Chapter - 9 DIFFERENTIAL EQUATIONS

STUDY NOTES

- **Differential Equation :** A relationship between an independent variable, say x, dependent variable, say y, and its derivatives $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$, etc., is called a differential equation, e.g., $\frac{dy}{dx} + 3y = \sin x$, $3\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 3y = 0$, etc.

- **Order and Degree :** The order of a differential equation is the order of the highest derivative present in the given differential equation. And the degree of a differential equation is the highest power of the highest order derivative in the differential equation . For example, the differential equation.

  (i) $y\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^4 = 0$, has order 3 and degree 1.

  (ii) $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y^4 = 0$, has order 2 and degree 3.

- **Solution of a differential equation :**

  (i) Solving a differential equation means finding an equivalent relationship between the independent and dependent variables by eliminating the derivatives.

  (ii) General solution of a differential equation is the solution which contains as many arbitrary constants as the order of the differential equation.

  (iii) Particular solution of a differential equation is obtained by substituting the given values of the arbitrary constants in the general solution of the differential equation.

- **Solution of a Differential Equation :**

  (i) **Variable Separable Type :** The differential equation of the form $\frac{dy}{dx} = f(x)\cdot g(y)$, where $f(x)$ is a function of x alone and $g(y)$ is a function of y alone can be written in the form $\frac{dy}{g(y)} = f(x)dx$.

  The solution is obtained by integrating both sides, i.e., $\int \frac{dy}{g(y)} = \int f(x)dx$.

  (ii) **Homogeneous Type :** A differential equation in x and y is called homogeneous if it is not possible to separate the variables. To solve homogeneous equation in the form $\frac{dy}{dx} = f(x, y)$ substitute $y = vx$; $\frac{dy}{dx} = v + x\frac{dv}{dx}$, which is separable in v and x and in the end, replace v by $\frac{y}{x}$.

- **First order Linear Differential Equations :**

  (i) $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x or constants.

  First we find I.F. $= e^{\int P dx}$ (Integrating factor)

  $\therefore$ The solution is given by $y \times$ I.F. $= \int(Q \times \text{I.F.})\, dx + C$.

(ii) $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y or constants.

First we find I.F. $= e^{\int P dy}$ (Integrating factor)

$\therefore$ The solution is given by $x \times \text{I.F.} = \int (Q \times \text{I.F.})\, dy + C$.

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is :

(a) 1 (b) 2 (c) 3 (d) not defined

2. The order and the degree of the differential equation : $\left[1 - \left(\frac{dy}{dx}\right)^3\right]^{3/4} = \left[\frac{d^2y}{dx^2}\right]^{1/5}$ is :

(a) (2, 4) (b) (2, 5) (c) (2, 10) (d) (2, 15)

3. The order of the differential equation of all circles of given radius a is :

(a) 1 (b) 2 (c) 3 (d) 4

4. Solution of the differential equation $\frac{dy}{y} + \frac{dx}{x} = 0$ is :

(a) $y + x = C$ (b) $\frac{1}{y} + \frac{1}{x} = C$ (c) $xy = C$ (d) $\log x \log y = C$

5. Integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{x} = 8x^3$ is :

(a) $\frac{1}{x}$ (b) x (c) $\log x$ (d) $\frac{1}{x^2}$

6. Which of the following is a homogeneous equation ?

(a) $\frac{dy}{dx} = \frac{x^3+y^3}{x^3+x^2y^2}$ (b) $(x + xy)dy = (y - yx^2)dx$

(c) $(x + y)dy - (x - y)dx = 0$ (d) $(e^{x/y})dx + e^{x/y}\left(1 - \frac{x^2}{y}\right)dy = 0$

7. The solution of the differential equation $x\frac{dy}{dx} + 2y = x^2$ is :

(a) $y = \frac{x^2+C}{4x^2}$ (b) $y = \frac{x^2}{4} + C$ (c) $y = \frac{x^4+C}{x^2}$ (d) $y = \frac{x^4+C}{4x^2}$

8. Solution of differential equation $xdy - ydx = 0$ represents :

(a) a rectangular hyperbola (b) parabola whose vertex is at origin

(c) straight line passing through origin (d) a circle whose centre is at origin

9. Family $y = Ax + A^3$ of curves will correspond to a differential equation of order :

(a) 3 (b) 2 (c) 1 (d) not defined

10. The radius of a circle is increasing at the rate of 1.4 cm/s. The rate of increasing of its circumference is :

(a) 1.4π cm/s (b) 2.8π m/s (c) 3.6π m/s (d) 2.8π cm/s

11. The solution of the differential equation $\frac{1+x^2}{1+y^2}\frac{dy}{dx} = 1$ is :

(a) $\tan^{-1}\left(\frac{y}{x}\right) = C$ (b) $y^2 + x^2 = C(y - x)$ (c) $y - x = A(1 + xy)$ (d) $\tan(x + y) + y^2 = C$

12. Solution of the differential equation $\frac{\tan y}{\cos^2 x}dx + \frac{\tan x}{\cos^2 y}dy = 0$ is :

(a) $\tan x \cdot \tan y = C$ (b) $\sec^3 x + \sec^2 y = C$ (c) $\tan x + \tan y = C$ (d) $\frac{\tan x}{\tan y} = C$

