Chapter - 7 INTEGRAL

STUDY NOTES

- **Indefinite Integral**

  If $\frac{d}{dx}[f(x)] = F(x)$, then $\int F(x)dx = f(x) + C$

  Here, $f(x) + C$ is called indefinite integral of $F(x)$, called the integrand, C is called the constant of integration.

- **Standard Results on Integration**

  The integrals of standard functions as known from the process of differentiation are as follows :

(i) $\int x^n dx = \frac{x^{n+1}}{n+1} + C \; (n \neq -1)$ (ii) $\int dx = \int 1 \cdot dx = \int x^0 dx = x + C$

(iii) $\int \frac{1}{x} dx = \log x + C$ (iv) $\int a^x dx = \frac{a^x}{\log a} + C$

(v) $\int e^x dx = e^x + C$ (vi) $\int \sin x \, dx = -\cos x + C$

(vii) $\int \cos x \, dx = \sin x + C$ (viii) $\int \sec^2 x dx = \tan x + C$

(ix) $\int \sec x \tan x \, dx = \sec x + C$ (x) $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

(xi) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$ (xii) $\int \tan x \, dx = \log |\sec x| + C$

(xiii) $\int \cot x \, dx = \log |\sin x| + C$ (xiv) $\int \sec x \, dx = \log |\sec x + \tan x| + C$

(xv) $\int \operatorname{cosec} x \, dx = \log |\operatorname{cosec} x - \cot x| + C$ (xvi) $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C = -\cos^{-1} x + C$

(xvii) $\int \frac{dx}{1+x^2} = \tan^{-1} x + C = -\cot^{-1} x + C$ (xviii) $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C = -\operatorname{cosec}^{-1} x + C$

(xix) $\frac{d}{dx}\left[\int f(x)dx\right] = f(x)$ (xx) $\int kf(x)dx = k\int f(x)dx, \; k \in R.$

(xxi) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

Note: On integrating function by two different methods, two different results may be obtained. However, these two results always differ by a constant. For example in (xvi) above.

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C \text{ and } -\cos^{-1} x + C_1$$

Now, $\sin^{-1} x + C - (-\cos^{-1} x + C_1) = \sin^{-1} x + \cos^{-1} x + C - C_1 = \frac{\pi}{2} + C - C_1 =$ constant.

- If $\int f(x)dx = F(x)$, then $\int f(ax+b)dx = \frac{F(ax+b)}{a} + C$

- **Extended Standard Forms**

(i) (a) $\int e^{ax} dx = \frac{e^{ax}}{a} + C$ (b) $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + C$

(c) $\int \sin(ax+b)\, dx = \frac{-\cos(ax+b)}{a} + C$ etc.

(ii) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

- Integral of the Form $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + C.$

- **Integration by Parts**

$$\int f_1(x) f_2(x) dx = f_1(x) \cdot \int f_2(x) dx - \int \left[\left\{ \frac{d}{dx} f_1(x) \right\} \cdot \int f_2(x) dx \right] dx$$

- **Integration by Partial Fractions**

Rational function : Rational function is defined as the ratio of two polynomials in the form of $\frac{P(x)}{Q(x)}$, where P(x) and Q(x) are polynomials in x.

If the degree of $P(x)$ is less than degree of Q(x), then it is said to be Proper, otherwise it is called an Improper Rational Function.

Thus, if $\frac{P(x)}{Q(x)}$ is improper, then by long division method it can be reduced to proper function, i.e., $\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$, where $T(x)$ is a function of x and $\frac{P_1(x)}{Q(x)}$ is a proper rational function.

Such fractions as integrands can be evaluated by breaking in factors given as follows :

S.No.	Form of the rational function	Form of the parial fraction
(i)	$\frac{px+q}{(x-a)(x-b)} (a \neq b)$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
(ii)	$\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
(iii)	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
(iv)	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
(v)	$\frac{px^2+qx+r}{(x-a)^3(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3} + \frac{D}{(x-b)}$
(vi)	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$ Where, $x^2 + bx + c$ cannot be factored further.

The constants A, B, C etc. are obtained by equating coefficients of like terms from both sides or by substituting any value for x on both sides.

- **Definite Integrals**

Let $f(x)$ be a continuous function defined on the internal $[a, b]$. If $\int f(x)dx = F(x)$ then $\int_a^b f(x)dx = F(b) - F(a)$ is called the definite integral of $f(x)$ between a and b.

- The value of a definite integral is unique. It does not depend upon the constant.

- **Properties of Definite Integrals**

(i) $\int_a^b f(x)dx = -\int_b^a f(x)\,dx$

(ii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)\,dx$

(iii) $\int_a^b f(x)dx = \int_a^b f(t)\,dt$

(iv) $\int_a^a f(x)dx = 0$.

(v) $\int_a^b f(x)dx = \int_a^b f(a+b-x)\,dx$

(vi) $\int_0^a f(x)dx = \int_0^a f(a-x)\,dx$

(vii) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)\,dx$, when $f(x)$ is an even function, i.e., when $f(-x) = f(x)$.

$= 0$, when $f(x)$ is an odd function, i.e, when $f(-x) = -f(x)$.

(viii) $\int_0^{2a} f(x) = 2\int_0^a f(x)$, if $f(2a - x) = f(x)$.

$= 0$ if $f(2a - x) = -f(x)$.

- **Some Important Results**

(i) $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cos x)dx = -\frac{\pi}{2}\log 2$

(ii) $\int_0^{\pi/2} \log(\sin x)dx = \int_0^{\pi/2} \log(\cot x)dx = 0$

(iii) $\int_0^{\pi/2} \log(\sec x)dx = -\int_0^{\pi/2} \log(\cos x)dx = \frac{\pi}{2}\log 2$

(iv) $\int_0^{\pi/2} \log(\operatorname{cosec} x)dx = -\int_0^{\pi/2} \log(\sin x)dx = \frac{\pi}{2}\log 2$

- **Standard substitutions**

S.No.	Integrals form	Substitution
(i)	$\sqrt{a^2 - x^2}$, $\frac{1}{\sqrt{a^2 - x^2}}$, $a^2 - x^2$	$x = a \sin\theta$, or $x = a\cos\theta$
(ii)	$\sqrt{x^2 + a^2}$, $\frac{1}{\sqrt{x^2 + a^2}}$, $x^2 + a^2$	$x = a\tan\theta$, $a\cot\theta$ or $x = a\sinh\theta$
(iii)	$\sqrt{x^2 - a^2}$, $\frac{1}{\sqrt{x^2 - a^2}}$, $x^2 - a^2$	$x = a\sec\theta$, $a\operatorname{cosec}\theta$ or $x = a\cosh\theta$
(iv)	$\sqrt{\frac{x}{a+x}}$, $\sqrt{\frac{a+x}{x}}$, $\sqrt{x(a+x)}$, $\frac{1}{\sqrt{x(a+x)}}$	$x = a\tan^2\theta$, $a\cot^2\theta$

(v)	$\sqrt{\frac{x}{a-x}}, \sqrt{\frac{a-x}{x}}, \sqrt{x(a-x)}, \frac{1}{\sqrt{x(a-x)}}$	$x = a \sin^2\theta, a \cos^2\theta$
(vi)	$\sqrt{\frac{x}{x-a}}, \sqrt{\frac{x-a}{x}}, \sqrt{x(x-a)} \frac{1}{\sqrt{x(x-a)}}$	$x = a \sec^2\theta, a \operatorname{cosec}^2\theta$
(vii)	$\sqrt{\frac{a-x}{a+x}}, \sqrt{\frac{a+x}{a-x}}$	$x = a \cos 2\theta$
(viii)	$\sqrt{\frac{x-\alpha}{\beta-x}}, \sqrt{(x-\alpha)(\beta-x)}, (\beta > \alpha)$	$x = \alpha \cos^2\theta + \beta \sin^2\theta$

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. $\int a^{3x+3} dx =$

(a) $\frac{a^{3x+3}}{\log a} + C$ (b) $\frac{a^{3x+3}}{3 \log a} + C$ (c) $3a^{3x+3} \log a + C$ (d) $a^{3x+3} \log a + C$

2. If $\int \frac{x}{(x-1)(x^2+4)} dx = \frac{1}{5(x-1)} - \frac{1}{5}\int \frac{x-P}{x^2+4}$, then the value of P is :

(a) 2 (b) 3 (c) 4 (d) 1

3. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}} dx =$

(a) $2a^{\sqrt{x}} \log_e a + C$ (b) $\frac{2a^{\sqrt{x}}}{\log_e a} + C$ (c) $2a^{\sqrt{x}} \log_{10} a + C$ (d) $2a^{\sqrt{x}} \log_a 10 + C$

4. $\int (\sin x + \cos x)^2 dx$ is:

(a) $x + \frac{\sin 2x}{2} + C$ (b) $x - \frac{\cos 2x}{2} + C$ (c) $x - 2\cos 2x + C$ (d) $x^2 - \frac{\cos 2x}{2} + C$

5. $\int \frac{x^2}{1+x^3} dx$ is:

(a) $\log |1 + x^3| + C$ (b) $3\log |1 + x^3| + C$ (c) $\frac{1}{3} \log |1 + x^3| + C$ (d) $3 \log |x^3| + C$

