Chapter - 13 PROBABILITY

STUDY NOTES

- **Sample space :** The set of all the possible outcomes of an experiment is called a sample space.
- **Events :** The outcomes of an experiment are called events. An event is a subset of the sample space.
- **Probability of an event :** Probability of an event E, denoted by P(E) is defined as

$$P(E) = \frac{\text{No. of outcomes favourable to E}}{\text{Total no. of outcomes}}$$

- **Mutually exclusive events :** If two or more events have no outcome in common, i.e., they cannot occur simultaneously, then they are said to be mutually exclusive events.
- The probability of any event A lies between 0 and 1, i.e., $0 \le P(A) \le 1$.
- $P(\text{not A}) = P(A') = 1 - P(A)$
- **Addition Law of Probability**
  - (i) If A and B are two events, then the probability that either A or B occurs is given by

    $P(\text{A or B}) = P(A \cup B) = P(A) + P(B) - P(\text{A and B}) = P(A) + P(B) - (P(A \cap B)$
  - (ii) If A and B are mutually exclusive events, then the joint occurrence of A and B is not possible, i.e., $P(\text{A and B}) = 0$ or $P(A \cap B) = 0$

    Then the addition law becomes, $P(\text{A or B}) = P(A) + P(B)$
- **Conditional Probability :** Let A and B be two events associated with the same random experiment. Then the probability of occurrence of A under the condition that B has already occurred over $P(B) \neq 0$, is called the conditional probability of A and is denoted by P(A/B)

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0.$$

- **Multiplication Law of Probability :**
  - (i) When A and B are dependent events, then the probability of simultaneous occurrence of A and B is given by

    $P(\text{A and B}) = P(A \cap B) = P(A) \cdot P(B/A)$
  - (ii) When A and B are independent events than,

    $P(B/A) = P(B)$ and $P(A/B) = P(A)$

    Thus, $P(A \cap B) = P(A) \cdot P(B)$
- **The Law of Total Probability :** If $E_1, E_2, E_3, ..., E_n$ are mutually exclusive and exhaustive events associated with a sample space S of a random experiment and A is any event associated with S, then

$P(A) = P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) + ... + P(E_n)\, P(A/E_n)$
- **Baye's theorem :** If $E_1, E_2, E_3, ..., E_n$ are n mutually exclusive and exhaustive events with non-zero probabilities and A is any of the n events, then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^{n} P(E_i)P(A/E_i)}, \; i = 1, 2, 3, ..., n.$$

- **Use of Binomial Coefficients in Finding Probability :**
  - (i) The total number of ways in which 4 different cards can be drawn from a pack of 52 cards is ${}^{52}C_4$.
  - (ii) The number of ways in which 4 spades can be drawn out of a pack of 52 cards is ${}^{13}C_4$.
  - (iii) The number of ways in which a king and a queen can be drawn from a pack of 52 cards is ${}^4C_1 \times {}^4C_1$. (Fundamental Principle of Multiplication)
  - (iv) The number of ways in which 2 kings and 3 queens can be drawn from a pack of 52 cards is ${}^4C_2 \times {}^4C_3$.
  - (v) The number of ways in which 2 kings or 2 queens can be drawn from a pack of 52 cards is ${}^4C_2 + {}^4C_2$.

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. Let A and B be two events. If P(A) = 0.2, P(B) = 0.4, P(A∪B) = 0.6, then P(A/B) is equal to

(a) 0.3 (b) 0 (c) 0.5 (d) 0.8

2. A and B are two events of a random experiment. If $P(A \cup B) = \frac{7}{8}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{5}{8}$, then $P(A \cap \overline{B})$ is equal to :

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$

3. A card is drawn at random from a well-shuffled deck of cards. The probability that the cards drawn is a king or a red card is:

(a) $\frac{7}{13}$ (b) $\frac{14}{36}$ (c) $\frac{7}{18}$ (d) $\frac{26}{52}$

4. We wish to choose one child out of 2 boys and 3 girls. A coin is tossed . If it comes up heads, a boy is chosen otherwise a girl is chosen. The number of sample space is :

(a) 4 (b) 5 (c) 6 (d) 7

5. The first 12 letters of English alphabet are written in a row at random. The probability that there are exactly four letters in between A and B is :

(a) $\frac{1}{22}$ (b) $\frac{15}{66}$ (c) $\frac{7}{66}$ (d) $\frac{1}{11}$

6. Two dice of different colours are thrown at a time. The probability that the sum of the faces appeared is either 7 or 11 is :

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{4}{3}$ (d) $\frac{9}{36}$

7. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, then the probability of its being defective is (if it is red) :

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{2}{7}$ (d) $\frac{3}{8}$

8. A problem in Mathematics is given to 3 students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$. The probability that the problem solved is :

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{3}{7}$ (d) $\frac{3}{5}$

9. A card is drawn at random from a well-shuffled deck of cards. Then the probability that a club or a diamond card drawn is :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

10. Let X be a discrete random variable. The probability distribution of X is given below :

X	30	10	–10
P(X)	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{1}{2}$

Then E(X) is equal to :

(a) 4 (b) 5 (c) 3 (d) –5

11. 20 cards are numbered from 1 to 20. One card is drawn at random. What is the probability that the number on the card will be not a multiple of 6?