13. The degree of the differential equation satisfying $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$ is :

(a) 1 (b) 2 (c) 3 (d) 4

14. The solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola when

(a) $a = 1, b = 2$ (b) $a = 0, b = 0$ (c) $a = 0, b \neq 0$ (d) $a = 2, b = 1$

15. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of

(a) First order and first degree (b) Second order and second degree

(c) First order and second degree (d) Second order and first degree

16. The solution of differential equation $\frac{dy}{dx} - y \tan x = -y^2 \sec x$ is :

(a) $y^{-1} \sec x = \tan x + C$ (b) $y^{-1} \cos x = \tan x + C$

(c) $y^{-1} \sec x = \cot x + C$ (d) $y^{-1} \tan x = \sec x + C$

17. The degree of the differential equation of all tangent lines to the parabola $y^2 = 4ax$ is :

(a) 1 (b) 2 (c) 3 (d) 4

18. If $ydx + y^2dy = xdy$ and $y(1) = 1$, then the particular solution of the equation is :

(a) $xy = C$ (b) $x^2 - y^2 = 2y$ (c) $x + y^2 = 2y$ (d) $\frac{x}{y} = C$

19. The differential equation for which $\sin^{-1}x + \sin^{-1}y = C$ is given by

(a) $\sqrt{1-x^2}dx+\sqrt{1-y^2}dy=0$ (b) $\sqrt{1-x^2}dy+\sqrt{1-y^2}dx=0$

(c) $\sqrt{1-x^2}dy-\sqrt{1-y^2}dx=0$ (d) $\sqrt{1-x^2}dx-\sqrt{1-y^2}dy=0$

20. Order of the differential equation of the family of all concentric circles centered at (h, k) is :

(a) 1 (b) 2 (c) 3 (d) 4

21. If m and n are the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^5 + 4\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1$, then

(a) $m = 3$ and $n = 5$ (b) $m = 3$ and $n = 1$ (c) $m = 3$ and $n = 3$ (d) $m = 3$ and $n = 2$

22. The differential equation of all straight lines passing through the origin is :

(a) $y = \sqrt{x\frac{dy}{dx}}$ (b) $\frac{dy}{dx} = y + x$ (c) $\frac{dy}{dx} = \frac{y}{x}$ (d) $y = \frac{d^2y}{dx^2}$

23. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = x + y$ is :

(a) $e^x + e^y = C$ (b) $e^x + e^{-y} = C$ (c) $e^{-x} + e^y = C$ (d) $e^{-x} + e^{-y} = C$

24. The solution of the equation $\frac{dy}{dx} = \frac{x+y}{x-y}$ is :

(a) $c(x^2 + y^2)^{1/2} + e^{\tan^{-1}(y/x)} = 0$ (b) $c(x^2 + y^2)^{1/2} = e^{\tan^{-1}(y/x)}$

(c) $c(x^2 - y^2) = e^{\tan^{-1}(y/x)}$ (d) $c(x^2 - y^2)^{1/2} = e^{\tan^{-1}(x/y)}$

25. Equation of curve through point (1, 0) which satisfies the differential equation $(1 + y^2)dx - xydy = 0$, is :

(a) $x^2 + y^2 = 1$ (b) $x^2 - y^2 = 1$ (c) $2x^2 + y^2 = 1$ (d) $x^2 + 2y^2 = -1$

26. The equation of the curve satisfying the differential equation $y_2(x^2 + 1) = 2xy_1$ passing through the point (0, 1) and having slope of tangent at $x = 0$ as 3 is :

(a) $y = x^3 + 3x + 1$ (b) $y = x^3 - 3x + 1$ (c) $y = x^2 + 3x + 1$ (d) $y = x^2 - 3x + 1$

27. A function $y = f(x)$ has a second order derivatives $f''(x) = 6(x - 1)$. If its graph passes through the point (2, 1) and at that point the tangent to the graph is $y = 3x - 5$, then the function is :

(a) $(x + 1)^3$ (b) $(x - 1)^3$ (c) $(x + 1)^2$ (d) $(x - 1)^2$

28. Integrating factor of the differential equation $(1 + x^2)\frac{dy}{dx} + e^{\tan^{-1}x}y = 2x$ is :

(a) $e^{\tan^{-1}x}$ (b) $e^{e^{\tan^{-1}x}}$ (c) $e^{e^{\tan^{-1}y}}$ (d) $e^{\tan^{-1}y}$

29. The particular solution of the differential equation $x^2dy + y(x + y)dx = 0$ is : (when $x = 1$ and $y = 1$)

(a) $y + 2x = 3xy^2$ (b) $y + 2x = 3x^2y$ (c) $2y + x = xy^2$ (d) $2y + x = 3x^2y$

30. The solution of the equation $\frac{dy}{dx} = \cos(x - y)$ is :

(a) $y + \cot\left(\frac{x-y}{2}\right) = C$ (b) $x + \cot\left(\frac{x-y}{2}\right) = C$ (c) $x + \tan\left(\frac{x-y}{2}\right) = C$ (d) $y + \tan\left(\frac{x-y}{2}\right) = C$

31. The general solution of $x^2\frac{dy}{dx} = 2$ is :

(a) $y = C + \frac{2}{x}$ (b) $y = C - \frac{2}{x}$ (c) $y = 2cx$ (d) $y = C - \frac{3}{x^3}$

32. What is the solution of $\frac{dy}{dx} + 2y = 1$ satisfying $y(0) = 0$?

(a) $y = \frac{1-e^{-2x}}{2}$ (b) $y = \frac{1+e^{-2x}}{2}$ (c) $y = \frac{1+e^{x}}{2}$ (d) $y = 1 + e^x$