6. $\int \frac{\cos^2 x - \sin^2 x}{7\cos^2 x \sin^2 x} dx$

(a) $\frac{-1}{7}(\cot x - 2\tan x) + C$ (b) $\frac{-1}{7}(2\cot x + \tan x) + C$ (c) $\frac{-1}{7}(\cot x + \tan x) + C$ (d) $\frac{-1}{7}(\cot x - \tan x) + C$

7. $\int \frac{5x^4}{\sqrt{x^5+9}} dx$ is:

(a) $\sqrt{x^5+9} + C$ (b) $2\sqrt{x^5-9} + C$ (c) $2(x^5 + 9) + C$ (d) $2\sqrt{x^5+9} + C$

8. $\int \frac{3}{9+x^2}\, dx$ is:

(a) $\tan^{-1}\frac{x}{2} + C$ (b) $\tan^{-1}\frac{x}{3} + C$ (c) $\tan^{-1}\frac{x}{5} + C$ (d) $\tan^{-1}\frac{x}{4} + C$

9. $\int_0^1 20e^{x^4} x^3\, dx$ is:

(a) $5(e + 1)$ (b) $5e$ (c) $(e - 1)$ (d) $5(e - 1)$

10. $\int_0^{\sqrt{\frac{\pi}{4}}} 2x \cos x^2\, dx$ is:

(a) 1 (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-1}{\sqrt{2}}$ (d) $\sqrt{2}$

11. If $\int e^x[f(x) + f'(x)]\, dx = e^x \sin x + C$, then $f(x)$ is equal to:

(a) $\sin x$ (b) $-\sin x$ (c) $\cos x - \sin x$ (d) $\sin x + \cos x$

12. $\int_1^3 \frac{3\cos(\log x)}{x}\, dx$ is :

(a) $3 \sin(\log 3)$ (b) $3 \cos(\log 3)$ (c) 1 (d) $\frac{\pi}{4}$

13. $\int_{a+c}^{b+c} f(x)\, dx$ is:

(a) $\int_a^b f(x+c)\, dx$ (b) $\int_a^b f(x-c)\, dx$ (c) $\int_a^b f(x)\, dx$ (d) $\int_{a-c}^{b-c} f(x)\, dx$

14. $\int \cos^{-1}(\sin x)\, dx$ is :

(a) $\frac{\pi}{2} - \frac{x^2}{2} + C$ (b) $\frac{\pi}{2}x + x^2 + C$ (c) $\frac{\pi}{2}x - \frac{x^2}{2} + C$ (d) $\pi x - \frac{x^2}{2} + C$

15. $\int \sec^4 x \tan x\, dx$ is:

(a) $\frac{\sec^2 x \tan^2 x}{4} + C$ (b) $\frac{(\sec x)^4}{4} + C$ (c) $\frac{\tan^3 x}{3} + C$ (d) $\frac{\tan x^4}{4} + C$

16. $\int \frac{dx}{x + x\log x}$ is:

(a) $\log |x| + C$ (b) $\log |1+\log x| + C$ (c) $\log |x+\log x| + C$ (d) $\log |1+x| + C$

17. $\int x^3 \sin^4(x^4) \cos(x^4)\, dx$ is :

(a) $\frac{1}{20} \sin^3(x^4) + C$ (b) $\frac{1}{20} \sin^4(x^4) + C$ (c) $\frac{1}{20} \sin^5(x^4) + C$ (d) $\frac{1}{20} \sin(x^4) + C$

18. $\int_0^{\pi/4} \frac{\tan^3 x}{1+\cos 2x}\,dx$ is :

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

19. $\int_0^1 \frac{\log(1+x)}{x^2}\,dx$ is:

(a) $4\log 2$ (b) $2\log 4$ (c) $2\log 2$ (d) $-2\log 2$

20. $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}\,dx$ is:

(a) $\tan^{-1}(\tan 2x) + C$ (b) $\tan^{-1}(\tan^2 x) + C$ (c) $\tan^{-1}(\sec x) + C$ (d) $\tan^{-1}(\sec^2 x) + C$

21. $\int_0^{\pi/4} \frac{2\cos 2x}{1+\sin 2x}\,dx$ is:

(a) $\log 4$ (b) $\log 2$ (c) $\cos^2 x$ (d) $\tan x$

22. If $\int_0^1 \frac{e^t}{1+t}\,dt = a$, then $\int_0^1 \frac{e^t}{(1+t)^2}\,dt$ is:

(a) $a - 1 + \frac{e}{2}$ (b) $a + 1 + \frac{e}{2}$ (c) $a - 1 - \frac{e}{2}$ (d) $a + 1 - \frac{e}{2}$

23. $\int \frac{1}{\sqrt{x-x^2}}\,dx$ is :

(a) $\sin^{-1}(x-1) + C$ (b) $\sin^{-1}(x+1) + C$ (c) $\sin^{-1}(2x-1) + C$ (d) $\sin^{-1}(2x+1) + C$

24. If $f'(x) = \sqrt{x}$ and $f(1) = 2$, then value of $f(x)$ is :

(a) $\frac{-2}{3}(x^{1/2} + 2)$ (b) $\frac{2}{3}(x^{3/2} + 2)$ (c) $\frac{3}{2}(x^{3/2} + 2)$ (d) $\frac{1}{2}(x^{1/2} + 2)$

25. $\int e^{e^x} e^x\,dx$ is:

(a) $e^x + C$ (b) $e^{e^x} + C$ (c) $e^{x^2} + C$ (d) $e^{2x} + C$

26. $\int e^{-\cot x} \operatorname{cosec}^2 x\,dx$ is :

(a) $e^{\operatorname{cosec} x} + C$ (b) $e^{-\cot x} + C$ (c) $2e^{-\cot x} + C$ (d) $2e^{-\operatorname{cosec} x} + C$

27. $\int_0^{\pi} \left(\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}\right) dx$ is:

(a) 1 (b) $\frac{\pi}{2}$ (c) π (d) 0

28. $\int \frac{\sin x}{\sin(x-\alpha)}\,dx =$

(a) $x\cos\alpha - \sin\alpha \log\sin(x-\alpha) + C$
(b) $x\cos\alpha + \sin\alpha \log\sin(x-\alpha) + C$
(c) $x\sin\alpha - \sin\alpha \log\sin(x-\alpha) + C$
(d) $x\sin\alpha + \sin\alpha \log\sin(x-\alpha) + C$

29. $\int \sqrt{\left(1+\sin\frac{x}{2}\right)}\,dx =$

(a) $\frac{1}{4}\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)+C$ (b) $4\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)+C$ (c) $4\left(\sin\frac{x}{4}-\cos\frac{x}{4}\right)+C$ (d) $4\left(\sin\frac{x}{4}+\cos\frac{x}{4}\right)+C$

30. $\int \frac{1+\cos^2 x}{\sin^2 x}\,dx =$

(a) $-\cot x - 2x + C$ (b) $-2\cot x - 2x + C$ (c) $-2\cot x - x + C$ (d) $-2\cot x + x + C$

31. $\int \frac{dx}{\sin x + \cos x} =$

(a) $\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+C$ (b) $\log\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)+C$ (c) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+C$ (d) $\frac{1}{2}\log\sec\left(\frac{\pi}{8}+\frac{x}{2}\right)+C$

32. $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}\,dx$ is equal to

(a) $2(\sin x + x\cos\theta) + C$ (b) $2(\sin x - x\cos\theta) + C$

(c) $2(\sin x + 2x\cos\theta) + C$ (d) $2(\sin x - 2x\cos\theta) + C$

33. $\int e^x\left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

(a) $\frac{e^x}{1+x^2}+C$ (b) $\frac{e^{-x}}{(1+x^2)^2}+C$ (c) $\frac{-e^x}{1+x^2}+C$ (d) $\frac{e^x}{(1+x^2)^2}+C$

34. $\int \frac{\sqrt{\tan x}}{\sin x\cos x}\,dx =$

(a) $2\sqrt{\sec x}+C$ (b) $2\sqrt{\tan x}+C$ (c) $\frac{2}{\sqrt{\tan x}}+C$ (d) $\frac{2}{\sqrt{\sec x}}+C$

35. $\int \frac{xe^x}{(1+x)^2}\,dx$ is equal to

(a) $\frac{e^{-x}}{1+x}+C$ (b) $-\frac{e^{-x}}{1+x}+C$ (c) $\frac{e^x}{1+x}+C$ (d) $-\frac{e^x}{1+x}+C$

36. If $\int xe^{2x}\,dx$ is equal to $x^{2e} f(x) + C$, where C is constant of integration, then $f(x)$ is :

(a) $\frac{(3x-1)}{4}$ (b) $\frac{(2x+1)}{2}$ (c) $\frac{(2x-1)}{4}$ (d) $\frac{x-4}{6}$

37. If $\int \frac{e^x(1+\sin x)}{1+\cos x}\,dx = e^x f(x) + C$, then $f(x) =$

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$ (c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

38. $I_1 = \int \sin^{-1}x\,dx$ and $I_2 = \int \sin^{-1}\sqrt{1-x^2}\,dx$, then

(a) $I_1 = I_2$ (b) $I_2 = \frac{\pi}{2} I_1$ (c) $I_1 + I_2 = \frac{\pi}{2}$ (d) $I_1 - I_2 = \frac{\pi}{2}$

39. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ is equal to

(a) $\frac{e^2}{2}+e$ (b) $e-\frac{e^2}{2}$ (c) $\frac{e^2}{2}-e$ (d) $\frac{e^2}{2}$

40. $\int_0^{\pi/2} x\sin^2 x\cos^2 x\,dx$ is equal to

(a) $\frac{\pi^2}{64}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi}{32}$ (d) $\frac{\pi}{16}$

41. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}\,dx$ is equal to :

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

42. $\int_0^{\pi/2} \log\tan x\,dx =$

(a) $\frac{\pi}{2}\log 2$ (b) $-\frac{\pi}{2}\log 2$ (c) $\pi\log 2$ (d) 0

43. The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ is :

(a) 1 (b) 0 (c) –1 (d) $\frac{1}{2}$

44. $\int_0^{\pi} \frac{x\,dx}{1+\sin x}$ is equal to

(a) $-\pi$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{1}{2}$

45. $\int_0^2 |x-1|\,dx =$

(a) 0 (b) 2 (c) $\frac{1}{2}$ (d) 1

46. $\int_0^{\pi/2} \sin 2x\log\tan x\,dx$ is equal to :

(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) 2π

47. $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}\,dx =$

(a) a (b) $\frac{a}{2}$ (c) $2a$ (d) 0

48. $\int_0^{\infty} \frac{\log(1+x^2)}{1+x^2}\,dx =$

(a) $\pi\log\frac{1}{2}$ (b) $\pi\log 2$ (c) $2\pi\log\frac{1}{2}$ (d) $2\pi\log 2$

49. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}\right]=0$

(a) 0 (b) $\log_e 4$ (c) $\log_e 3$ (d) $\log_e 2$

50. If $\int \frac{\cos x - 1}{\sin x + 1} e^x \, dx$ is equal to :

(a) $\frac{e^x \cos x}{1+\sin x} + C$ (b) $C - \frac{e^x \sin x}{1+\sin x}$ (c) $C - \frac{e^x}{1+\sin x}$ (d) $C - \frac{e^x \cos x}{1+\sin x}$

51. Evaluate : $\int \frac{1}{1+3\sin^2 x + 8\cos^2 x} dx$

(a) $\frac{1}{6}\tan^{-1}(2\tan x) + C$ (b) $\tan^{-1}(2\tan x) + C$ (c) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + C$ (d) None of these

52. Evaluate : $\int \sqrt{\frac{x}{4-x^3}} dx$

(a) $\frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{2}\right) + C$ (b) $\frac{2}{3}\sin^{-1}(x^{3/2}) + C$ (c) $2\sin^{-1}\left(\frac{x^{3/2}}{2}\right) + C$ (d) $\frac{1}{3}\sin^{-1}\left(\frac{x^{3/2}}{2}\right) + C$

53. Evaluate : $\int \frac{x^2}{x^2-1} dx$

(a) $x - \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + C$ (b) $x + \frac{1}{2}\log\left(\frac{x+1}{x-1}\right) + C$ (c) $x + \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + C$ (d) None of these

54. If $\int x \log\left(1+\frac{1}{x}\right) dx = f(x)\log(x+1) + g(x)x^2 + Lx + C$, then

(a) $f(x) = \frac{1}{2}x^2$ (b) $g(x) = \log x$ (c) $L = 1$ (d) None of these

55. $\int \frac{(x^2-1)}{(x^2+1)\sqrt{x^4+1}} dx$ is equal to :

(a) $\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + C$ (b) $\frac{1}{\sqrt{2}} \sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + C$ (c) $\frac{1}{\sqrt{2}} \sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right) + C$ (d) None of these

56. $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx$ equals :

(a) 2 (b) π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

57. Let $f(x)$ be a polynomial of degree three satisfying $f(0) = -1$ and $f(1) = 0$. Also, 0 is a stationary point of $f(x)$. If $f(x)$ does not have an extremum at $x = 0$, then which of the following is equal to $\int \frac{f(x)}{x3-1} dx$

(a) $\frac{x^2}{2} + C$ (b) $x + C$ (c) $\frac{x^3}{6} + C$ (d) None of these

58. $\int \frac{1}{x\sqrt{x^2-1}} dx =$

(a) $\cos^{-1} x + C$ (b) $\sec^{-1} x + C$ (c) $\cot^{-1} x + C$ (d) $\tan^{-1} x + C$

59. $\int_0^{10} \frac{x^{10}}{(10-x)^{10} + x^{10}} dx$ is equal to :

(a) 10 (b) 5 (c) 2 (d) $\frac{1}{2}$

60. $\lim_{x\to\infty} \dfrac{\int_0^{2x} xe^{x^2}}{e^{4x^2}} dx$ equals :

(a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

61. Evaluate $\int_0^{\pi/2} \dfrac{\sin x}{1+\cos^2 x} dx$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) π

62. Evaluate : $\int x \tan^{-1} x \, dx$

(a) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2}x + C$ (b) $\frac{1}{2}(x^2 + 1) \tan^{-1} x + \frac{1}{2}x + C$

(c) $\frac{1}{2}(x^2 - 1) \tan^{-1} x - \frac{1}{2}x + C$ (d) None of these

63. Find the value of $\int_0^{4\pi} |\sin x| \, dx$

(a) 8 (b) 6 (c) 4 (d) 2

64. $\int_a^{\infty} \dfrac{dx}{(a^2+x^2)^3}$ is equal to :

(a) $\dfrac{(3\pi-8)}{32a^5}$ (b) $\dfrac{3\pi}{16a^5}$ (c) $\dfrac{3\pi}{32a^5}$ (d) $\dfrac{3\pi-4}{16a^5}$

65. $\int 4\cos\left(x+\frac{\pi}{6}\right)\cos 2x.\cos\left(\frac{5\pi}{6}+x\right)dx$ is equal to :

(a) $-\left(x+\dfrac{\sin 4x}{4}+\dfrac{\sin 2x}{2}\right)+C$ (b) $-\left(x+\dfrac{\sin 4x}{4}-\dfrac{\sin 2x}{2}\right)+C$

(c) $-\left(x-\dfrac{\sin 4x}{4}+\dfrac{\sin 2x}{2}\right)+C$ (d) $-\left(x-\dfrac{\sin 4x}{4}+\dfrac{\cos 2x}{2}\right)+C$

66. $\int_0^{\infty} \dfrac{dx}{1+e^x}$ equals

(a) log 2 – 1 (b) log 2 (c) log 4 – 1 (d) – log 2

67. Evaluate : $\int_0^1 \dfrac{dx}{\sqrt{2-x^2}}$

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

68. Let $I_1 = \int_1^2 \dfrac{1}{\sqrt{1+x^2}} dx$ and $I_2 = \int_1^2 \dfrac{1}{x} dx$, then

(a) $I_1 > I_2$ (b) $I_2 > I_1$ (c) $I_1 = I_2$ (d) None of these

69. If $I_m = \int_1^e (\ln x)^m \, dx$, where $m \in N$, then $I_{10} + 19\, I_9$ is equal to :

(a) e^{10} (b) $\frac{e^{10}}{10}$ (c) e (d) $e - 1$

70. $\int \frac{dx}{1-\sin x} =$

(a) $x + \cos x + C$ (b) $1 + \sin x + C$ (c) $\sec x - \tan x + C$ (d) $\sec x + \tan x + C$

71. If $\int \frac{2\cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx = A \ln |(\cos x + \sin x - 2)| + Bx + C$

Then the ordered triplet A, B, λ is :

(a) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$ (b) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$ (c) $\left(\frac{1}{2}, -1, \frac{3}{2}\right)$ (d) $\left(\frac{3}{2}, -1, \frac{1}{2}\right)$

72. $\int \frac{x + \sin x}{1 + \cos x} dx =$

(a) $\cot \frac{x}{2} + C$ (b) $x \tan \frac{x}{2} + C$ (c) $\log(1 + \cos x) + C$ (d) $x \tan \frac{x}{2} + \tan x + C$

73. The value of integral $\int_0^{\pi/2} \frac{\phi(x)}{\phi(x) + \phi(\pi/2 - x)} dx$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) None of these

74. Evaluate $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx$

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 2

75. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$ and $I_4 = \int_1^2 2^{x^3} dx$ then

(a) $I_2 > I_1$ (b) $I_1 > I_2$ (c) $I_3 = I_4$ (d) $I_3 > I_4$

76. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then $\int_0^{\pi/2} f(x)\, dx$ is equal to :

(a) $\frac{1}{4}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1

77. If $\int_0^n [x] dx = 66$, then $n =$

(a) 24 (b) 9 (c) 12 (d) 7

78. $\int \frac{1}{(x+1)\sqrt{x^2 - 1}} dx =$

(a) $\sqrt{\frac{x+1}{x-1}} + C$ (b) $\sqrt{\frac{x^2-1}{x+1}} + C$ (c) $\sqrt{\frac{x-1}{x+1}} + C$ (d) $\sqrt{\frac{x-1}{x^2+1}} + C$