(a) $\frac{5}{13}$ (b) $\frac{17}{20}$ (c) $\frac{13}{20}$ (d) $\frac{5}{7}$

12. A boy remembers all but the last digit of his friend's mobile number. He randomly chooses a digit from 0 to 9 (including 0 and 9). If he attempts two times, the probability that he reaches his at least once is :

(a) 0.2 (b) 0.3 (c) 0.02 (d) 0.04

13. Three faces of a fair die are yellow, two faces are green and one face is blue. If the die is tossed three times the probability that the colours yellow, green and blue appear in the first, second and third toss, respectively is :

(a) $\frac{11}{12}$ (b) $\frac{1}{12}$ (c) $\frac{1}{36}$ (d) $\frac{5}{36}$

14. The probability that India winning a cricket T-20 match against England is $\frac{1}{2}$. In a 5-match series, India surely wins the third match is :

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

15. A bag contains 4 balls of unknown colours. A ball is drawn at random from it and is found to be white. The probability that all the balls in the bag are white is :

(a) $\frac{2}{5}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

16. In a college, 70% students pass in Physics, 75% pass in Mathematics and 10% students fail in both. One student is chosen at random. What is the probability that he passes in Physics and Mathematics?

(a) $\frac{7}{20}$ (b) $\frac{11}{20}$ (c) $\frac{11}{30}$ (d) $\frac{3}{4}$

17. A, B and C are three athletes running in the race. If the probability of A winning is twice as likely to win as probability of B and that of B is as likely to win as of C, then the probability of A's win is :

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

18. If three vertices of a regular hexagon are chosen at random, then the chance that they form an equilateral triangle is :

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{1}{10}$ (d) $\frac{1}{2}$

19. Five dice are tossed. The probability that the five numbers shown will be different is :

(a) $\frac{5}{18}$ (b) $\frac{5}{27}$ (c) $\frac{5}{54}$ (d) $\frac{5}{81}$

20. If m rupee coins and n ten paise coins are placed in a line, then the probability that the extreme coins are ten paise coins is :

(a) $\frac{{}^{m+n}C_n}{n^m}$ (b) $\frac{n(n-1)}{(m+n)(m+n-1)}$ (c) $\frac{{}^{m+n}P_m}{m^n}$ (d) $\frac{{}^{m+n}P_n}{n^m}$

21. Two numbers a and b are chosen at random from the set of first 30 natural numbers. The probability that $a^2 - b^2$ is divisible by 3 is :

(a) $\frac{43}{87}$ (b) $\frac{47}{87}$ (c) $\frac{12}{87}$ (d) $\frac{15}{87}$

22. There are 5 volumes of Mathematics among 25 books. They are arranged on a shelf in a random order. The probability that the volumes of Mathematics stand in increasing order from left to right (the volumes are not necessarily kept side by side) is :

(a) $\frac{1}{5!}$ (b) $\frac{50!}{55!}$ (c) $\frac{1}{50^5}$ (d) $\frac{5!}{5^5}$

23. 3 integers are chosen at random from the set of first 20 natural numbers. The chance that their product is a multiple of 3, is :

(a) $\frac{1}{17}$ (b) $\frac{234}{385}$ (c) $\frac{194}{285}$ (d) $\frac{131}{191}$

24. Three coins are tossed all together. The probability of getting at least two heads is :

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$ (d) $\frac{2}{3}$

25. A dice is thrown two times. If getting the odd number is considered as success, then the probability of two successes is :

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

26. An anti air craft gun take a maximum of four shots at an enemy plane moving away from it. The probability of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2 and 0.1 respectively. The probability that the gun hits the plane is :

(a) 0.2576 (b) 0.2176 (c) 0.1676 (d) 0.6976

27. Six boys and six girls sit in a row. The probability that the boys and girls sit alternatively is :

(a) $\frac{1}{462}$ (b) $\frac{1}{2}$ (c) $\frac{1}{924}$ (d) $\frac{1}{234}$

28. In four schools B_1, B_2, B_3, B_4 the percentage of girls students is 12, 20, 13, 17 respectively. From a school selected at random one student is picked up at random and it is found that the student is a girl. The probability that the school selected is B_2 is :

(a) $\frac{6}{31}$ (b) $\frac{10}{31}$ (c) $\frac{13}{62}$ (d) $\frac{17}{62}$

29. At a telephone enquiry system the number of phone calls regarding relevant enquiry follow poisson distribution with an average of 5 phone calls during 10-minute time intervals. The probability that there is at the most are phone call during a 10-minute time period is :

(a) $\frac{6}{5^e}$ (b) $\frac{5}{6}$ (c) $\frac{6}{55}$ (d) $\frac{6}{e^5}$

30. A rifle man is firing at a distant target and has only 10% chance of hitting it. The minimum number of rounds he must fire in order to have 50% chance of hitting it at least once is :

(a) 7 (b) 8 (c) 9 (d) 6

31. The probability of drawing a jack or an ace from a pack of playing cards is :

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{2}{13}$ (d) $\frac{1}{8}$

32. A black die and a white die are rolled. Then the probability that the number shown by the black die will be more than twice that shown by the white die is :

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{8}$

33. If A and B are mutually exclusive events and if $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{13}{21}$, then P(A) is equal to :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{4}{7}$ (d) $\frac{5}{7}$