33. The general solution of the differential equation $\frac{dy}{dx} + \sin(x + y) = \sin(x - y)$ is :

(a) $\log \tan y + \sin x = C$ (b) $\log \tan\frac{y}{2} + \sin x = C$ (c) $\tan\frac{y}{2} + \log \sin x = C$ (d) $\log \sec\frac{y}{2} + 3\sin x = C$

34. The solution of $x^3\frac{dy}{dx} + 4x^2 \tan y = e^x \sec y$ satisfying $y(1) = 0$, is :

(a) $\tan y = (x - 2)e^x \log x$ (b) $\tan y = (x - 1)e^x \cdot x^{-3}$ (c) $\sin y = e^x(x - 1)x^{-3}$ (d) $\sin y = e^x(x - 1)x^{-4}$

35. What is the solution of $\frac{dy}{dx} + 2y = 1$ satisfying $y(0) = 0$?

(a) $y = \frac{1-e^{-2x}}{2}$ (b) $y = \frac{1+e^{-2x}}{2}$ (c) $y = 1 + e^x$ (d) $y = \frac{1+e^{x}}{2}$

36. The solution of differential equation $2x\frac{dy}{dx} - y = 3$ represents a family of.

(a) circles (b) straight lines (c) ellipses (d) parabola

37. If $y(x)$ is a solution of $\left[\frac{(2+\sin x)}{(1+y)}\right]\frac{dy}{dx} = -\cos x$ and $y(0) = 1$, then the value of $y\left(\frac{\pi}{2}\right)$ is :

(a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{5}$

38. Integrating factor of $(1 - x^2)\frac{dy}{dx} - xy = 1$ is :

(a) $\sqrt{1-x^2}$ (b) $1 - x^2$ (c) $\frac{1}{1-x^2}$ (d) $\frac{1}{\sqrt{1-x^2}}$

39. Integrating factor of the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is :

(a) $e^{\sin x}$ (b) $\frac{1}{\sin x}$ (c) $\frac{1}{\cos x}$ (d) $e^{\cos x}$

40. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x \log_e x} = \frac{1}{x}$ under the condition $y = 1$ when $x = e$ is:

(a) $2y = \log_e x + \frac{1}{\log_e x}$ (b) $y = \log_e x + \frac{2}{\log_e x}$

(c) $y \log_e x = \log_e x + 1$ (d) $y = \log_e x + e$

41. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is :

(a) $\frac{1}{4} e^{-2x}$ (b) $\frac{1}{4} e^{-2x} + cx + d$ (c) $\frac{1}{4} e^{-2x} + cx^2 + d$ (d) $\frac{1}{4} e^{-2x} + c + d$

42. If $xdy = y(dx + ydy)$, $y > 0$ and $y(1) = 1$, then $y(-3)$ is equal to :

(a) 1 (b) 3 (c) 5 (d) −1

43. Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is :

(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$

44. The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is :

(a) none (b) one (c) two (d) infinite

45. The differential equation $y\frac{dy}{dx} + x = C$ represents :

(a) Family of hyperbolas (b) Family of parabolas (c) Family of ellipses (d) Family of circles

46. The order of differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c_5}$ Where c_1, c_2, c_3, c_4 and c_5 are arbitrary constants, is

(a) 2 (b) 4 (c) 3 (d) 5

47. The order and degree of the differential equation $\left[x + \left(\frac{dy}{dx}\right)^2\right]^{-3/2} = a\frac{d^2y}{dx^2}$ are respectively.

(a) 2, 2 (b) 2, 3 (c) 2, 1 (d) 2, 4

48. The order and degree of the differential equation of all straight lines in the *xy*-plane which are at constant distance *p* from the origin are, respectively.

(a) 1, 1 (b) 2, 1 (c) 2, 2 (d) 1, 2

49. The differential equation of the family of curves represented by the equation $y = Ae^{3x} + Be^{5x}$ is:

(a) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$ (b) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 15y = 0$

(c) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} - 15y = 0$ (d) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} - 15y = 0$

50. The differential equation for which $Ax^2 + By^2 = 1$ (A and B are arbitrary constants) is the general solution, is :

(a) $\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^2 = y\frac{dy}{dx}$ (b) $x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = y\frac{dy}{dx}$

(c) $x\left[y\frac{d^2y}{dx^2} + \frac{dy}{dx}\right]^2 = y\frac{dy}{dx}$ (d) $y\left[x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] = x\frac{dy}{dx}$

51. If the substitution $x = \tan z$ is used, then the transformed form of the equation

$(1 + x^2)^2 \dfrac{d^2y}{dx^2} + 2x(1 + x^2)\dfrac{dy}{dx} + y = 0$ is :

(a) $\dfrac{d^2y}{dz^2} + 2y = 0$ (b) $\dfrac{d^2y}{dz^2} + 2\dfrac{dy}{dz} - y = 0$ (c) $\dfrac{d^2y}{dz^2} - 2\dfrac{dy}{dz} + y = 0$ (d) $\dfrac{d^2y}{dz^2} + y = 0$