79. The value of $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is :

(a) $\tan x - \cot x + C$ (b) $\tan x + \cot x + C$ (c) $-\tan x - \cot x + C$ (d) None of these

80. If $\int \frac{dx}{(x+2)(x^2+1)} = a \log |1 + x^2| + b \tan^{-1} x + \frac{1}{5} \log |x + 2| + C$, then,

(a) $a = -\frac{1}{10}, b = -\frac{2}{5}$ (b) $a = \frac{1}{10}, b = \frac{-2}{5}$ (c) $a = \frac{-1}{10}, b = \frac{2}{5}$ (d) $a = \frac{1}{10}, b = \frac{2}{5}$

81. $\int_0^{\pi/2} \sqrt{1-\sin 2x}\, dx$ is equal to :

(a) $2\sqrt{2}$ (b) $2(\sqrt{2}+1)$ (c) 2 (d) $2(\sqrt{2}-1)$

82. If $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, then $a =$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

83. $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is equal to :

(a) $\frac{\pi}{2} - \tan^{-1} e$ (b) $\tan^{-1} e - \frac{\pi}{4}$ (c) $\tan^{-1} \frac{1}{e} - \frac{\pi}{2}$ (d) $\tan^{-1} e$

84. $\int \frac{10x^9 + 10^x \log 10}{x^{10} + 10^x} dx$ equals :

(a) $10x - x^{10} + C$ (b) $10x + x^{10} + C$ (c) $(10x - x^{10})^{-1} + C$ (d) $\log (10^x + x^{10}) + C$

85. $\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$ equals :

(a) $-\cot (ex^x) + C$ (b) $\tan (xe^x) + C$ (c) $\tan(e^x) + C$ (d) $\cot(e^x) + C$

86. $\int \frac{dx}{x^2 + 2x + 2}$ equals :

(a) $x \tan^{-1} (x + 1) + C$ (b) $\tan^{-1} (x + 1) + C$ (c) $(x + 1) \tan^{-1} x + C$ (d) $\tan^{-1} x + C$

87. $\int \frac{dx}{\sqrt{9x - 4x^2}}$ equals :

(a) $\frac{1}{9} \sin^{-1}\left(\frac{9x-8}{8}\right) + C$ (b) $\frac{1}{9} \sin^{-1}\left(\frac{8x-9}{9}\right) + C$ (c) $\frac{1}{3} \sin^{-1}\left(\frac{9x-8}{8}\right) + C$ (d) $\frac{1}{2} \sin^{-1}\left(\frac{9x-8}{8}\right) + C$

88. $\int \sqrt{x^2 - 8x + 7}\, dx$ is equal to :

(a) $\frac{1}{2} (x - 4)\sqrt{x^2 - 8x + 7} + 9 \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$

(b) $\frac{1}{2} (x + 4)\sqrt{x^2 - 8x + 7} + 9 \log |x + 4 + \sqrt{x^2 - 8x + 7}| + C$

(c) $\frac{1}{2} (x - 4)\sqrt{x^2 - 8x + 7} + 3\sqrt{2} \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$

(d) $\frac{1}{2} (x - 4)\sqrt{x^2 - 8x + 7} - \frac{9}{2} \log |x - 4 + \sqrt{x^2 - 8x + 7}| + C$

89. The value of integral $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$ is :

(a) 6 (b) 0 (c) 3 (d) 4

90. The value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is :

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) –2

91. The value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx$

(a) 0 (b) 2 (c) π (d) 1

92. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is equal to :

(a) 1 (b) 0 (c) –1 (d) $\frac{\pi}{4}$

93. If $f(a + b - x) = f(x)$, then $\int_a^b x f(x)\, dx$ is equal to :

(a) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b+x)dx$ (c) $\frac{b-a}{2}\int_a^b f(x)dx$ (d) $\frac{a+b}{2}\int_a^b f(x)dx$

94. $\int \frac{a^x}{\sqrt{1-a^{2x}}} dx =$

(a) $\frac{1}{\log_e a}\sin^{-1}(a^x) + C$ (b) $\log_e a \sin^{-1}(a^x) + C$ (c) $\frac{1}{\log_e a}\sin^{-1}(a^x) + C$ (d) $\frac{1}{\log_e a}\cos^{-1}(a^x) + C$

95. $\int \frac{x^2 dx}{(x\sin x + \cos x)^2} =$

(a) $\frac{-x\sec x}{x\sin x + \cos x} + \tan x + C$ (b) $\frac{-x\sec x}{x\sin x + \cos x} - \tan x + C$

(c) $\frac{x\sec x}{x\sin x + \cos x} + \tan x + C$ (d) None of these

96. If $y^2 = x^2 - x + 1$ and $I_n = \int \frac{x^n}{y} dx$ and $AI_3 + BI_2 + CI_1 = x^2 y$, then ordered triplet A, B, C is :

(a) $\left(\frac{1}{2}, -\frac{1}{2}, 1\right)$ (b) (3, 1, 0) (c) (1, – 1, 2) (d) $\left(3, -\frac{5}{2}, 1\right)$

97. $\int 4\cos\left(x + \frac{\pi}{6}\right)\cos 2x.\cos\left(\frac{5\pi}{6} + x\right) dx =$

(a) $-\left(x + \frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + C$ (b) $-\left(x + \frac{\sin 4x}{4} - \frac{\sin 2x}{2}\right) + C$

(c) $-\left(x - \frac{\sin 4x}{4} + \frac{\sin 2x}{2}\right) + C$ (d) $-\left(x - \frac{\sin 4x}{4} + \frac{\cos 2x}{2}\right) + C$

98. $\int x^{2/3}(1 + x^{1/2})^{-13/3}\, dx =$

(a) $\frac{3}{5}(1 + x^{-1/2})^{-10/3} + C$ (b) $\frac{3}{5}(1 + x^{1/2})^{-10/3} + C$ (c) $-\frac{(1+x^{1/2})}{13} + C$ (d) none of these

99. $\int \frac{\log(x+1) - \log x}{x(x+1)}\, dx$ is equal to :

(a) $-\frac{1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2 + C$ (b) $C - [\{\log(x+1)\}^2 - (\log x)^2]$

(c) $-\log\left[\log\left(\frac{x+1}{x}\right)\right] + C$ (d) $-\log\left(\frac{x+1}{x}\right) + C$

100. $\int \frac{\sqrt{x}}{\sqrt{a^3 - x^3}}$ is equal to :

(a) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + C$ (b) $\frac{2}{6}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + C$ (c) $\frac{2}{3}\cos^{-1}\left(\frac{x}{a}\right)^{3/2} + C$ (d) None of these

101. $\int \frac{x^8 + 4}{x^4 - 2x^2 + 2} dx =$

(a) $\frac{x^5}{5} - \frac{2x^3}{3} + 2x + C$ (b) $\frac{x^5}{5} - \frac{2x^3}{3} - 2x + C$ (c) $\frac{x^5}{5} + \frac{2x^3}{3} - 2x + C$ (d) $\frac{x^5}{5} + \frac{2x^3}{3} + 2x + C$

INPUT TEXT BASED MCQ's

102. A function f is continuous for all $x \geq 0$. If f is continuous on $[a, b]$, Then $\int_a^b f(x) = 0$

Then answer the following questions.

Answer the following questions :

(i) If $g:[0, a]$ is continuous and $f(x) = f(a - x)$, $g(x) + g(a - x) = 2$, then $\int_0^a f(x)g(x)dx =$

(a) $\int_0^a g(x)dx$ (b) $\int_0^a f(x)dx$ (c) $2\int_0^a f(x)dx$ (d) 0

(ii) $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x} dx =$

(a) $\pi(\sqrt{2} - 1)$ (b) $\pi(\sqrt{2} + 1)$ (c) $2\pi\sqrt{2}$ (d) $\pi\sqrt{2}$

(iii) If $f(a + b - x) = f(x)$, then $\int_a^b x\, f(x)dx =$

(a) $\frac{a+b}{2}\int_a^b f(a+b+x)dx$ (b) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (c) $\frac{a+b}{2}\int_a^b f(x)dx$ (d) $\frac{b-a}{2}\int_a^b f(x)dx$

(iv) $\int_0^{\pi} e^{|\cos x|}\left[\left[2\sin\left(\frac{1}{2}\cos x\right)dx\right]\right] =$

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi^2}{2}$ (d) 0

(v) The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is :

(a) $\frac{22}{7} - \pi$ (b) $\frac{2}{105}$ (c) 0 (d) $\frac{71}{15} - \frac{3\pi}{2}$

103. Using the formula $\int u\,dv = uv - \int v\,dv$,

There are always exceptions, but these are generally helpful (ILATE).

The steps of applying ILATE rule :

- Identity the type of each function as inverse trigonometric (I), Logarithmic (L), Algebraic (A), Trigonometric (T), or Exponential (E).
- See which of the given functions comes first in the order of ILATE and choose it as the first function.
- Select the remaining function as the second function.
- Then apply the integration by parts formula.