34. A die is loaded such that the probability of throwing the number I is proportional to its reciprocal. The probability that 3 appears in a single throw is :

(a) $\frac{3}{11}$ (b) $\frac{3}{22}$ (c) $\frac{9}{22}$ (d) $\frac{20}{147}$

35. A bag contains 5 brown and 4 white socks. A man pulls out 2 socks. Then the probability that they are of the same colour is :

(a) $\frac{4}{9}$ (b) $\frac{2}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{9}$

36. A class contains 20 boys and 20 girls of which half the boys and half the girls have cat eyes. If one student is selected from the class, the probability that either the student is a boy or has cat eyes is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{2}{3}$

37. A positive integer is selected at random from the first 200 natural numbers. The probability that it is divisible by 4 or 5 is :

(a) $\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{3}{5}$ (d) $\frac{3}{10}$

38. Bag A contains 5 red and 7 white balls and bag B contains 3 red and 12 white balls. One of the bag is selected at random and one ball is drawn from it. The probability that the drawn ball is red is :

(a) $\frac{37}{120}$ (b) $\frac{83}{120}$ (c) $\frac{63}{120}$ (d) $\frac{17}{120}$

39. A letter is two come from either LONDON or CLIFTON. The postal mark on the letter legibly shows consecutive letters "ON". The probability that the letter has come from LONDON is :

(a) $\frac{12}{17}$ (b) $\frac{13}{17}$ (c) $\frac{5}{17}$ (d) $\frac{4}{17}$

40. A person goes to office either by car, scooter, bus or train whose probability are, respectively, $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}$ and $\frac{1}{7}$. The probability that he reaches office on time, if he takes car, scooter bus or train is $\frac{7}{9}, \frac{8}{9}, \frac{5}{9}$ and $\frac{8}{9}$, respectively. Given that he reached office in time the probability that he travelled by car is :

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{6}{7}$ (d) $\frac{5}{7}$

41. A lot consists of 12 good pencils, 6 with minor defects and 2 with major defects. A pencil is choosen at random. The probability that this pencil is not defective is :

(a) $\frac{3}{5}$ (b) $\frac{3}{10}$ (c) $\frac{4}{5}$ (d) $\frac{1}{2}$

42. One coin is thrown 100 times. The probability of coming tail in odd number is :

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

43. The probability that a contractor will get a plumbing contract is $\frac{2}{3}$ and an electric contract is $\frac{4}{9}$. If the probability of getting at least one contract is $\frac{4}{5}$, then the probability that he will get both the contracts is :

(a) $\frac{4}{45}$ (b) $\frac{7}{45}$ (c) $\frac{7}{23}$ (d) $\frac{14}{45}$

44. A card is drawn from a well shuffled pack of playing cards. The probability that it is either a spade or an ace or both is :

(a) $\frac{4}{13}$ (b) $\frac{4}{17}$ (c) $\frac{3}{13}$ (d) $\frac{3}{18}$

45. Three ships A, B and C sail from England to India. If the ratio of their arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively then the probability of all the ships for arriving safely is :

(a) $\frac{18}{595}$ (b) $\frac{6}{17}$ (c) $\frac{3}{10}$ (d) $\frac{2}{7}$

46. Probability that a student will succeed in entrance test is 0.2 and that he will succeed in other entrance test is 0.5. If the probability that he will be successful at both the places is 0.3, than the probability that he does not succeed at both the places is :

(a) 0.4 (b) 0.3 (c) 0.2 (d) 0.6

47. If the probability of a horse A winning a race is $\frac{1}{4}$ and the probability of a horse B winning the same race is $\frac{1}{5}$, then the probability that either of them will win the race is :

(a) $\frac{1}{20}$ (b) $\frac{9}{20}$ (c) $\frac{11}{20}$ (d) $\frac{19}{20}$

48. If A and B are independent events of a random experiment such that $P(A \cap B) = \frac{1}{6}$ and $P(\overline{A} \cap \overline{B}) = \frac{1}{3}$, then P(A) is equal to (Here, $\overline{E}$ is the complement of the event E) :

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ or $\frac{1}{2}$ (c) $\frac{1}{2}$ or $\frac{1}{3}$ (d) $\frac{2}{3}$

49. One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is seated adjacent to his wife is :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

50. In an entrance test there are multiple choice questions. There are four possible answers to each questions of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing is:

(a) $\frac{37}{40}$ (b) $\frac{1}{37}$ (c) $\frac{36}{37}$ (d) $\frac{1}{9}$

51. The probability that in 10 throws of a fair a score which is a multiple of 3 will be obtained in at least 8 of the throws is :

(a) $\frac{199}{3^{10}}$ (b) $\frac{201}{3^{11}}$ (c) $\frac{201}{3^{10}}$ (d) $\frac{201}{3^9}$

52. A discrete random variable X has the probability distribution given as below :

X	0.5	1	1.5	2
P(X)	k	k^2	$2k^2$	k

Then the value of k is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

53. Ten coins are tossed. The probability of getting at least 8 heads is :

(a) $\frac{5}{128}$ (b) $\frac{7}{128}$ (c) $\frac{7}{125}$ (d) $\frac{8}{125}$

54. Suppose that 6% of the people with blood group O are left handed and 10% of those with other blood groups are left handed 30% of the people have blood group O. If a left handed person is selected at random, what is the probability that he will have blood group O?