52. The differential equation whose general solution is $y = A \sin x + B \cos + x \sin x$, is :

(a) $\dfrac{d^2y}{dx^2} + y = \cos x$ (b) $\dfrac{d^2y}{dx^2} + y = 2\cos x$ (c) $\dfrac{d^2y}{dx^2} - y = 2\sin x$ (d) $\dfrac{d^2y}{dx^2} - y = 2\cos x$

53. The equation of the curve passing through the point (1, 1) and satisfying the differential equation $\dfrac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is given by

(a) $e^y = e^x + \dfrac{x}{2} + \dfrac{1}{2}$ (b) $e^y = e^x + \dfrac{x}{3} - \dfrac{1}{2}$ (c) $e^y = e^x + \dfrac{x^3}{3} - \dfrac{1}{3}$ (d) $e^y = e^x - \dfrac{x}{3} + \dfrac{1}{3}$

54. By substituting $y = vx$, the solution of the differential equation, $\dfrac{dy}{dx} = \dfrac{x^2+y^2}{xy}$, is :

(a) $x^2y^2 = \log x + C$ (b) $\dfrac{y^2}{2x^2} = \log x + C$ (c) $\dfrac{2y^2}{x^2} = \log x + C$ (d) $\dfrac{y^2}{x^2} = \log x + C$

55. Solution of the equation $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$ is given by

(a) $y = x + \sqrt{1-x^2}\sin C$ (b) $y = x\cos C + \sqrt{1-x^2}\sin C$

(c) $xy = \sqrt{1-x^2} + C$ (d) $y = xy + \sqrt{(1-x^2)(1-y^2)} + C$

56. The general solution of $y^2\,dx + (x^2 - xy + y^2)\,dy = 0$, is :

(a) $\tan^{-1}\dfrac{x}{y} + \log y + C = 0$ (b) $2\tan^{-1}\dfrac{x}{y} + \log x + C = 0$

(c) $\log\left(y + \sqrt{x^2 + y^2}\right) + \log y + C = 0$ (d) $\log y = \tan^{-1}\dfrac{y}{x} + C$

57. The solution of the differential equation $x\dfrac{dy}{dx} + y = x^3y^6$, is :

(a) $x^7 = 5y^5 + cx^2y^5$ (b) $2x^7 = 5y^5 + cx^2y^5$ (c) $5x^7 = 2y^5 + cx^2y^5$ (d) $2x^7 = 5y^5 + cx^5y^2$

58. The differential equation for the family of curves $x^2 - y^2 - 2ay = 0$, where a is an arbitrary constant, is :

(a) $(x^2 + y^2)\dfrac{dy}{dx} = 2xy$ (b) $2(x^2 + y^2)\dfrac{dy}{dx} = xy$ (c) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$ (d) $2(x^2 - y^2)\dfrac{dy}{dx} = xy$

59. The solution of the differential equation $(x + y)(dx - dy) = dx + dy$, is :

(a) $x - y = ke^{x-y}$ (b) $x + y = ke^{x+y}$ (c) $x + y = k(x - y)$ (d) $x + y = ke^{x-y}$

60. Solution of the differential equation $x\left(\dfrac{dy}{dx}\right)^2 + 2\sqrt{xy}\cdot\dfrac{dy}{dx} + y = 0$, is :

(a) $x + y = a$ (b) $\sqrt{x} - \sqrt{y} = a$ (c) $x^2 + y^2 = a^2$ (d) $\sqrt{x} + \sqrt{y} = \sqrt{a}$

INPUT TEXT BASED MCQ's

61. General solution of the differential equation $f(x)dx + g(y)dy = 0$ is $\int f(x)dx + \int g(y)dy = C$

Answer the following questions :

(i) General solution of the equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ is :

(a) $\tan(x+y) = x + C$ (b) $1 + \tan\left(\frac{x+y}{2}\right) = Ce^x$ (c) $\tan(x+y) = ke^x$ (d) $\tan\left(\frac{x+y}{2}\right) = Ce^{x+y}$

(ii) Solution of $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ is :

(a) $\sin y = x^2 \log x + C$ (b) $y = x^2 \log x + C$ (c) $y\sin y = x\log x + C$ (d) $y\sin y = x^2\log x + C$

(iii) Solution of $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{(1+\log x + \log y)^2}$ is :

(a) $xy[1+(\log xy)^2] = \frac{x^2}{2} + C$ (b) $1 + (\log xy)^2 = \frac{x^2}{2} + y + C$

(c) $xy[1 + \log(xy)] = \frac{x^2}{2} + C$ (d) $xy[1 + \log x + \log y) = \frac{x}{2} + C$

(iv) General solution of the differential equation $\frac{ydx - xdy}{y^2} = 0$ is :

(a) $xy = C$ (b) $x^2 = Cy$ (c) $y^2 = Cx$ (d) $y = Cx$

(v) By a suitable substitution, the equation $y^3\frac{dy}{dx} + x + y^2 = 0$ can be transformed to

(a) Variables separable (b) Homogeneous (c) Linear (d) Bernoulli's equation

ANSWERS

1. (d)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (c)	**9.** (c)	**10.** (d)
11. (c)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (d)	**16.** (a)	**17.** (b)	**18.** (c)	**19.** (b)	**20.** (a)
21. (d)	**22.** (c)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (a)	**27.** (b)	**28.** (b)	**29.** (b)	**30.** (b)
31. (b)	**32.** (a)	**33.** (b)	**34.** (d)	**35.** (a)	**36.** (d)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (a)
41. (b)	**42.** (b)	**43.** (c)	**44.** (b)	**45.** (d)	**46.** (c)	**47.** (a)	**48.** (d)	**49.** (a)	**50.** (b)
51. (d)	**52.** (b)	**53.** (c)	**54.** (b)	**55.** (b)	**56.** (d)	**57.** (b)	**58.** (c)	**59.** (d)	**60.** (d)
51. (i) (b)	(ii) (d)	(iii) (a)	(iv) (d)	(v) (b)					

Hints to Some Selected Questions

1. (d) The given differential equations is not a polynomial equation in terms of its derivatives, so its degree is not defined.