Answer the following questions :

(i) $\int x^3 \log x\, dx =$

(a) $\frac{x^4}{4}\log x + \frac{x^4}{16} + C$ (b) $\frac{x^4}{4}\log x - \frac{x^4}{16} + C$ (c) $\frac{x^2}{2}\log x - \frac{x^4}{16} + C$ (d) $\frac{x^4}{4}\log x + C$

(ii) $\int x^2 \sin x\, dx =$

(a) $-x^2 \cos x + 2x \sin x + 2\cos x + C$ (b) $x^2 \cos x - 2x \sin x + 2\cos x + C$

(c) $-x^2 \cos x - 2x \sin x - 2\cos x + C$ (d) $x^2 \cos x + 2x \sin x + 3\sin x + C$

(iii) $\int \sin x \log(\cos x)dx =$

(a) $\cos x \log \sin x + \cos x + C$ (b) $-\cos x \log \sin x + \cos x + C$

(c) $-\cos x \log \cos x - \sin x + C$ (d) $-\cos x \log (\cos x) + \cos x + C$

(iv) $\int \sec^3 x\, dx$ is equal to :

(a) $\frac{1}{2}(\sec x \tan x + \log_e|\sec x + \tan x|) + C$ (b) $\frac{1}{2}(\sec x + \log_e|\sec x + \tan x|) + C$

(c) $\frac{1}{2}(\sec x + \tan x + \log_e|\sec x + \tan x|) + C$ (d) $\sec x \tan x + 2\log_e|\sec x + \tan x| + C$

(v) $\int e^x \sin x\, dx$ is equal to

(a) $\frac{e^x}{2}(\sin x - \cos x) + C$ (b) $\frac{-e^x}{2}(\sin x - \cos x) + C$

(c) $\frac{e^x}{4}(\sin x - \cos x) + C$ (d) $\frac{e^x}{4}(\cos x - \sin x) + C$

ANSWERS

1. (b)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (b)	**9.** (d)	**10.** (b)
11. (a)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (d)	**20.** (b)
21. (b)	**22.** (d)	**23.** (c)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (d)	**28.** (b)	**29.** (c)	**30.** (c)
31. (c)	**32.** (a)	**33.** (a)	**34.** (b)	**35.** (c)	**36.** (c)	**37.** (c)	**38.** (c)	**39.** (c)	**40.** (a)

41. (c)	**42.** (d)	**43.** (d)	**44.** (c)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (b)	**49.** (d)	**50.** (a)
51. (c)	**52.** (a)	**53.** (c)	**54.** (d)	**55.** (b)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (b)
61. (c)	**62.** (b)	**63.** (a)	**64.** (a)	**65.** (a)	**66.** (b)	**67.** (a)	**68.** (b)	**69.** (c)	**70.** (d)
71. (b)	**72.** (b)	**73.** (a)	**74.** (c)	**75.** (b)	**76.** (b)	**77.** (c)	**78.** (c)	**79.** (a)	**80.** (c)
81. (d)	**82.** (a)	**83.** (b)	**84.** (d)	**85.** (b)	**86.** (c)	**87.** (b)	**88.** (d)	**89.** (a)	**90.** (c)
91. (c)	**92.** (b)	**93.** (d)	**94.** (a)	**95.** (a)	**96.** (d)	**97.** (a)	**98.** (a)	**99.** (a)	**100.** (a)
101. (d)									

102. (i) (b) (ii) (a) (iii) (c) (iv) (d) (v) (a) **103.** (i) (b) (ii) (a) (iii) (a) (iv) (a) (v) (a))

Hints to Some Selected Questions

1. (b) $\int a^{3x+3}dx = \dfrac{a^{3x+3}}{3\log a} + C$

3. (b) Let $\sqrt{x} = t \Rightarrow \dfrac{1}{2\sqrt{x}}\,dx = dt \Rightarrow 2\int a^t dt = 2\,\dfrac{a^t}{\log_e a} + C = 2\dfrac{a^{\sqrt{x}}}{\log_e a} + C.$

4. (b) $\int(1 + 2\sin x\cos x)dx = x - \dfrac{\cos 2x}{2} + C$

5. (c) Let $1 + x^3 = t \Rightarrow 3x^2\,dx = dt$. So the given integral becomes $\dfrac{1}{3}\int\dfrac{1}{t}\,dt = \dfrac{1}{3}\log(1 + x^3) + C.$

6. (c) $\dfrac{1}{7}\int(\operatorname{cosec}^2 x - \sec^2 x)dx = \dfrac{1}{7}(-\cot x - \tan x) + C$

7. (d) Let $x^5+9 = t \Rightarrow 5x^4\,dx = dt$. So, the given integral becomes $\int\dfrac{1}{t^{1/2}}\,dt = 2t^{1/2} = 2\sqrt{x^5+9} + C$

8. (b) $\int\dfrac{3}{9+x^2}dx = \tan^{-1}\dfrac{x}{3} + C$

9. (d) Let $x^4 = t \Rightarrow 4x^3dx = dt$. So, the given integral becomes $\dfrac{20}{4}\int_0^1 e^t dt = 5\left[e^t\right]_0^1 = 5(e - 1)$

10. (b) Let $x^2 = t \Rightarrow 2xdx = dt$. So the given integral become $\int_0^{\pi/4}\cos t\,dt = [\sin t]_0^{\pi/4} = \dfrac{1}{\sqrt{2}}.$

11. (a) $\int e^x f(x)dx + \int e^x f'(x)dx = f(x)e^x - \int e^x f'(x)dx + \int e^x f'(x)dx + C = f(x)\,e^x + C$

So, $f(x) = \sin x$ (on comparison)

12. (a) Let $\log x = t \Rightarrow \dfrac{1}{x}\,dx = dt$. So the given integral becomes $\int_0^{\log 3} 3\cos t\,dt = 3\sin(\log 3) + C$

13. (c) $\int_{a+c}^{b+c} f(x)dx = F(b + c - a - c) + C = F(b - a) + C = \int_a^b f(x)dx$

14. (c) $\int\cos^{-1}(\sin x)dx = \int\cos^{-1}\left[\cos\left(\dfrac{\pi}{2} - x\right)\right]dx = \dfrac{\pi}{2}x - \dfrac{x^2}{2} + C$

15. (b) Let $\sec x = t \Rightarrow \sec x\tan x\,dx = dt$. Then, $\int t^3 dt = \dfrac{(\sec x)^4}{4} + C$

16. (b) Let $(1 + \log x) = t \Rightarrow \dfrac{1}{x}\,dx = dt$. So, the given integral becomes $\int\dfrac{1}{t}dt = \log|1 + \log x| + C$

18. (a) $\int_0^{\pi/4}\dfrac{\tan^3 x}{2\cos^2 x}dx = \dfrac{1}{2}\int_0^{\pi/4}\tan^3 x\sec^2 x\,dx = \dfrac{1}{2}\left[\dfrac{t^4}{4}\right]_0^1 + C = \dfrac{1}{8}$ $[\tan x = t \Rightarrow \sec^2 x dx = dt]$

20. (b) Dividing Nr and Dr by $\cos^4 x$ and putting $\tan x = t$ we get $\int \frac{2t}{1+t^4}dt$

$$= \int \frac{du}{1+u^2} = \tan^{-1}u = \tan^{-1}t^2 = \tan^{-1}(\tan^2 x) + C$$

21. (b) Let $1 + \sin 2x = t \Rightarrow 2\cos 2x\,dx = dt$. Then, $\int_1^2 \frac{1}{t}dt = [\log t]_1^2 = \log 2$.

22. (d) $\int_0^1 \frac{e^t}{(1+t^2)}dt = \int_0^1 e^t\left[\frac{1}{1+t} - \frac{t}{(1+t)^2}\right]dt = a - \int_0^1 \frac{te^t}{(1+t)^2} = a - \left(\frac{e}{2} - 1\right) = a + 1 - \frac{e}{2}$.

23. (c) $\int \frac{dx}{\sqrt{-(x^2 - x)}} = \int \frac{dx}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}} = \sin^{-1}(2x - 1) + C$

24. (b) $f(x) = \frac{2}{3}x^{3/2} + C \Rightarrow f(1) = \frac{2}{3}(1)^{3/2} + C \Rightarrow 2 - \frac{2}{3} = C \Rightarrow C = \frac{4}{3}$

Then, $f(x) = \frac{2}{3}(x^{3/2} + 2)$.

25. (b) Let $e^x = t \Rightarrow e^x dx = dt$. Then $\int e^t dt = e^{e^x} + C$

26. (b) Let $-\cot x = t \Rightarrow \text{cosec}^2 x\,dx = dt$. Then, $\int e^t dt = e^{-\cot x} + C$

27. (d) $-\int_0^\pi \cos x\,dx = -[\sin x]_0^\pi = 0$

29. (c) $\int \sqrt{\left(1 + \sin\frac{x}{2}\right)}\,dx = \int \sqrt{\left(\sin^2\frac{x}{4} + \cos^2\frac{x}{4} + 2\sin\frac{x}{4}\cos\frac{x}{4}\right)}\,dx = \int\left(\sin\frac{\pi}{4} + \cos\frac{x}{4}\right)dx = 4\left(\sin\frac{x}{4} - \cos\frac{x}{4}\right) + C$

30. (c) $\int \frac{1 + \cos^2 x}{\sin^2 x}dx = \int(\text{cosec}^2 x + \cot^2 x)\,dx = \int(2\text{cosec}^2 x - 1)\,dx = -2\cot x - x + C$.

