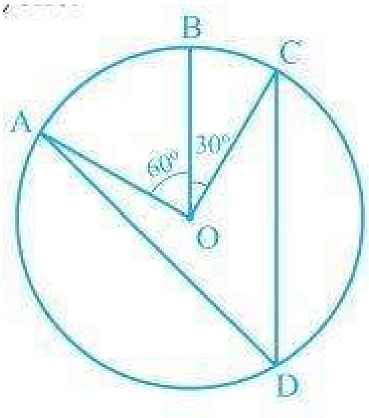
CHAPTER 9 CIRCLES

Exercise: 9.3 **Page No: 127**

1. In Fig. 9.23, A, B and C are three points on a circle with centre O, such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Solution:

It is given that,

∠AOC = ∠AOB+∠BOC

So, ∠AOC = 60°+30°

∴ ∠AOC = 90°

It is known that an angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

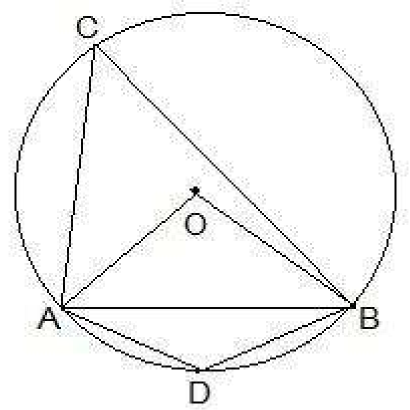
So,

∠ADC = (½)∠AOC

= (½)× 90° = 45°

2. A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.

Solution:



Here, the chord AB is equal to the radius of the circle. In the above diagram, OA and OB are the two radii of the circle.

Now, consider the ΔOAB. Here,

AB = OA = OB = radius of the circle

So, it can be said that ΔOAB has all equal sides, and thus, it is an equilateral triangle.

∴ ∠AOC = 60°

And, ∠ACB = ½ ∠AOB

So, ∠ACB = ½ × 60° = 30°

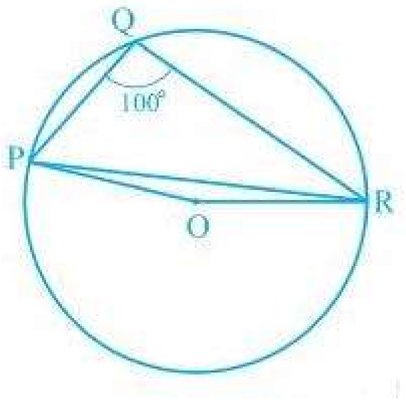
Now, since ACBD is a cyclic quadrilateral,

∠ADB +∠ACB = 180° (They are the opposite angles of a cyclic quadrilateral)

So, ∠ADB = 180°-30° = 150°

So, the angle subtended by the chord at a point on the minor arc and also at a point on the major arc is 150° and 30°, respectively.

3. In Fig. 9.24, ∠PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠OPR.



Solution:

Since the angle which is subtended by an arc at the centre of the circle is double the angle subtended by that arc at any point on the remaining part of the circle.

So, the reflex ∠POR = 2×∠PQR

We know the values of angle PQR as 100°.

So, ∠POR = 2×100° = 200°

∴ ∠POR = 360°-200° = 160°

Now, in ΔOPR,

OP and OR are the radii of the circle.

So, OP = OR

Also, ∠OPR = ∠ORP

Now, we know the sum of the angles in a triangle is equal to 180 degrees.

So,

∠POR+∠OPR+∠ORP = 180°

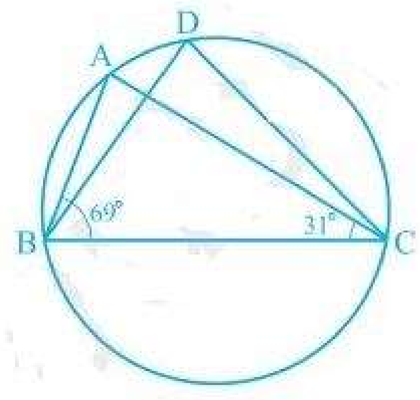
∠OPR+∠OPR = 180°-160°

As ∠OPR = ∠ORP

2∠OPR = 20°

Thus, ∠OPR = 10°

4. In Fig. 9.25, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.



Solution:

We know that angles in the segment of the circle are equal, so,

∠BAC = ∠BDC

Now. in the ΔABC, the sum of all the interior angles will be 180°.