2. (a) We have : $\left[1-\left(\frac{dy}{dx}\right)^3\right]^{3/4} = \left[\frac{d^2y}{dx^2}\right]^{1/5} \Rightarrow \left[1-\left(\frac{dy}{dx}\right)^3\right]^{15} = \left(\frac{d^2y}{dx^2}\right)^4$

$\therefore$ order = 2, degree = 4.

3. (b) Let the equation of given family be $(x-h)^2 + (y-k)^2 = a^2$. It has two arbitrary constants h and k. Therefore, the order of the given differential equation will be 2.

4. (c) From the given equation, $\frac{dy}{y} + \frac{dx}{x} = 0$, we get after integration.

$\Rightarrow \log y + \log x = \log c \Rightarrow \log xy = \log c \Rightarrow xy = c.$

5. (b) In differential equation, $P = \frac{1}{x}$

$\therefore$ I.F. $= e^{\int P dx} = e^{\int 1/x dx} = e^{\log x} = x.$

7. (d) I.F. $= e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$.

Therefore, the solution is $y \cdot x^2 = \int x^2 \cdot x dx = \frac{x^4}{4} + k$, i.e., $y = \frac{x^4 + C}{4x^2}$

8. (c) Here, $xdy - ydx = 0 \Rightarrow \frac{dy}{y} = \frac{dx}{x}$

On integrating both sides, we get, $y = cx$.

$\therefore$ Which is a straight line passing through origin.

9. (c) Given, $y = Ax + A^3 \Rightarrow \frac{dy}{dx} = A$

Putting the value of A in equation of family curve.

Then, $y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$

$\therefore$ order = 1.

10. (d) ATQ, $\frac{dr}{dt} = 1.4$ cm/s

As, $C = 2\pi r$ [diff. w.r. to t, we have]

$\frac{dc}{dt} = 2\pi\frac{dr}{dt} = 2\pi \times 1.4$ cm/s $= 2.8\pi$ cm/s.

12. (a) Given differential eq^n : $\frac{\tan y}{\cos^2 x}dx + \frac{\tan x}{\cos^2 y}dy = 0$

$\Rightarrow \tan y \sec^2 x\, dx = -\tan x \sec^2 y\, dy \Rightarrow \frac{\sec^2 y}{\tan y}dy = -\frac{\sec^2 x}{\tan x}dx$

Integrating, $\int \frac{\sec^2 y}{\tan y}dy = -\int \frac{\sec^2 x}{\tan x}dx$

$\Rightarrow \log|\tan y| = -\log|\tan x| + \log C$

$\Rightarrow \log|\tan x \tan y| = \log C \Rightarrow \tan x \tan y = C$.

14. (c) $\frac{dy}{dx} = \frac{ax+h}{by+k} \Rightarrow (by + k)dy = (ax + h)dx$

$\Rightarrow by^2 + ky = \frac{a}{2}x^2 + hx + C$

For this to represent a parabola, one of the two terms x^2 or y^2 is zero. Therefore, either $a = 0, b \neq 0$ or $a \neq 0, b = 0$.

16. (a) $\frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$,

$\Rightarrow \frac{1}{y} = v; \frac{-1}{y^2}\frac{dy}{dx} = \frac{dV}{dx}$ $\therefore \frac{-dv}{dx} - v\tan x = -\sec x$

$\Rightarrow \frac{dV}{dx} + v\tan x = \sec x$, Here, $P = \tan x$, $Q = \sec x$

I.F. $= e^{\int \tan x dx} = \sec x$; $v \sec x = \int \sec^2 x dx + C$

Hence, the solution is $y^{-1} \sec x = \tan x + C$.

17. (b) Equation of the tangent $y = mx + \frac{a}{m}$

Where m is arbitrary constant $\therefore$ order = 1

$$\frac{dy}{dx} = m \cdot 1 + 0 = m$$

$$\therefore y = \frac{dy}{dx}x + \frac{a}{\frac{dy}{dx}} \Rightarrow x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + a = 0 \qquad \therefore \text{ degree} = 2.$$

19. (b) Here, $\sin^{-1}x + \sin^{-1}y = C$

On differentiating w.r. to x, we get

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \Rightarrow \sqrt{1-x^2}\,dy + \sqrt{1-y^2}\,dx = 0.$$

20. (a) $(x-h)^2 + (y-k)^2 = r^2$. Here r is arbitrary constant

$\therefore$ Order of differential equation = 1

21. (d) The highest order (m) of the given equation is $\frac{d^3y}{dx^3} = 3$ and degree (n) of the given equation is $\left(\frac{d^3y}{dx^3}\right)^2 = 2.$

Therefore, $m = 3$ and $n = 2$.

