left(\frac{1}{3600}\right)^2 \text{ h}^2} = 3 \times 3600 \times 3600 \times 10^{-3} \text{ km h}^{-2}$

$= 3.888 \times 10^4 \text{ km h}^{-2}$.

(d) $G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2} = 6.67 \times 10^{-11} \frac{\text{kg m}}{s^2} \text{ m}^2 \text{ kg}^{-2}$

$= 6.67 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$

$= 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2} = \frac{6.67 \times 10^{-11} \times (10^2)^3}{(10^3)^2}$

$= 6.67 \times 10^{-8} \text{ cm}^{-3} \text{ s}^{-2} \text{ g}^{-1}$.

Question 3 . A calorie is a unit of heat or energy and it equals about 4.2 J where 1 J = 1 kgm^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals a kg, the unit of length equals j8 m, the. unit of time is ys. Show that a calorie has a magnitude 4.2 $\alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

Answer:

$1 \text{ cal} = 4.2 \text{ kg m}^2 \text{ s}^{-2}$

SI	New system
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 1 \text{ kg}$	$M_2 = \alpha \text{ kg}$
$L_1 = 1 \text{m}$	$L_2 = \beta m$
$T_1 = 1 \text{ s}$	$T_2 = y$ second

Dimensional formula of energy is $[M^1 L^2 T^{-2}]$

Comparing with $[M^a L^b T^c]$, we get

$$a = 1, \quad b = 2, \quad c = -2$$

Now, $$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$= 4.2 \left[\frac{1\text{ kg}}{\alpha\text{ kg}}\right]^1 \left[\frac{1\text{ m}}{\beta\, m}\right]^2 \left[\frac{1\text{ s}}{\gamma\, s}\right]^{-2}$$

or, $$n_2 = 4.2\ \alpha^{-1}\ \beta^{-2}\ \gamma^2.$$

Question 4. Explain this statement clearly:
"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary:
(a) atoms are very small objects
(b) a jet plane moves with great speed
(c) the mass of Jupiter is very large
(d) the air inside this room contains a large number of molecules
(e) a proton is much more massive than an electron
(f) the speed of sound is much smaller than the speed of light.

Answer: Physical quantities are called large or small depending on the unit (standard) of measurement. For example, the distance between two cities on earth is measured in kilometres but the distance between stars or inter —galactic distances are measured in parsec. The later standard parsec is equal to 3.08 x 10^{16} m or 3.08 x 10^{12} km is certainly larger than metre or kilometre. Therefore, the inter-stellar or intergalactic distances are certainly larger than the distances between two cities on earth.
(a) The size of an atom is much smaller than even the sharp tip of a pin.
(b) A Jet plane moves with a speed greater than that of a super fast train.
(c) The mass of Jupiter is very large compared to that of the earth.
(d) The air inside this room contains more number of molecules than in one mole of air.
(e) This is a correct statement.
(f) This is a correct statement.

Question 5. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if

light takes 8 min and 20 s to cover this distance?

Answer: Distance between Sun and Earth
= Speed of light in vacuum x time taken by light to travel from Sim to Earth = 3×10^8 m/ s x 8 min 20 s = 3×10^8 m/s x 500 s = $500 \times 3 \times 10^8$ m.
In the new system, the speed of light in vacuum is unity. So, the new unit of length is 3×10^8 m.
.·. distance between Sun and Earth = 500 new units.

Question 6. Which of the following is the most precise device for measuring length:
(a) a vernier callipers with 20 divisions on the sliding scale.
(b) a screw gauge of pitch 1 mm and 100 divisions on the circular scale.
(c) an optical instrument that can measure length to within a wavelength of light?

Answer: (a) Least count of vernier callipers = 1/20 = 0.05 mm = 5×10^{-5} m
(b) Least count of screw gauge =Pitch/No. of divisions on circular scale = $1 \times 10^{-3}/100 = 1 \times 10^{-5}$ m
(c) Least count of optical instrument = 6000 A (average wavelength of visible light as 6000 A) = 6×10^{-7}m As the least count of optical instrument is least, it is the most precise device out of three instruments given to us.

Question .7. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of hair?
Answer: As magnification, m =thickness of image of hair/ real thickness of hair = 100
and average width of the image of hair as seen by microscope = 3.5 mm
.·. Thickness of hair =3.5 mm/100 = 0.035 mm

Question 8. Answer the following:
(a) You are given a thread and a metre scale. How will you estimate the diameter of the thread?
(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?
(c) The mean diameter of a thin brass rod is to be measured by vernier callipers.

Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Answer: (a) Wrap the thread a number of times on a round pencil so as to form a coil having its turns touching each other closely. Measure the length of this coil, mode by the thread, with a metre scale. If n be the number of turns of the coil and l be the length of the coil, then the length occupied by each single turn i.e., the thickness of the thread = 1/n .
This is equal to the diameter of the thread.
(b) We know that least count = Pitch/number of divisions on circular scale When number of divisions on circular scale is increased, least count is decreased. Hence the accuracy is increased. However, this is only a theoretical idea.Practically speaking, increasing the number of 'turns would create many difficulties.
As an example, the low resolution of the human eye would make observations difficult. The nearest divisions would not clearly be distinguished as separate. Moreover, it would be technically difficult to maintain uniformity of the pitch of the screw throughout its length.
(c) Due to random errors, a large number of observation will give a more reliable result than smaller number of observations. This is due to the fact that the probability (chance) of making a positive random error of a given magnitude is equal to that of making a negative random error of the same magnitude. Thus in a large number of observations, positive and negative errors are likely to cancel each other. Hence more reliable result can be obtained.