(a) $\frac{7}{42}$ (b) $\frac{9}{42}$ (c) $\frac{7}{44}$ (d) $\frac{9}{44}$

55. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P(A'/B') \cdot P(B'/A')$ is equal to :

(a) $\frac{5}{6}$ (b) $\frac{5}{7}$ (c) $\frac{25}{42}$ (d) 1

56. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability that both cards are queens is :

(a) $\frac{1}{13} \times \frac{1}{13}$ (b) $\frac{1}{13} + \frac{1}{13}$ (c) $\frac{1}{13} \times \frac{1}{17}$ (d) $\frac{1}{13} \times \frac{4}{51}$

57. A die is thrown. Let A be the event that the number obtained is greater than 3. Let B be the event that the number obtained is less than 5. Then, $P(A \cup B)$ is :

(a) 0 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$

58. Let A, B and C be three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.8$, $P(A \cap B) = 0.08$, $P(A \cap C) = 0.28$, $P(A \cap B \cap C) = 0.09$. If $P(A \cup B \cup C) \geq 0.75$, then $P(B \cap C)$ satisfies :

(a) $P(B \cap C) \leq 0.48$ (b) $0.23 \leq P(B \cap C) \leq 0.48$ (c) $0.23 \leq P(B \cap C) \geq 0.48$ (d) $P(B \cap C) \leq 0.23$

59. One ticket is selected at random from 50 tickets numbered 00, 01, 02, ..., 49. Then the probability that the sum of the digits on the selected ticket is 8, gives that the product of these digits is zero, equals :

(a) $\frac{1}{7}$ (b) $\frac{1}{14}$ (c) $\frac{5}{14}$ (d) $\frac{1}{50}$

60. In an entrance examination there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to the question, then the probability that he was guessing is :

(a) $\frac{1}{9}$ (b) $\frac{1}{37}$ (c) $\frac{36}{37}$ (d) $\frac{3}{37}$

INPUT TEXT BASED MCQ's

61. A class of 90 students of class 11th had a sports competition. For this, they had to be divided into weight and gender categories. $\frac{4}{9}$ of the students are girls and the ratio of students above and below the 50 kg mark is 2:1. $\frac{3}{4}$ of the girls weigh more than 50 kg.

(i) What is the probability that a boy picked at random, weighs less that 50 kg ?

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{2}{7}$ (d) $\frac{2}{9}$

(ii) What is the probability that a girl will win the competition in the above 50 kg category?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

(iii) If 15 boys did not qualify to participate in the event, then what is the probability that a boy will win ?

(a) $\frac{3}{10}$ (b) $\frac{4}{10}$ (c) $\frac{6}{10}$ (d) $\frac{7}{10}$

(iv) If $\frac{1}{2}$ of the girls above and $\frac{1}{5}$ of the girls below the 50 kg mark did not qualify to participate, what is the probability that a girl will win ?

(a) $\frac{21}{40}$ (b) $\frac{22}{40}$ (c) $\frac{23}{40}$ (d) $\frac{24}{40}$

(v) What is the probability that from the group of students a boy weighing more than 50 kg will win?

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

62. Urns p, q and r respectively, contain 2 white and 3 black balls, 1 white and 4 black balls, 4 white and 1 black balls. The probabilities of choosing the urns are, respectively $\frac{2}{5}$, $\frac{2}{5}$ and $\frac{1}{5}$. One of the urns is chosen at random and a ball is drawn from it.

Answer the following questions :

(i) Probability that the drawn ball is black is :

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{10}{12}$ (d) $\frac{12}{25}$

(ii) Probability that the drawn ball is white is :

(a) $\frac{3}{5}$ (b) $\frac{12}{25}$ (c) $\frac{2}{5}$ (d) $\frac{7}{12}$

(iii) If the drawn ball is black, then the probability that it is from urn p is :

(a) $\frac{7}{5}$ (b) $\frac{12}{5}$ (c) $\frac{2}{5}$ (d) $\frac{5}{12}$

(iv) If the drawn ball is white, then the probability that it is from urn r is :

(a) $\frac{13}{25}$ (b) $\frac{12}{25}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

(v) If the drawn ball is white, then the probability that it is from urn q is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{5}{12}$

63. Mohit, a shopkeeper sells three types of seeds - A_1, A_2 and A_3. He sells them in a packet containing the 3 seeds in the ratio 5 : 3 : 2. He undertakes several experiments over the monsoon season and finds out that the germination rates of the 3 seeds are 40%, 70% and 20% respectively. The probability that:

(i) A seed does not germinate given it is of type A_2 is :

(a) $\frac{7}{10}$ (b) $\frac{3}{5}$ (c) $\frac{6}{10}$ (d) $\frac{3}{10}$

(ii) A randomly chosen seed germinates is :

(a) $\frac{45}{100}$ (b) $\frac{13}{100}$ (c) $\frac{55}{100}$ (d) $\frac{50}{100}$

(iii) If it germinates, it is of type A_3 is :

(a) $\frac{14}{100}$ (b) $\frac{14}{45}$ (c) $\frac{28}{45}$ (d) $\frac{40}{100}$

(iv) If it does not germinate, it is not of type A_3 is :