So, ∠ABC+∠BAC+∠ACB = 180°

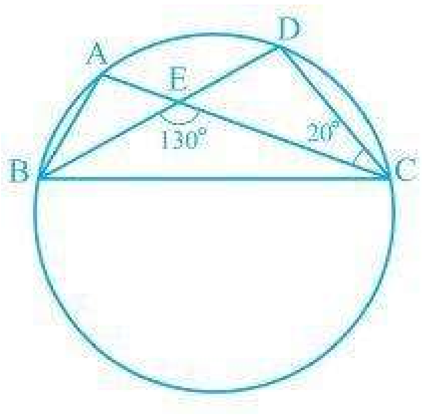
Now, by putting the values,

∠BAC = 180°-69°-31°

So, ∠BAC = 80°

∴ ∠BDC = 80°

5. In Fig. 9.26, A, B, C and D are four points on a circle. AC and BD intersect at a point E, such that ∠ BEC = 130° and ∠ ECD = 20°. Find BAC.



Solution:

We know that the angles in the segment of the circle are equal.

So,

∠ BAC = ∠ CDE

Now, by using the exterior angles property of the triangle,

In ΔCDE, we get

∠ CEB = ∠ CDE+∠ DCE

We know that ∠ DCE is equal to 20°.

So, ∠ CDE = 110°

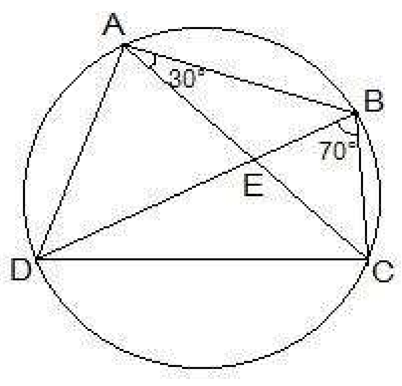
∠ BAC and ∠ CDE are equal

∴ ∠ BAC = 110°

6. ABCD is a cyclic quadrilateral whose diagonals intersect at point E. If ∠ DBC = 70°, ∠ BAC is 30°, find ∠ BCD. Further, if AB = BC, find ∠ ECD.

Solution:

Consider the following diagram.



Consider the chord CD.

We know that angles in the same segment are equal.

So, ∠ CBD = ∠ CAD

∴ ∠ CAD = 70°

Now, ∠ BAD will be equal to the sum of angles BAC and CAD.

So, ∠ BAD = ∠ BAC+∠ CAD

= 30°+70°

∴ ∠ BAD = 100°

We know that the opposite angles of a cyclic quadrilateral sum up to 180 degrees.

So,

∠ BCD+∠ BAD = 180°

It is known that ∠ BAD = 100°

So, ∠ BCD = 80°

Now, consider the ΔABC.

Here, it is given that AB = BC

Also, ∠ BCA = ∠ CAB (They are the angles opposite to equal sides of a triangle)

∠ BCA = 30°

also, ∠ BCD = 80°

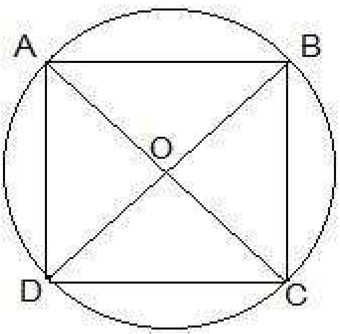
∠ BCA +∠ ACD = 80°

Thus, ∠ ACD = 50° and ∠ ECD = 50°

7. If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

Solution:

Draw a cyclic quadrilateral ABCD inside a circle with centre O, such that its diagonal AC and BD are two diameters of the circle.



We know that the angles in the semi-circle are equal.

So, ∠ ABC = ∠ BCD = ∠ CDA = ∠ DAB = 90°

So, as each internal angle is 90°, it can be said that the quadrilateral ABCD is a rectangle.

8. If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Solution:

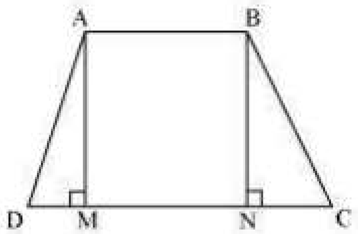
Construction:

Consider a trapezium ABCD with AB | |CD and BC = AD.

Draw AM ⊥ CD and BN ⊥ CD

In ΔAMD and ΔBNC,

The diagram will look as follows:



In ΔAMD and ΔBNC,

AD = BC (Given)

∠AMD = ∠BNC (By construction, each is 90°)

AM = BM (Perpendicular distance between two parallel lines is same)

△AMD ≅ △ BNC (RHS congruence rule)

∠ADC = ∠BCD (CPCT) ... (1)

∠BAD and ∠ADC are on the same side of transversal AD.