Question 9. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement?

Answer: Here area of the house on slide = 1.75 cm^2 = 1.75 x 10^{-4} m^2 and area of the house of projector-screen = 1.55 m^2
.•. Areal magnification =Area on screen/Area on slide = 1.55 m^2 / 1.75 x 10^{-4} m^2 = 8.857 x 10^3
.•. Linear magnification

$$= \sqrt{\text{Areal magnification}}$$

$$= \sqrt{(8.857) \times 10^3}$$

$$= 94.1.$$

Question 10. State the number of significant figures in the following:
(a) 0.007 m^2 (b) 2.64 x 10^4 kg
(c) 0.2370 g cm^{-3} (d) 6.320 J
(e) 6.032 N m^{-2} (f) 0.0006032 m^2

Answer: (a) 1 (b) 3 (c) 4 (d) 4 (e) 4 (f) 4.

Question 11. 'The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Answer: As Area = (4.234 x 1.005) x 2 = 8.51034 = 8.5 m^2
Volume = (4.234 x 1.005) x (2.01 x 10^{-2}) = 8.55289 x 10^{-2} = 0.0855 m^3.

Question 12. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) the total mass of the box (b) the difference in the masses of the pieces to correct significant figures?

Answer: (a) Total mass of the box = (2.3 + 0.0217 + 0.0215) kg = 2.3442 kg
Since the least number of decimal places is 1, therefore, the total mass of the box = 2.3 kg.
(b) Difference of mass = 2.17 – 2.15 = 0.02 g
Since the least number of decimal places is 2 so the difference in masses to the correct significant figures is 0.02 g.

Question 13. A famous relation in physics relates 'moving mass' m to the 'rest mass' m0 of a particle in terms of its speed v and the speed of light c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes:

$$m = \frac{m_0}{\left(1 - v^2\right)^{1/2}}$$

Guess where to put the missing c.

Answer:

From the given equation, $\frac{m_0}{m} = \sqrt{1 - v^2}$

Left hand side is dimensionless.

Therefore, right hand side should also be dimensionless.

It is possible only when $\sqrt{1-v^2}$ should be $\sqrt{1-\frac{v^2}{c^2}}$.

Thus, the correct formula is $m = m_0 \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$

Question 14. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by A: 1 A = 10^{-10} m. The size of a hydrogen atom is about 0.5 A. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

Answer: Volume of one hydrogen atom = 4/3 πr3 (volume of sphere)
= 4/3 x 3.14 x (0.5 x 10^{-10}) m^3 = 5.23 x 10^{-31} m^3
According to Avagadro's hypothesis, one mole of hydrogen contains 6.023 x 10^{23} atoms.
Atomic volume of 1 mole of hydrogen atoms
= 6.023 x 1023 x 5.23 x 10^{-31} = 3.15 x $10^{-7}m^3$.

Question 15. One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 A.) Why is this ratio so large?

Answer: Volume of one mole of ideal gas, V_g
= 22.4 litre = 22.4 x 10^{-3} m^3
Radius of hydrogen molecule = 1A/2
= 0.5 A = 0.5 x 10^{-10} m
Volume of hydrogen molecule = 4/3 πr^3
=4/3 x 22/7 (0.5 x $10^{-10})^3$ m^3
= 0.5238 x 10^{-30} m^3
One mole contains 6.023 x 10^{23} molecules.
Volume of one mole of hydrogen, VH
= 0.5238 x 10^{-30} x 6.023 x 10^{23} m^3 = 3.1548 x 10^{-7} m^3
Now V_g/V_H=22.4 x 10^{-3}/3.1548 x 10^{-7} =7.1 x 10^4

The ratio is very large. This is because the interatomic separation in the gas is very large compared to the size of a hydrogen molecule.

Question 16 Explain this common observation clearly: If you look out of the window of a fast moving train, the nearby trees, houses etc., seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hill tops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

Answer: The line joining a given object to our eye is known as the line of sight. When a train moves rapidly, the line of sight of a passenger sitting in the train for nearby trees changes its direction rapidly. As a result, the nearby trees and other objects appear to run in a direction opposite to the train's motion. However, the line of sight of distant and large size objects e.g., hill tops, the Moon, the stars etc., almost remains unchanged (or changes by an extremely small angle). As a result, the distant object seems to be stationary.

Question .17.The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 107 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^{8} m.

Answer: Given $M = 2 \times 10^{30}$ kg, $r = 7 \times 10^{8}$ m

.-. Volume of Sun = $4/3\pi r^{3} \times 3.14 \times (7 \times 10^{8})^{3} = 1.437 \times 10^{27}$ m^{3}

As p = M/V, .'. $p = 2 \times 10^{30}/1.437 \times 10^{27} = 1391.8$ kg $m^{-3} = 1.4 \times 10^{3}$ kg m^{-3}

Mass density of Sun is in the range of mass densities of solids/liquids and not gases.