(a) $\frac{16}{55}$ (b) $\frac{16}{110}$ (c) $\frac{39}{55}$ (d) $\frac{16}{45}$

(v) Seed of type A_1 or A_2 will germinate is :

(a) $\frac{18}{100}$ (b) $\frac{28}{100}$ (c) $\frac{40}{100}$ (d) $\frac{82}{100}$

ANSWERS

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (c)	**14.** (c)	**15.** (a)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (b)
21. (b)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (d)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (d)	**30.** (a)
31. (c)	**32.** (b)	**33.** (b)	**34.** (d)	**35.** (a)	**36.** (b)	**37.** (d)	**38.** (a)	**39.** (a)	**40.** (a)
41. (a)	**42.** (a)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (d)	**47.** (b)	**48.** (b)	**49.** (c)	**50.** (b)
51. (c)	**52.** (a)	**53.** (b)	**54.** (d)	**55.** (c)	**56.** (a)	**57.** (b)	**58.** (c)	**59.** (b)	**60.** (b)

61. (i) (b) (ii) (a) (iii) (d) (iv) (c) (v) (b)

62. (i) (b) (ii) (c) (iii) (c) (iv) (d) (v) (a)

63. (i) (d) (ii) (a) (iii) (b) (iv) (d) (v) (d)

Hints to Some Selected Questions

1. (b) We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. But $P(A \cap B) = 0$

Thus, $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = 0$

2. (c) We have : $P(A \cap \overline{B}) = P(A - B) = P(A) - P(A \cap B)$

$\Rightarrow P(A \cap \overline{B}) = \left(1 - \dfrac{5}{8}\right) - \dfrac{1}{4} = \dfrac{3}{8} - \dfrac{1}{4} = \dfrac{1}{8}$.

3. (a) P(a king or red card) = P(a king) + P(a red card) – P(king of red card)

$P(\text{king or red card}) = \dfrac{4}{52} + \dfrac{26}{52} - \dfrac{2}{52} = \dfrac{28}{52} = \dfrac{7}{13}$.

4. (b) Sample space = $\{HB_1, HB_2, TG_1, TG_2, TG_3\}$

5. (c) Required probability = $\dfrac{14}{11 \times 12} = \dfrac{7}{66}$.

7. (b) Given, $P(A) = 10\% = \dfrac{10}{100} = \dfrac{1}{10}$ and $P(A \cap B) = 2\% = \dfrac{2}{100} = \dfrac{1}{50}$.

$\therefore P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{\frac{1}{50}}{\frac{1}{10}} = \dfrac{1}{5}$.

8. (b) Here, $P(A) = \dfrac{1}{2}$, $P(B) = \dfrac{1}{3}$ and $P(C) = \dfrac{1}{4}$

And A, B and C are independent events

$\therefore$ Required probability = $P(A \cup B \cup C) = 1 - P(\overline{A})\, P(\overline{B})\, P(\overline{C}) = 1 - \dfrac{1}{2} \times \dfrac{2}{3} \times \dfrac{3}{4} = \dfrac{3}{4}$.

10. (a) $E(X) = 30 \times \dfrac{1}{5} + 10 \times \dfrac{3}{10} - 10 \times \dfrac{1}{2} = 4$

11. (b) Numbers that are multiples of 6 are 6, 12, 18, i.e., 3 numbers.

There are 20 – 3 = 17 numbers that are not multiples of 6.

$\therefore$ P(not a multiples of 6) = $\dfrac{17}{20}$.

12. (a) $P(\overline{E}) = \dfrac{9}{10} \times \dfrac{8}{9} = \dfrac{4}{5}$

$\therefore P(E) = 1 - P(\overline{E}) = 1 - \dfrac{4}{5} = \dfrac{1}{5} = 0.2$.

13. (c) P(yellow) in first toss = $\dfrac{3}{6}$

P(green) in second toss = $\dfrac{2}{6}$

P(blue) in third toss = $\dfrac{1}{6}$

There are all events are independent,

$\therefore$ The required probability = $\dfrac{3}{6} \times \dfrac{2}{6} \times \dfrac{1}{6} = \dfrac{1}{36}$.

14. (c) India winning the third match is independent of the results of the first two matches. Hence, the probability that India surely winning the third match is $\dfrac{1}{2}$.

16. (b) $P(A) = 70\% = \dfrac{7}{10}$, $P(B) = 75\% = \dfrac{3}{4}$ and $P(\overline{A} \cap \overline{B}) = 10\% = \dfrac{1}{10}$

$\Rightarrow P(\overline{A \cup B}) = \frac{1}{10} \Rightarrow 1 - P(A \cup B) = \frac{1}{10} \Rightarrow P(A \cup B) = \frac{9}{10}$

Then, $P(A \cap B)\ \frac{7}{10} + \frac{3}{4} - \frac{9}{10} = \frac{11}{20}$.

17. (c) ATQ, $P(A) = 2P(B)$ and $P(B) = 2P(C)$

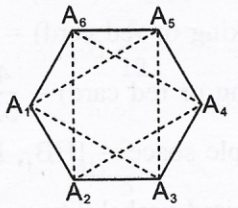
$\therefore P(A) + P(B) + P(C) = 1 \Rightarrow 4P(C) + 2P(C) + P(C) = 1$

$\Rightarrow P(C) = \frac{1}{7}$ so, $P(A) = \frac{4}{7}$.