22. (c) The equation of all straight lines passing through the origin is $y = mx$.

where m is arbitrary constant, differentiating *w.r.* to x, we get $\frac{dy}{dx} = m \Rightarrow \frac{dy}{dx} = \frac{y}{x}$

23. (b) $\log\left(\frac{dy}{dx}\right) = x + y \Rightarrow e^{x+y} = \frac{dy}{dx} \Rightarrow e^x e^y = \frac{dy}{dx}$

$$\Rightarrow \int e^x dx = \int \frac{1}{e^y} dy \Rightarrow e^x = -e^{-y} + C \Rightarrow e^x + e^{-y} = C$$

25. (b) We have, $\frac{dx}{x} = \frac{y\,dy}{1+y^2}$

Integrating, we get $\log|x| = \frac{1}{2}\log(1+y^2) + \log C$ or $|x| = C\sqrt{(1+y^2)}$

But it passes through (1, 0), so we get C = 1

$\therefore$ Solution is $x^2 = y^2 + 1$ or $x^2 - y^2 = 1$

26. (a) Given, $y_2(x^2+1) = 2xy_1 \Rightarrow \frac{y_2}{y_1} = \frac{2x}{x^2+1}$

Integrating both sides, we get : $\log y_1 = \log(x^2+1) + \log C \Rightarrow y_1 = C(x^2+1)$

Given, $y_1 = 3$ at $x = 0 \Rightarrow C = 3$

$\therefore y_1 = 3(x^2+1)$.

Again, integrating we get, $y = \frac{3x^3}{3} + 3x + C_1$

This passes through (0, 1) $\therefore C_1 = 1$

$\therefore$ Equation of curve is $y = x^3 + 3x + 1$

28. (b) $\frac{dy}{dx} + \frac{e^{\tan^{-1}x}}{1+x^2}y = \frac{2x}{1+x^2}$

$$\text{I.F.} = e^{\int \frac{e^{\tan^{-1}x}}{1+x^2}dx}$$

Put $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}dx = dt$

I.F. $= e^{e^{\tan^{-1}x}}$.

30. (b) Put $u = x - y$, then, $\frac{du}{dx} = 1 - \frac{dy}{dx} \Rightarrow 1 - \cos u = \frac{du}{dx} \Rightarrow \int \frac{du}{1-\cos u} = \int dx$

$\Rightarrow \frac{1}{2}\int \operatorname{cosec}^2\left(\frac{u}{2}\right)du = \int dx \Rightarrow x + \cot\left(\frac{u}{2}\right) = \text{constant} \Rightarrow x + \cot\left(\frac{x-y}{2}\right) = \text{C}$

31. (b) $\frac{dy}{dx} = \frac{2}{x^2} \Rightarrow dy = \frac{2}{x^2}dx$

Now, integrating, we get $y = -\frac{2}{x} + \text{C}$

32. (a) $\frac{dy}{dx} + 2y = 1 \Rightarrow \frac{dy}{dx} = 1 - 2y \Rightarrow \int \frac{dy}{1-2y} = \int dx \Rightarrow -\frac{1}{2}\log|1-2y| = x + \text{C}$

at $x = 0$, $y = 0$; $\frac{-1}{2}\log 1 = 0 + \text{C} \Rightarrow \text{C} = 0$

$\Rightarrow 1 - 2y = e^{-2x} \Rightarrow y = \frac{1-e^{-2x}}{2}$.

33. (b) $\frac{dy}{dx} = \sin(x - y) - \sin(x + y) = 2\cos x \sin(-y) \Rightarrow \frac{dy}{\sin y} + 2\cos x\, dx = 0$

$\Rightarrow \int \operatorname{cosec} y\,dy + 2\int \cos x\,dx = \text{C} \Rightarrow \log\tan\frac{y}{2} + 2\sin x = \text{C}$

34. (d) $x^4 \cos y \frac{dy}{dx} + 4x^3 \sin y = xe^x$

$\Rightarrow \frac{d}{dx}(x^4 \sin y) = xe^x \Rightarrow x^4 \sin y = \int xe^x dx + \text{C} = (x - 1)e^x + \text{C}$

Since, $y(1) = 0$, so, $\text{C} = 0$

Hence, $\sin y = x^{-4}(x - 1)e^x$.

35. (a) $\frac{dy}{dx} + 2y = 1 \Rightarrow \frac{dy}{dx} = 1 - 2y$

$\int \frac{dy}{1-2y} = \int dx \Rightarrow -\frac{1}{2}\log|1 - 2y| = x + \text{C}$ at $x = 0$, $y = 0$; $\frac{-1}{2}\log 1 = 0 + \text{C} \Rightarrow \text{C} = 0$

$\Rightarrow 1 - 2y = e^{-2x} \Rightarrow y = \frac{1-e^{-2x}}{2}$.

36. (d) We have, $2x\frac{dy}{dx} = y + 3 \Rightarrow \frac{2}{y+3}dy = \frac{dx}{x}$

Integrating, $2\log(y + 3) = \log x + \log c = \log cx$

$\Rightarrow \log(y + 3)^2 = \log cx \Rightarrow (y + 3)^2 = cx$.

Which is a family of parabolas.

38. (a) $(1 - x^2)\frac{dy}{dx} - xy = 1 \Rightarrow \frac{dy}{dx} + \frac{-xy}{1-x^2} = \frac{1}{1-x^2}$

Here, $\text{P}(x) = \frac{-x}{1-x^2}$ and $\text{Q}(x) = \frac{1}{1-x^2}$

Then, I.F. $= e^{\int \text{P}dx} = e^{\frac{1}{2}\int \frac{-2x}{1+x^2}dx} = e^{1/2\log(-x^2+1)} = \sqrt{1-x^2}$.