31. (c) $\int \frac{dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sin x\cos\frac{\pi}{4} + \cos x\sin\frac{\pi}{4}} = \frac{1}{\sqrt{2}}\int \text{cosec}\left(x + \frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + C$

32. (a) Let $I = \int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} = \int \frac{2\cos^2 x - 2\cos^2\theta}{\cos x - \cos\theta} = 2\int(\cos x + \cos\theta)dx \Rightarrow I = 2(\sin x + \cos\theta \cdot x) + C$

34. (b) $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx = \int \frac{\tan x}{\sqrt{\tan x}\sin x\cos x}dx = \int \frac{\sin x\sec x}{\sqrt{\tan x}\sin x\cos x}dx = \int \frac{\sec^2 x}{\sqrt{\tan x}}dx$

Put Put $t = \tan x \Rightarrow dt = \sec^2 x\,dx$, $\Rightarrow \int \frac{1}{\sqrt{t}}dt = 2t^{1/2} + C = 2\sqrt{\tan x} + C$.

35. (c) $\int \frac{xe^x}{(1+x)^2}dx = \int \frac{(x+1-1)}{(1+x)^2}e^x dx = \int e^x\left[\frac{1}{1+x} - \frac{1}{(1+x)^2}\right]dx = \frac{e^x}{1+x} + C$.

36. (c) $\int xe^{2x}dx = \frac{xe^{2x}}{2} - \int 1\cdot\frac{e^{2x}}{2}dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C = e^{2x}\left(\frac{2x-1}{4}\right) + C \Rightarrow f(x) = \frac{(2x-1)}{4}$.

37. (c) $\mathrm{I}=\int e^{x}\left(\frac{1+\sin x}{1+\cos x}\right)dx=\int e^{x}\left[\frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^{2}\frac{x}{2}}\right]dx \Rightarrow \mathrm{I}=\int e^{x}\left[\frac{1}{2}\sec^{2}\left(\frac{x}{2}\right)+\tan\frac{x}{2}\right]dx=e^{x}\cdot\tan\frac{\pi}{2}+\mathrm{C}$

39. (c) $\int_{1}^{2}e^{x}\left(\frac{1}{x}-\frac{1}{x^{2}}\right)dx=\left(\frac{1}{x}e^{x}\right)_{1}^{2}=\frac{e^{2}}{2}-e.$

42. (d) $\int_{0}^{\pi/2}\log\tan x\,dx=\int_{0}^{\pi/2}\log\left(\frac{\sin x}{\cos x}\right)dx=\int_{0}^{\pi/2}\log\sin x\,dx-\int_{0}^{\pi/2}\log\cos x\,dx=0$

45. (d) $\mathrm{I}=\int_{0}^{2}|x-1|\,dx=\int_{0}^{1}(-x+1)dx+\int_{1}^{2}(x-1)dx=\left(\frac{-x^{2}}{2}+x\right)_{0}^{1}+\left(\frac{x^{2}}{2}-x\right)_{1}^{2}=1$

46. (c) $\mathrm{I}=\int_{0}^{\pi/2}\sin 2x\log\tan x\,dx=\int_{0}^{\pi/2}\sin 2\left(\frac{\pi}{2}-x\right)\log\tan\left(\frac{\pi}{2}-x\right)dx$

$=\int_{0}^{\pi/2}\sin 2x\log\cot x\,dx=-\int_{0}^{\pi/2}\sin 2x\log\tan x\,dx \Rightarrow 2\mathrm{I}=0\Rightarrow \mathrm{I}=0$

47. (a) $\mathrm{I}=\int_{0}^{2a}\frac{f(x)}{f(x)+f(2a-x)}dx$ ...(i)

$\mathrm{I}=\int_{0}^{2a}\frac{f(2a-x)}{f(2a-x)+f(x)}dx$ ...(ii)

Adding (i) and (ii), we get $2\mathrm{I}=\int_{0}^{2a}dx=2a\Rightarrow \mathrm{I}=a.$

48. (b) Let $\mathrm{I}=\int_{0}^{\infty}\frac{\log\left(1+x^{2}\right)}{1+x^{2}}dx$ Put $x=\tan\theta\Rightarrow dx=\sec^{2}\theta\,d\theta$

$\therefore \mathrm{I}=\int_{0}^{\pi/2}\log(\sec\theta)^{2}d\theta=2\int_{0}^{\pi/2}\log\sec\theta\,d\theta=-2\int_{0}^{\pi/2}\log\cos\theta\,d\theta=-2\cdot\frac{\pi}{2}\log\frac{1}{2}=-\pi\log\frac{1}{2}=\pi\log 2.$

49. (d) $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+...+\frac{1}{2n}\right]$

$=\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+1}+\frac{1}{n+2}...+\frac{1}{n+n}\right]=\frac{1}{n}\lim_{n\to\infty}\left[1+\frac{1}{1+\frac{1}{n}}+...+\frac{1}{1+\frac{n}{n}}\right]$

$=\frac{1}{n}\lim_{n\to\infty}\sum_{r=0}^{n}\left[\frac{1}{1+\frac{r}{n}}\right]=\int_{0}^{1}\frac{1}{1+x}dx=\left[\log_{e}(1+x)\right]_{0}^{1}=\log_{e}2-\log_{e}1=\log_{e}2$

52. (a) $\mathrm{I}=\int\sqrt{\frac{x}{4-x^{3}}}dx=\int\frac{\sqrt{x}dx}{\sqrt{4-x^{3}}}$, Here integral of $\sqrt{x}=\frac{2}{3}x^{3/2}$ and $4-x^{3}=4-(x^{3/2})^{2}$

Put $x^{3/2}=t\Rightarrow \sqrt{x}\,dx=\frac{2}{3}dt$ So, $\mathrm{I}=\frac{2}{3}\int\frac{dt}{\sqrt{4-t^{2}}}=\frac{2}{3}\sin^{-1}\left(\frac{x^{3/2}}{2}\right)+\mathrm{C}$

53. (c) Given integral $I = \int\left(1+\frac{1}{x^2-1}\right)dx = \int dx + \int\frac{dx}{(x-1)(x+1)}$

$$= x+\frac{1}{2}\int\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx = x+\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)+C$$

54. (d) $\int x\log\left(1+\frac{1}{x}\right)dx = \log\left(1+\frac{1}{x}\right)\frac{x^2}{2} - \int\frac{x}{x+1}\cdot\left(-\frac{1}{x^2}\right)\cdot\frac{x^2}{2}dx$

$$= \frac{x^2}{2}\log\left(\frac{x+1}{x}\right)+\frac{1}{2}\int\frac{x+1}{x+1}dx = \frac{x^2}{2}\log\left(\frac{x+1}{x}\right)+\frac{1}{2}x-\frac{1}{2}\log(x-1)+C = \left(\frac{x^2-1}{2}\right)\log(x+1)-\frac{x^2}{2}\log x+\frac{1}{2}x+C$$

55. (b) $I = \int\frac{x^2\left(1-\frac{1}{x^2}\right)dx}{x^2\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)^{1/2}}$

Let $x+\frac{1}{x} = p \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dp$ $= \int\frac{dp}{p\sqrt{p^2-2}} = \frac{1}{\sqrt{2}}\sec^{-1}\frac{p}{\sqrt{2}} = \frac{1}{\sqrt{2}}\sec^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+C$

56. (C) $I = \int_0^{\pi/2}\frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx = \int_0^{\pi/2}\frac{2^{\sin(\pi/2-x)}}{2^{\sin(\pi/2-x)}+2^{\cos(\pi/2-x)}}dx = \int\frac{2^{\cos x}}{2^{\cos x}+2^{\sin x}}dx \Rightarrow 2I = \int_0^{\pi/2}dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$.

58. (b) $\int\frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}x+C$

59. (b) Let $I = \int_0^{10}\frac{x^{10}}{(10-x)^{10}+x^{10}}dx$ ...(i)

$I = \int_0^{10}\frac{(10-x)^{10}}{(10-x)^{10}+x^{10}}dx$ ...(ii)

Adding (i) and (ii), we get $2I = \int_0^{10}dx \Rightarrow 2I = 10 \Rightarrow I = 5.$

60. (b) Consider $\lim_{x\to\infty}\frac{\int_0^{2x}xe^{x^2}dx}{e^{4x^2}}$

$$\lim_{x\to\infty}\frac{2\int_0^{2x}xe^{x^2}dx}{2e^{4x^2}} = \lim_{x\to\infty}\frac{2\int_0^{2x}e^{x^2}d(x^2)}{2e^{4x^2}} = \lim_{x\to\infty}\frac{[e^{x^2}]_0^{2x}}{e^{4x^2}} = \lim_{x\to\infty}\frac{e^{4x^2}-1}{2e^{4x^2}} = \lim_{x\to\infty}\left(\frac{1}{2}-\frac{1}{2e^{4x^2}}\right) = \frac{1}{2}.$$

61. (c) Let $I = \int_0^{\pi/2}\frac{\sin x}{1+\cos^2 x}dx$, Let $\cos x = t$ and $-\sin x\, dx = dt$.