18. (c) Three vertices can be selected in ${}^6C_3$ ways.

The only equilateral triangles possible are $A_1A_3A_5$ and $A_2A_4A_6$

$P(C) = \frac{2}{{}^6C_3} = \frac{2}{20} = \frac{1}{10}$.



19. (c) Total number of cases $= 6^5$

Number of favourable cases $= 6! = 720$

$\therefore$ Required probability $= \frac{720}{6^5} = \frac{5}{54}$.

21. (b) The total numbers of ways of choosing two numbers out of 1, 2, 3, ... 30 is ${}^{30}C_2 = 435$.

Since, $a^2 - b^2$ is divisible by 3 if either a and b both are divisible by 3 or none of a and b is divisible by 3.

Thus, the favourable number of cases $= {}^{10}C_2 + {}^{20}C_2 = 235$.

Hence, the required probability $= \frac{235}{435} = \frac{47}{87}$.

22. (a) Five places for 5 volumes of Mathematics from 25 places of books can be select in ${}^{25}C_5$ ways. In these places, we are not to permute the 5 volumes since order of these 5 volumes is fixed. Remaining 20 books can be arranged in 20! ways.

$\therefore$ Favourable ways $= {}^{25}C_5 \cdot 20! = \frac{25! \cdot 20!}{5!\ 20!} = \frac{25!}{5!}$

Also total number of ways $= 25!$

$\therefore$ Probability $= \frac{25!}{5!\ 25!} = \frac{1}{5!}$

23. (c) Total number of ways of selecting 3 integers from 20 natural numbers $= {}^{20}C_3 = 1140$

Their product is a multiple of 3 means at least one number is divisible by 3.

The numbers which area divisible by 3 are 3, 6, 9, 12, 15, 18 and the number of ways of selecting at least one of them is ${}^6C_1 \times {}^{14}C_2 + {}^6C_2 \times {}^{14}C_1 + {}^6C_3 = 776$

$\therefore$ Required Probability $= \frac{776}{1140} = \frac{194}{285}$.

24. (a) Required probability $= \left(\frac{1}{2}\right)^3 \cdot {}^3C_2 + \left(\frac{1}{3}\right)^3 \cdot {}^3C_3 = \frac{4}{8} = \frac{1}{2}$

25. (d) Probability of getting odd number $= \frac{3}{6} = \frac{1}{2}$

Hence, required probability $= {}^2C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^0 = \frac{1}{4}$.

26. (d) Let $P_1 = 0.4$, $P_2 = 0.3$, $P_3 = 0.2$ and $P_4 = 0.1$

P(the gun hits the plane) = P(the plane is hit in one)

$= 1 - P$(the plane is hit in none of the shots)

$= 1 - (1 - P_1)(1 - P_2)(1 - P_3)(1 - P_4)$

$= 1 - (1 - 0.4)(1 - 0.3)(1 - 0.2)(1 - 0.1) = 0.6976.$

27. (a) Let n = total number of ways = 12!

and m = favourable number of ways = $2 \times 6! \cdot 6!$

Since the boys and girls can sit alternately in $6! \cdot 6!$ ways if we begin with a boy and similarly they can sit alternately in $6! \cdot 6!$ ways if we begin with a girl.

Hence, required probability = $\frac{m}{n} = \frac{2 \times 6! \cdot 6!}{12!} = \frac{1}{462}$.

28. (b) Favorable number of cases = ${}^{20}C_1 = 20$

Sample space = ${}^{62}C_1 = 62$.

$\therefore$ Required probability = $\frac{20}{62} = \frac{10}{31}$.

31. (c) As there are four jacks and four aces the number of favourable cases = 8

$\therefore$ The required probability (P) = $\frac{8}{52} = \frac{2}{13}$.

33. (b) For mutually exclusive events

$P(A \cup B) = P(A) + P(B) \Rightarrow P(A) = \frac{2}{7}$.

34. (d) $P(I) = \frac{K}{I} \Rightarrow 1 = \sum_{I=1}^{6} P(I) = K \sum_{I=1}^{6} \frac{1}{I} = K \frac{49}{20}$

$\Rightarrow K = \frac{20}{49} \Rightarrow P(3) = \frac{20}{147}$

35. (a) $P(A) = \frac{{}^5C_2}{{}^9C_2} = \frac{5 \times 4}{9 \times 8} = \frac{5}{18}$

$P(B) = \frac{{}^4C_2}{{}^9C_2} = \frac{4 \times 3}{9 \times 8} = \frac{3}{18}$

Since A and B are mutually exclusive events,

So, required probability = $P(A) + P(B) = \frac{5}{18} + \frac{3}{18} = \frac{4}{9}$.

36. (b) Here, $P(A) = \frac{20}{40} = \frac{1}{2}$ and $P(B) = \frac{20}{40} = \frac{1}{2}$

Now, $P(A \cap B) = \frac{10}{40} = \frac{1}{4}$

Therefore, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$.

37. (d) Any integer divisible by 4 or 5 must be divisible by 20.

Number of multiples of 4 lying between 1 and 200 is 50. (including 200)

Number of multiples of 5 is 20.

number of multiples of 20 is 10.