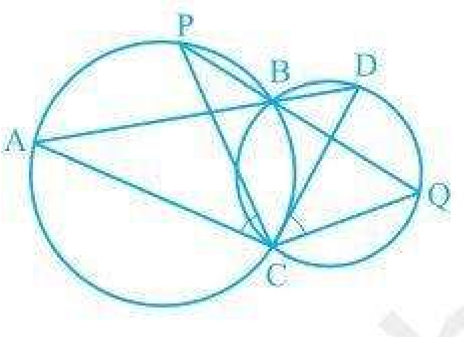
∠BAD + ∠ADC = 180° ... (2)

∠BAD + ∠BCD = 180° [Using equation (1)]

This equation shows that the opposite angles are supplementary.

Therefore, ABCD is a cyclic quadrilateral.

9. Two circles intersect at two points, B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q, respectively (see Fig. 9.27). Prove that ∠ ACP = ∠ QCD.



Solution:

Construction:

Join the chords AP and DQ.

For chord AP, we know that angles in the same segment are equal.

So, ∠ PBA = ∠ ACP — (i)

Similarly, for chord DQ,

∠ DBQ = ∠ QCD — (ii)

It is known that ABD and PBQ are two line segments which are intersecting at B.

At B, the vertically opposite angles will be equal.

∴ ∠ PBA = ∠ DBQ — (iii)

From equation (i), equation (ii) and equation (iii), we get

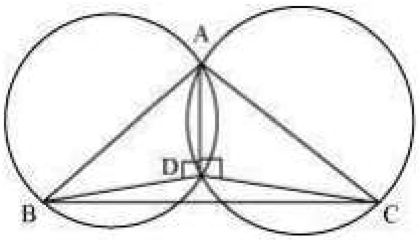
∠ ACP = ∠ QCD

10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lies on the third side.

Solution:

First, draw a triangle ABC and then two circles having diameters of AB and AC, respectively.

We will have to now prove that D lies on BC and BDC is a straight line.



Proof:

We know that angles in the semi-circle are equal.

So, ∠ ADB = ∠ ADC = 90°

Hence, ∠ ADB+∠ ADC = 180°

∴ ∠ BDC is a straight line.

So, it can be said that D lies on the line BC.

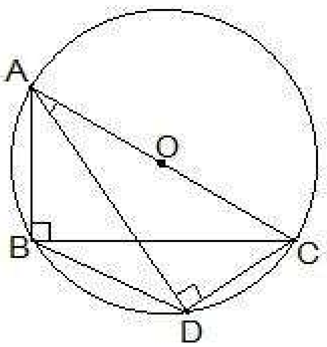
11. ABC and ADC are two right triangles with common hypotenuse AC.

Prove that ∠ CAD = ∠CBD.

Solution:

We know that AC is the common hypotenuse and ∠ B = ∠ D = 90°.

Now, it has to be proven that ∠ CAD = ∠ CBD



Since ∠ ABC and ∠ ADC are 90°, it can be said that they lie in a semi-circle.

So, triangles ABC and ADC are in the semi-circle, and the points A, B, C and D are concyclic.

Hence, CD is the chord of the circle with centre O.

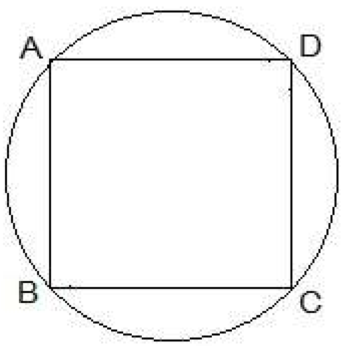
We know that the angles which are in the same segment of the circle are equal.

∴ ∠ CAD = ∠ CBD

12. Prove that a cyclic parallelogram is a rectangle.

Solution:

It is given that ABCD is a cyclic parallelogram, and we will have to prove that ABCD is a rectangle.



Proof:

Let ABCD be a cyclic parallelogram.

$\angle A + \angle C = 180^\circ$ (Opposite angle of cyclic quadrilateral) ... (1)

We know that opposite angles of a parallelogram are equal

$\angle A = \angle C$ and $\angle B = \angle D$

From equation (1)

$\angle A + \angle C = 180^\circ$

$\angle A + \angle A = 180^\circ$

$2\angle A = 180^\circ$

$\angle A = 90^\circ$

Parallelogram ABCD has one of its interior angles as 90°.

Thus, ABCD is a rectangle.