Now, $x = 0 \Rightarrow t = \cos 0 = 1$ and $x = \frac{\pi}{2} \Rightarrow t = \cos\frac{\pi}{2} = 0$

$$\therefore I = \int_1^0\frac{\sin x}{1+t^2}\left(\frac{-dt}{\sin x}\right) = -\int_1^0\frac{dt}{1+t^2} = \left[\tan^{-1}t\right]_1^0 = -\left[0-\frac{\pi}{4}\right] = \frac{\pi}{4}$$

62. (b) $\int x \tan^{-1} x\,dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int\left[1-\frac{1}{1+x^2}\right]dx = \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{1}{2}x + C$

63. (a) We know that $|\sin x|$ is a periodic function of π.

Hence, $\int_0^{4\pi}|\sin x|\,dx = 4\int_0^{\pi}|\sin x|\,dx = 4\int_0^{\pi}\sin x\,dx = 4[-\cos x]_0^{\pi} = 8$

64. (a) $\int_a^{\infty}\frac{dx}{(a^2+x^2)^3} = \int_{\pi/4}^{\pi/2}\frac{a\sec^2\theta\,d\theta}{a^6\sec^6\theta}$ on putting $x = a\tan\theta$

$= \frac{1}{a^5}\int_{\pi/4}^{\pi/2}\cos^4\theta\,d\theta = \frac{1}{a^5}\int_{\pi/4}^{\pi/2}\left(\frac{1+\cos 2\theta}{2}\right)d\theta = \frac{1}{4a^5}\int_{\pi/2}^{\pi/2}\left(1+\cos 2\theta+\frac{1+4\cos 4\theta}{2}\right)d\theta$

$= \frac{1}{4a^5}\left[\frac{3}{2}\theta+\sin 2\theta+\frac{\sin 8\theta}{8}\right]_{\pi/4}^{\pi/2} = \frac{3\pi-8}{32a^5}.$

66. (b) $I = \int_0^{\infty}\frac{e^{-x}}{e^{-x}+1}dx = -\left[\log(e^{-x}+1)\right]_0^{\infty} = -[\log 1 - \log 2] = \log 2.$

67. (a) $\int\frac{dx}{\sqrt{2-x^2}} = \sin^{-1}\frac{x}{\sqrt{2}}+C$

So $\int_0^1\frac{dx}{\sqrt{2-x^2}} = \left[\sin^{-1}\frac{x}{\sqrt{2}}\right]_0^1 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+C-\sin^{-1}(0)-C = \frac{\pi}{4}-0 = \frac{\pi}{4}.$

68. (b) If $I_1 = \int_1^2\frac{dx}{\sqrt{1+x^2}}, I_2 = \int_1^2\frac{dx}{x} \Rightarrow I_1 = \log\left(\frac{2+\sqrt{5}}{1+\sqrt{2}}\right), I_2 = \log_2 \Rightarrow I_1 < I_2$

69. (c) $I_{10} = \int_1^e 1\cdot(\ln x)^{10}dx = \left[(\ln x)^{10}x\right]_1^e - \int_1^e 10(\ln x)^9\cdot\frac{1}{x}\cdot x\,dx$

$= e - 0 - 10\int_1^e(\ln x)^9 dx = e - 10I_9 + 10I_9 = e$

70. (d) $\int\frac{dx}{1-\sin x} = \int\frac{(1+\sin x)}{1-\sin^2 x}dx = \int\sec^2 x\,dx + \int\tan x\cdot\sec dx = \tan x + \sec x + C$

71. (b) $\frac{d}{dx}(A\ln|\cos x+\sin x-2| + Bx + C)$

$= A\frac{\cos x-\sin x}{\cos x+\sin x-2} + B = \frac{A\cos x - A\sin x + B\cos x + B\sin x - 2B}{\cos x+\sin x-2}$

$\therefore 2 = A + B$ or $-1 = -A + B$; $\lambda = -2B$ $\therefore A = \frac{3}{2}, B = \frac{1}{2}, \lambda = -1.$

72. (b) $\int\frac{x+\sin x}{1+\cos x}dx = \int\frac{x+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx = \int\left(\frac{1}{2}x\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx = x\tan\frac{x}{2}+C$

73. (a) Let $I = \int_0^{\pi/2} \frac{\phi(x)}{\phi(x)+\phi\left(\frac{\pi}{2}-x\right)} dx$, then $I = \int_0^{\pi/2} \frac{\phi\left(\frac{\pi}{2}-x\right)}{\phi\left(\frac{\pi}{2}-x\right)+\phi(x)} dx$

Adding, $2I = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2} = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$.

76. (b) $f(x) = 4\cos^3 x - \cos x - 2\cos x = \cos 3x$

$\therefore \int_0^{\pi/2} f(x)dx = \left[\frac{\sin 3x}{3}\right]_0^{\pi/2} = -\frac{1}{3}$

77. (c) $\int_0^n [x]dx = \int_0^1 0dx + \int_1^2 1dx + \int_2^3 2dx + \ldots.. + \int_{n-1}^n (n-1)dx$

$= 1 + 2 + 3 + \ldots.. + (n-1) = \frac{(n-1)n}{2} = 66 \Rightarrow n(n-1) = 132 \Rightarrow n = 12$

78. (c) Let $I = \int \frac{1}{(x+1)\sqrt{x^2-1}} dx$ Put $x + 1 = \frac{1}{t}$ so that $dx = -\frac{1}{t^2}$

$\therefore I = -\int \frac{dt}{\sqrt{1-2t}} = -\int (1-2t)^{-1/2} dt = -\frac{(1-2t)^{1/2}}{(-2)(1/2)} + C = \sqrt{1-2t} + C = \sqrt{1-\frac{2}{x+1}} + C = \sqrt{\frac{x-1}{x+1}} + C$

79. (a) $\int \frac{1}{\sin^2 x \cos^2 x} dx$

$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\right) dx = \int \sec^2 x \, dx + \int \operatorname{cosec}^2 x \, dx = \tan x - \cot x + C$

82. (a) We have, $\int_0^a \frac{1}{1+4x^2} dx = \frac{\pi}{8} \Rightarrow \frac{1}{4}\int_0^a \frac{1}{\left(\frac{1}{4}+x^2\right)} dx = \frac{\pi}{8}$

$\Rightarrow \int_0^a \frac{1}{\left[\left(\frac{1}{2}\right)^2 + x^2\right]} dx = \frac{\pi}{2} \Rightarrow \frac{1}{1/2}\left[\tan^{-1}\frac{x}{1/2}\right]_0^a = \frac{\pi}{2}$

$\Rightarrow 2[\tan^{-1} 2a - \tan^{-1} 0] = \frac{\pi}{2} \Rightarrow \tan^{-1} 2a = \frac{\pi}{4} \Rightarrow 2a = \tan\frac{\pi}{4} \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$.

83. (b) Let $I = \int_0^1 \frac{dx}{e^x + e^{-x}} = \int_0^1 \frac{dx}{e^x + \frac{1}{e^x}} = \int_0^1 \frac{e^x dx}{e^{2x}+1}$

Put $e^x = t \Rightarrow e^x \, dx = dt$

Changing the limit, we have when $x = 0$; $t = e^0 = 1$; when $x = 1$ $\therefore t = e^1 = e$

$\therefore I = \int_1^e \frac{dt}{t^2+1} = \left[\tan^{-1} t\right]_1^e = \left(\tan^{-1} e - \frac{\pi}{4}\right)$.

84. (d) Let $x^{10} + 10^x = t$

Differentiating both sides w.r. to x we get

$(10x^{10-1} + 10^x \log 10)dx = dt \Rightarrow (10x^9 + 10^x \log 10)dx = dt$

$\therefore \int \frac{1}{t}\, dt = \log|t| + C \Rightarrow \log|10^x + x^{10}| + C$

85. (b) We have : $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$

Put $xe^x = t \Rightarrow (e^x + e^x \cdot x)dx = dt \Rightarrow e^x|1 + x|dx = dt$

$\Rightarrow \int \sec^2 t\,dt = \tan t + C = \tan(xe^x) + C$

86. (c) We have, $\int \frac{dx}{x^2+2x+2} = \int \frac{1}{(x+1)^2+1^2}dx = \tan^{-1}(x+1) + C$

87. (b) $\int \frac{dx}{\sqrt{9x-4x^2}} = \int \frac{dx}{\sqrt{4\left[\left(\frac{9}{8}\right)^2 - \left(x-\frac{9}{8}\right)^2\right]}}$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{\left(\frac{9}{8}\right)^2 - \left(x-\frac{9}{8}\right)^2}} = \frac{1}{2}\sin^{-1}\left[\frac{x-\frac{9}{8}}{\frac{9}{8}}\right] + C = \frac{1}{2}\sin^{-1}\frac{8x-9}{2} + C$$

88. (d) We have, $\int \sqrt{(x^2-8x+7)}dx = \int \sqrt{(x-4)^2-9}dx = \frac{x-4}{2}\sqrt{(x-4)^2-(3)^2} - \frac{(3)^2}{2}\log\left|x-4+\sqrt{(x-4)^2-(3)^2}\right| + C$

$$= \frac{x-4}{2}\sqrt{x^2-8x+7} - \frac{9}{2}\log\left|x-4+\sqrt{x^2-8x+7}\right| + C$$