Therefore required probability = $\frac{50+20-10}{200} = \frac{60}{200} = \frac{3}{10}$.

38. (a) Let A and B denotes bag A and bag B, respectively and R denote drawing a red ball.

Then, $R = (A \cup B) \cap R = (A \cap R) \cup (B \cap R)$

Therefore, $P(R) = P(A \cap R) + P(B \cap R) = P(A)\ P(R/A) + P(B)\ P(R/B)$

$= \frac{1}{2} \times \frac{5}{12} + \frac{1}{2} \times \frac{3}{15} = \frac{25+12}{2 \times 60} = \frac{37}{120}$

40. (a) $P(E_1) = \frac{1}{7}$, $P(E_2) = \frac{3}{7}$, $P(E_3) = \frac{2}{7}$, $P(E_4) = \frac{1}{7}$

$P(E/E_1) = \frac{7}{9}$, $P(E/E_2) = \frac{8}{9}$, $P(E/E_3) = \frac{5}{9}$, $P(E/E_4) = \frac{8}{9}$

By Bayes' theorem

$$P(E_1/E) = \frac{P(E_1)\,P(E/E_1)}{\sum_{i=1}^{4} P(E_i)P(E/E_i)} = \frac{\frac{1}{7}\times\frac{7}{9}}{\frac{1}{7}\times\frac{7}{9}+\frac{3}{7}\times\frac{8}{9}+\frac{2}{7}\times\frac{5}{9}+\frac{1}{7}\times\frac{8}{9}} = \frac{7}{49} = \frac{1}{7}.$$

41. (a) Required probability $= \frac{^{12}C_1}{^{20}C_1} = \frac{3}{5}$.

43. (d) Let A be the events of getting contractor plumbing contract and B be the events of getting contractor electric contract.

$P(A) = \frac{2}{3}$, $P(B) = \frac{4}{9}$, $P(A\cup B) = \frac{4}{5}$

$P(A\cap B) = \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{30+20-36}{45} = \frac{14}{45}$.

44. (a) Let A = drawing card of spade and B = drawing an ace,

then $A\cap B$ = an ace or spade

$P(A) = \frac{13}{52}$, $P(B) = \frac{4}{52}$ and $P(A\cap B) = \frac{1}{52}$

$\therefore$ P(either a spade or an ace or both) = $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}.$$

45. (a) We have ratio of the ships A, B and C for arriving safely are 2 : 5, 3 : 7 and 6 : 11 respectively.

$\therefore$ The probability of ship A for arriving safely $= \frac{2}{2+5} = \frac{2}{7}$.

Similarly, for B $= \frac{3}{3+7} = \frac{3}{10}$ and for C $= \frac{6}{6+11} = \frac{6}{17}$.

$\therefore$ Probability of all the ships for arriving safely $= \frac{2}{7}\times\frac{3}{10}\times\frac{6}{17} = \frac{18}{595}$.

46. (d) Let A denotes the event that the student is selected in entrance test and B denotes the event that he is selected in other entrance test. Then

$P(A) = 0.2$, $P(B) = 0.5$ and $P(A\cap B) = 0.3$

Required probability $= P(\bar{A}\cap\bar{B}) = 1 - P(A\cup B) = 1 - (0.2 + 0.5 - 0.3) = 0.6$.

47. (b) $P(A\cup B) = P(A) + P(B) = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}$

$\therefore$ Events are mutually exclusive, so $P(A\cap B) = 0$.

49. (c) Let A be the event that the Indian man is seated adjacent to his wife and B be the event that the American man is seated adjacent to his wife.

$P(A) = \frac{2\times 8!}{9!}$; $P(B) = \frac{2^4\times 5!}{9!}$

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{2^5\times 4!}{9!}}{\frac{2^4\times 5!}{9!}} = \frac{2}{5}.$$

50. (b) Let A_1 : He knows the answer. A_2 : He does not know the answer and E: He gets the correct answer.

Then, $P(A_1) = \frac{9}{10}$, $P(A_2) = 1 - \frac{9}{10} = \frac{1}{10}$, $P(E/A_1) = 1$, $P(E/A_2) = \frac{1}{4}$

$\therefore$ Required probability $= P(A_2/E) = \frac{A(A_2)P(E/A_2)}{P(A_1)P(E/A_1) + P(A_2)P(E/A_2)} = \frac{1}{37}$.

52. (a) Sum of all probabilities = 1

$\Rightarrow k + k + 2k + k = 1 \Rightarrow 5k = 1 \Rightarrow k = \frac{1}{5}$.

54. (d)

	Blood group O	other than blood group
Number of people	30%	70%
Percentage of left handed people	6%	10%

$P(E_1) = 0.3$, $P(E_2) = 0.7$

$P(E/E_1) = 0.06$ and $P(E/E_2) = 0.1$

using Bayes's theorem

$$P(E_1/E) = \frac{0.3 \times 0.06}{0.3 \times 0.06 + 0.7 \times 0.01} = \frac{9}{44}.$$

55. (c) We know that, $P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - \left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)}{1 - \frac{3}{10}} = \frac{5}{7}$

$$P(B'/A') = \frac{P(B' \cap A')}{P(A')} = \frac{1 - \frac{1}{2}}{1 - \frac{2}{5}} = \frac{5}{6}$$

$$\therefore P(A'/B') \cdot P(B'/A') = \frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}$$

56. (a) Required probability $= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$.