40. (a) $\frac{dy}{dx} + \frac{y}{x\log_e x} = \frac{1}{x} \Rightarrow \frac{dy}{dx} + \frac{1}{x\log_e e^x}y = \frac{1}{x}$

I.F. $= e^{\int \frac{1}{x\log_e x}dx} = e^{\log_e(\log_e x)} = \log_e x$. $\Rightarrow y\log_e x = \int \frac{1}{x}\log_e x\,dy \Rightarrow y\log_e x = \frac{(\log x)^2}{2} + \text{C}$

$y = 1, x = e \Rightarrow 1(1) = \frac{(1)^2}{2} + C \Rightarrow C = \frac{1}{2}$

Solution : $y \log_e x = \frac{(\log_e x)^2}{2} + \frac{1}{2} \Rightarrow 2y = \log_e x + \frac{1}{\log_e x}$.

41. (b) $\frac{d^2y}{dx^2} = e^{-2x}$ Integrating both sides, we get $\frac{dy}{dx} = \frac{e^{-2x}}{-2} + C$

Again integrate, we get $y = \frac{e^{-2x}}{4} + Cx + d$

42. (b) $x\,dy = y(dx + y\,dy) \Rightarrow \frac{xdy - ydx}{y^2} = dy \Rightarrow -d\left(\frac{x}{y}\right) = dy$

Integrating both sides, we get $\frac{x}{y} + y = C$

$\because y(1) = 1 \Rightarrow C = 2 \therefore \frac{x}{y} + y = 2$ For $x = -3$.

$y^2 - 2y - 3 = 0 \Rightarrow y = -1$ or $3 \Rightarrow y = 3$ $(\because y > 0)$

43. (c) $\frac{dy}{dx} - \frac{y}{x} = x^3 - 3$, $P = \frac{-1}{x}$ and $Q = x^3 - 3$. IF $= e^{\int \frac{-1}{x}dx} = e^{-\log x} = \frac{1}{x}$.

44. (b) Here, $\int \frac{dy}{y+1} = \int \frac{dy}{x-1} \Rightarrow \log(y + 1) = \log(x - 1) - \log C$

$\Rightarrow \log C(y + 1) = \log(x + 1) \Rightarrow \frac{x-1}{y+1} = C$

Now it is given $y(1) = 2$ which means $x = 1, y = 2$

$\Rightarrow \frac{1-1}{2+1} = C \Rightarrow C = 0 \Rightarrow \frac{x-1}{y+1} = 0 \Rightarrow x - 1 = 0$. So, only one solution exists.

45. (d) Here, $y\frac{dy}{dx} = c - x \Rightarrow ydy = (c - x)dx$

Integrate, $\int ydy = \int (c - x)dx \Rightarrow \frac{y^2}{2} = cx - \frac{x^2}{2} + k$

$\Rightarrow \frac{y^2}{2} + \frac{x^2}{2} = cx + k \Rightarrow x^2 + y^2 = 2cx + k \Rightarrow x^2 + y^2 - 2cx - k = 0$

$\therefore$ This equation represents family of circles for different values of c and k we will get different circles.

46. (c) We have, $y = a\cos(x + c_3) - c_4e^{c_5} \cdot e^x$ where $a = c_1 + c_2$.

$y = a\cos(x + c_3) - be^x$

Where $a = c(c_1 + c_2)$ and $b = c_4\, e^{c_5}$. Since there are only 3 arbitrary constants, the order of the differential equation is 3.

47. (a) We have, $\left[x + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

Therefore, order is 2 and degree is 2.

50. (b) Given, $Ax^2 + By^2 = 1$ ...(i)

$Ax + By\frac{dy}{dx} = 0$ ...(ii)

And $A + B\left(\frac{dy}{dx}\right)^2 + By\frac{d^2y}{dx^2} = 0$ ...(iii)

From eq^n (ii) and (iii), we get

$$\frac{y\left(\frac{dy}{dx}\right)}{x} = \frac{-A}{B} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$$

Therefore, $y\frac{dy}{dx} = x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]$.

52. (b) We have, $y = A\sin x + B\cos x + x\sin x$ ...(i)

Differentiating *w.r.t.* x, we get

$y_1 = A\cos x - B\sin x + \sin x + x\cos x$ ...(ii)

Again, differentiating *w.r.* to x, we get

$y_2 = -A\sin x - B\cos x + \cos x + \cos x - x\sin x$

$y_2 = -(A\sin x + B\cos x + x\sin x) + 2\cos x \Rightarrow y_2 = -y + 2\cos x$

Therefore, $y_2 + y = 2\cos x$

53. (c) We have, $\frac{dy}{dx} = e^{x-y} + x^2e^{-y} \Rightarrow e^y dy = (e^x + x^2)dx$

Integrating we get :

$\int e^y dy = \int(e^x + x^2)dx + C \Rightarrow e^y = e^x + \frac{1}{3}x^3 + C$

The curve passing through (1, 1) then , $e = e + \frac{1}{3} + C \Rightarrow C = \frac{-1}{3}$

Therefore, the curve equation is, $e^y = e^x + \frac{x^3}{3} - \frac{1}{3}$.