89. (a) We have, $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx = \int_{1/3}^{1} \frac{(x)^{1/3}\left(\frac{1}{x^2}-1\right)^{1/3}}{x^4}dx = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2}-1\right)^{1/3}}{x^3}dx$

Let $t = \frac{1}{x^2} - 1 \Rightarrow dt = \frac{-2}{x^3}dx \quad \therefore \frac{1}{2}\int_8^0 t^{1/3}dt = \frac{-1}{2}\left[\frac{t^{4/3}}{\frac{4}{3}}\right]_8^0 = \frac{1}{2}\left(\frac{3}{4}\right)(8)^{4/3} = 6.$

90. (c) Let $I = \int_0^{\pi/2} \log\left[\frac{4+3\sin x}{4+3\cos x}\right]dx$ ...(i)

$I = \int_0^{\pi/2} \log\left[\frac{4+3\cos x}{4+3\sin x}\right]dx$ ...(ii)

Add (i) and (ii) we get $\Rightarrow 2I = \int_0^{\pi/2} \log\left[\frac{4+3\sin x}{4+3\cos x} \times \frac{4+3\cos x}{4+3\sin x}\right]dx \Rightarrow 2I = \int_0^{\pi/2} \log 1\, dx = 0 \Rightarrow I = 0$

91. (c) $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)dx = \int_0^{\pi/2} 0dx + \int_0^{\pi/2} 0dx + \int_0^{\pi/2} 0dx + 2\int_0^{\pi/2} dx$

Using $\int_{-a}^{a} f(x)dx = \begin{cases} 0 & , \text{ if } f(-x) = -f(x) \\ 2\int_0^a f(x)dx & , \text{ if } f(-x) = f(x) \end{cases} = 2\int_0^{\pi/2} dx = 2[x]_0^{\pi/2} = \pi.$

93. (d) Let $I = \int_a^b x f(x)dx$ ...(i)

$I = \int_a^b (a+b-x) f(a+b-x)dx$ Given $f(a+b-x) = f(x)$ Then, $I = \int_a^b (a+b-x) f(x)dx$ ...(ii)

Add eqn (i) and (ii) we get $2I = (a+b)\int_a^b f(x)dx \Rightarrow I = \frac{a+b}{2}\int_a^b f(x)dx$.

94. (a) $I = \int \frac{a^x}{\sqrt{1-a^{2x}}}dx$, Put $a^x = t$ So that $a^x \log_e a\, dx = dt$ or $dx = \frac{dt}{a^x \log_e a}$

$\therefore I = \frac{1}{\log_e a}\int \frac{dt}{\sqrt{1^2 - t^2}} = \frac{1}{\log_e a}\sin^{-1}(t) + C = \frac{1}{\log_e a}\sin^{-1}(a^x) + C$

95. (a) $I = \int \frac{x^2}{(x\sin x + \cos x)^2}dx = \int x\cdot\sec x\left(\frac{x\cos x}{(x\sin x+\cos x)^2}\right)dx = \frac{-x\sec x}{x\sin x + \cos x} + \tan x + C$

98. (a) $\int x^{-2/3}\cdot x^{-13/6}(x^{-1/2}+1)^{-13/3}dx = \int x^{-3/2}(x^{-1/2}+1)^{-13/3}dx$

Let $(x^{-1/2}+1) = t;\ -\frac{1}{2}x^{-3/2}\,dx = dt$

Integral will becomes $-2\int t^{-13/3}\,dt = -2\frac{t^{-10/3}}{-10/3} + C = \frac{3}{5}(x^{-1/2}+1)^{-10/3} + C$

99. (a) $\int \frac{\log(x+1)-\log x}{x(x+1)}dx = -\int (\log(x+1) - \log x)\cdot\frac{1}{-x(x+1)}dx$

$= \frac{-1}{2}\left[\log\left(\frac{x+1}{x}\right)\right]^2 + C$ $\left[\because \frac{d}{dx}[\log(x+1) - \log x = \frac{1}{x+1} - \frac{1}{x} = -\frac{1}{(x+1)x}\right]$

100. (a) Integral of the numerator $= \frac{x^{3/2}}{3/2}$ Put $x^{3/2} = t$, we get

$I = \frac{2}{3}\int \frac{dt}{\sqrt{a^3 - t^2}} = \frac{2}{3}\sin^{-1}\frac{t}{a^{3/2}} + C = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + C$

101. (d) $\int \frac{(x^8+4+4x^4)-4x^4}{(x^4-2x^2+2)}dx = \int \frac{(x^4+2)^2-(2x^2)^2}{(x^4-2x^2+2)}dx = \frac{x^5}{5} + \frac{2x^3}{3} + 2x + C$

102. (i) (b) Since f and g are continuous for $x \geq 0$ if follows that $\int_0^a f(x)dx, \int_0^a g(x)dx$ and $\int_0^a f(x)g(x)dx$ exits.

Now $\int_0^a f(x)g(x)dx = \int_0^a f(a-x)g(a-x)dx = \int_0^a f(x)[2-g(x)]dx$ $[\because f(a-x) = f(x)$ and $g(x) + g(a-x) = 2]$

Therefore, $2\int_0^a f(x)g(x)dx = 2\int_0^a f(x)dx \Rightarrow \int_0^a f(x)g(x)dx = \int_0^a f(x)dx$

(ii) (a) Let $I = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}dx$

It exist because $\dfrac{x}{1+\sin x}$ is continuous on $\left[\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right]$.

$$\text{Now I} = \int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}\,dx = \int_{\pi/4}^{3\pi/4} \frac{\frac{\pi}{4}+\frac{3\pi}{4}-x}{1+\sin\left(\frac{\pi}{4}+\frac{3\pi}{4}-x\right)} = \int_{\pi/4}^{3\pi/4} \frac{\pi - x}{1+\sin x}\,dx = \pi \int_{\pi/4}^{3\pi/4} \frac{1}{1+\sin x}\,dx - \text{I}$$

$$\Rightarrow 2\text{I} = \pi \int_{\pi/4}^{3\pi/4} \frac{1-\sin x}{\cos^2 x}\,dx \Rightarrow \text{I} = \frac{\pi}{2} \int_{\pi/4}^{3\pi/4} (\sec^2 x - \sec x \tan x)\,dx$$

$$= \frac{\pi}{2}\left\{[\tan x]_{\pi/4}^{3\pi/4} - [\sec x]_{\pi/4}^{3\pi/4}\right\} = \frac{\pi}{2}\left[(-1-1)-(-\sqrt{2}-\sqrt{2}\right] = \pi(\sqrt{2}-1)$$

(iii) (c) Let $\text{I} = \int_a^b x f(x)\,dx = \int_a^b (a+b-x) f(x)\,dx$ $\qquad [\because f(a+b-x) = f(x)]$

$2\text{I} = \int_a^b (a+b) f(x)\,dx = (a+b)\int_a^b f(x)\,dx \Rightarrow \text{I} = \dfrac{a+b}{2}\int_a^b f(x)\,dx$ $\qquad [\because f(a+b-x) = f(x)]$

(iv) (d) Let $f(x) = e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)\right)$

Since, f is continuous for all real x, it follows that $\int_0^a f(x)\,dx$ exists. Also $f(\pi - x)$ $\left(2\sin\left(\frac{1}{2}\cos(\pi - x)\right)\right)$

Therefore, $\int_0^\pi f(x)\,dx = \int_0^\pi f(\pi - x)\,dx = -\int_0^\pi f(x)\,dx \Rightarrow \int_0^\pi f(x)\,dx = 0$

(v) (a) Let $\text{I} = \int_0^1 \dfrac{x^4(1-x)^4}{1+x^2}\,dx$

$$= \int_0^1 \frac{1}{1+x^2}\left(x^8 - 4x^7 + 6x^6 - 4x^5 + x^4\right)dx = \frac{1}{7} - \frac{4}{6} + 1 - \frac{4}{3} + 4 - 4\tan^{-1}1 = \frac{22}{7} - \pi$$

103. (i) (b) $\int x^3 \log x\,dx = (\log x)\dfrac{x^4}{4} - \dfrac{1}{4}\int x^3\,dx = \dfrac{x^4}{4}\log x - \dfrac{x^4}{16} + \text{C}$

(ii) (a) $\int x^2 \sin x\,dx = x^2(-\cos x) - \int(-\cos x)2x\,dx = -x^2\cos x + 2x\sin x + 2\cos x + \text{C}$

(iii) (a) $\int \sin x \log(\cos x)\,dx = \log(\cos x)\,(-\cos x) - \int(-\cos x)(-\tan x)\,dx = -\cos x\,\log(\cos x) + \cos x + \text{C}.$

(iv) (a) $\text{I} = \int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx \Rightarrow 2\text{I} = \sec x\tan x + \log(\sec x + \tan x) + \text{C}$

(v) (a) $\text{I} = \int e^x \sin x = e^x \sin x - \int e^x \cos x\,dx = e^x\sin x - \left[e^x\cos x - \int e^x(-\sin x)\,dx\right] = e^x\sin x - e^x\cos x - \text{I} + \text{C}$

$\Rightarrow 2\text{I} = e^x(\sin x - \cos x) + \text{C} \Rightarrow \text{I} = \dfrac{1}{2}e^x(\sin x - \cos x) + \text{C}$