57. (b) We have, $P(A) = \frac{3}{6} = \frac{1}{2}$, $P(B) = \frac{4}{6} = \frac{2}{3}$

And $P(A \cap B)$ = Probability of getting a number 3 and less than 5.

= Probability of getting $4 = \frac{1}{6}$.

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{2}{3} - \frac{1}{6} = 1$$

58. (c) We know that the probability of occurrence of an event is always less than or equal to 1 and it is given that $P(A \cup B \cup C) \geq 0.75$

$\therefore 0.75 \leq P(A \cup B \cup C) \leq 1$

$\Rightarrow 0.75 \leq P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C) \leq 1$

$\Rightarrow 0.75 \leq 0.3 + 0.4 + 0.8 - 0.08 - P(B \cap C) - 0.28 + 0.09 \leq 1$

$\Rightarrow 0.75 \leq 1.59 - 0.36 - P(B \cap C) \leq 1$

$\Rightarrow -0.48 \leq -P(B \cap C) \leq -0.23 \Rightarrow 0.23 \leq P(B \cap C) \leq 0.48$

59. (b) Let A = sum of the digits on the selected tickets is 8. and B = Product of the digits on the selected ticket is zero.

There are 14 tickets having product of digits appearing on them as zero. The numbers on such tickets are 00, 01, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40.

$\therefore P(B) = \frac{14}{50}$ and $P(AB) = \frac{1}{50}$.

$$\therefore \text{Required probability} = P(A \cap B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4}.$$

60. (b) Let the Event E_1 = He knows the answer, E_2 = He guesses the answer and A= He gets the correct answer.

We have, $P(E_1) = \frac{90}{100} = \frac{9}{10}$, $P(E_2) = \frac{1}{10}$

$P(A/E_1) = 1,\ P(A/E_2) = \frac{1}{4}$.

$\therefore$ Required probability $= P(E_2/A) = \dfrac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)} = \dfrac{\frac{1}{10}\times\frac{1}{4}}{\frac{9}{10}\times 1+\frac{1}{10}\times\frac{1}{4}} = \frac{1}{37}$.

61.

Category	Above 50 kg	Below 50 kg	Total
Boys	30	20	50
Girls	30	10	40
Total	60	30	90

(i) (b) Out of a total of 50 boys, 20 weigh less than 50 kg. Therefore the probability is $\frac{2}{5}$.

(ii) (a) Out of a total of 40 girls, 30 weigh more than 50 kg. Therefore the probability is $\frac{1}{2}$.

(iii) (d) New number of boys = 50 – 15 = 35. Therefore, the probability of a boy winning is now $\frac{7}{10}$.

(iv) (c) $40-\left(\frac{1}{2}\times 30\right)-\left(\frac{1}{5}\times 10\right)=23$. Therefore, the probability is $\frac{23}{40}$.

(v) (b) Out of a total of 50 boys, 30 weigh more than 50 kg. Therefore the probability is $\frac{1}{3}$.

62. (i) (b) Let B denote the black ball drawn,

Then, $B = (p\cup q\cup r)\cap B$

Therefore, $P(B) = P(p\cap B) + P(q\cap B) + P(r\cap B) = P(p)P(B/p) + P(q)P(B/q) + P(r)\ P(B/r)$

$= \frac{2}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}+\frac{1}{5}\times\frac{1}{5} = \frac{6+8+1}{25} = \frac{3}{5}$.

(ii) (c) Let W denote the white ball drawn.

Then, $W = (p\cup q\cup r)\cap W$

Therefore, $P(W) = P(p\cap W) + P(q\cap W) + p(r\cap W) = P(p)P(W/p) + P(q)P(W/q) + P(W/r)$

$= \frac{2}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{1}{5}+\frac{1}{5}\times\frac{4}{5} = \frac{10}{25} = \frac{2}{5}$.

(iii) (c) $P(p/B) = \dfrac{P(p)\cdot P(B/p)}{P(p)P(B/p)+P(q)P(B/q)+P(r)P(B/r)} = \dfrac{\frac{2}{5}\times\frac{3}{5}}{\frac{2}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}+\frac{1}{5}\times\frac{1}{5}} = \dfrac{\frac{6}{25}}{\frac{15}{25}} = \frac{2}{5}$.

(iv) (d) $P(r/W) = \dfrac{P(r)\cdot P(W/r)}{P(p)P(W/p)+P(q)P(W/q)+P(r)P(W/r)} = \dfrac{\frac{1}{5}\times\frac{4}{5}}{\frac{2}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{1}{5}+\frac{1}{5}\times\frac{4}{5}} = \dfrac{\frac{4}{25}}{\frac{10}{25}} = \frac{2}{5}$.

(v) (a) $P(q/W) = \dfrac{P(q)\cdot P(W/q)}{P(p)P(W/p)+P(q)P(W/q)+P(r)P(W/r)} = \dfrac{\frac{2}{5}\times\frac{1}{5}}{\frac{2}{5}\times\frac{2}{5}+\frac{2}{5}\times\frac{1}{5}+\frac{1}{5}\times\frac{4}{5}} = \dfrac{\frac{2}{25}}{\frac{10}{25}} = \frac{1}{5}$.