54. (b) Substituting $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in the given differential equation, we get

$\Rightarrow v + x\frac{dy}{dx} = \frac{1+v^2}{v} \Rightarrow x\frac{dv}{dx} = \frac{1}{v} \Rightarrow vdv = \frac{1}{x}dx$

On integrating, we get $\Rightarrow \frac{v^2}{2} = \log x + C \Rightarrow \frac{y^2}{2x^2} = \log x + C$

55. (b) We have, $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}} \Rightarrow \frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$

Integrating, we get $\Rightarrow \sin^{-1}y = \sin^{-1}x + C$

So, $y = \sin(\sin^{-1}x + C) = \sin(\sin^{-1}x)\cos C + \cos(\sin^{-1}x)\sin C$

$y = x\cos C + \sqrt{1-x^2}\,\sin C$.

58. (c) We have, $x^2 + y^2 - 2ay = 0$ ...(i)

Differentiating *w.r.* to x, we get, $2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0 \Rightarrow a = \frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}$

Substituting this value of a in (i), we get

$(x^2 + y^2)\frac{dy}{dx} - 2y\left(x + y\frac{dy}{dx}\right) = 0 \Rightarrow (x^2 - y^2)\frac{dy}{dx} = 2xy$.

59. (d) We have, $(x + y)(dx - dy) = dx + dy$

$\Rightarrow dx - dy = \frac{dx+dy}{x+y} \Rightarrow d(x - y) = \frac{d(x+y)}{x+y}$

On integrating

$\Rightarrow x - y = \log(x + y) + \log c \Rightarrow x + y = ke^{x-y}$, where, $k = \frac{1}{c}$.

60. (d) We have, $x\left(\frac{dy}{dx}\right)^2 + 2\sqrt{xy}\frac{dy}{dx} + y = 0$

$\Rightarrow \left(\sqrt{x}\frac{dy}{dx} + \sqrt{y}\right)^2 = 0 \Rightarrow \sqrt{x}\frac{dy}{dx} + \sqrt{y} = 0 \Rightarrow \frac{1}{\sqrt{x}}dx + \frac{1}{\sqrt{y}}dy = 0 \Rightarrow 2\sqrt{x} + 2\sqrt{y} = C$

$\Rightarrow \sqrt{x} + \sqrt{y} = \sqrt{a}$, where, $\sqrt{a} = 2C$.

61. (i) (b) We have, $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$

Put $x + y = u$. Then, $\frac{dy}{dx} = 1 + \sin u + \cos u = 2\cos^2\frac{u}{2} + 2\sin\frac{u}{2}\cos\frac{u}{2}$

Therefore, $\frac{1}{2\cos^2\frac{u}{2} + 2\sin\frac{u}{2}\cos\frac{u}{2}}du = dx \Rightarrow \frac{\frac{1}{2}\sec^2\frac{u}{2}}{1 + \tan\frac{u}{2}}du = dx$

Integrating we get, $\int \frac{\frac{1}{2}\sec^2\frac{u}{2}}{1 + \tan\frac{u}{2}}dx = x + C$

$\Rightarrow \log\left(1 + \tan\frac{u}{2}\right) = x + C \Rightarrow 1 + \tan\left(\frac{x+y}{2}\right) = ce^x$

(ii) (d) We have, $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$

$\Rightarrow (\sin y + y\cos y)dy = x(2\log x + 1)dx$

Integrated we get, $\int(\sin y + y\cos y)dy = \int(2x\log x + x)dx + C$

$-\cos y + \cos y + y\sin y = x^2\log x - \int x^2\cdot\frac{1}{x}dx + \int xdx + C = x^2\log x + C$

Therefore, $y\sin y = x^2\log x + C$

(iii) (a) We have, $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{(1+\log x+\log y)^2} = \frac{1}{(1+\log xy)^2}$

Put $xy = u$ so that $\frac{dy}{dx} = \frac{1}{(1+\log u)^2}$

Therefore, $(1 + \log u)^2\, du = xdx$

$\Rightarrow \int(1+\log u)^2 du = \int xdx + C \Rightarrow u(1 + \log u)^2 - \int\frac{2(1+\log u)}{u}\cdot udu = \frac{x^2}{2} + C$

$\Rightarrow u(1 + \log u)^2 - 2u - 2u\log u + 2u = \frac{x^2}{2} + C \Rightarrow u[1 + 2\log u + (\log u)^2] - 2u\log u = \frac{x^2}{2} + C$

$\Rightarrow u[1 + (\log u)^2] = \frac{x^2}{2} + C \Rightarrow xy[1 + (\log(xy))^2] = \frac{x^2}{2} + C$

(iv) (d) We have, $\frac{dy}{x} - \frac{dy}{x} = 0 \Rightarrow \log|y| - \log|x| + C \Rightarrow y = kx$.

(v) (b) Putting $z = x + y^2$ in the given equation, we get, $\frac{dz}{dx} = 1 + 2y\frac{dx}{dx} \Rightarrow (z - x)\frac{1}{2}\left(\frac{dz}{dx} - 1\right) + z = 0$

$\Rightarrow (z - x)\left(\frac{dz}{dx} - 1\right) + 2z = 0 \Rightarrow (z - x)\frac{dz}{dx} - (z - x) + 2z = 0 \Rightarrow (z - x)\frac{dz}{dx} + z + x = 0 \Rightarrow \frac{dz}{dx} + \frac{z+x}{z-x} = 0$

Which is homogeneous equation.