Chapter - 4 DETERMINANTS

STUDY NOTES

- A determinant of order three consisting of 3 rows and 3 columns is written as

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ is equal to } a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix} = a_1 (b_2c_3 - c_2b_3) - b_1 (a_2c_3 - c_2a_3) - c_1 (a_2b_3 - b_2a_3)$$

- **Minor :** The determinant obtained by deleting the i^{th} row and j^{th} column is called minor of element at the i^{th} row and j^{th} column. The cofactors of this element is $(-1)^{i+j}$ (minor).

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1A_1 + b_1B_1 + c_1C_1\text{; where } A_1, B_1, C_1 \text{ are the cofactors of } a_1, b_1, c_1 \text{ respectively.}$$

- **Properties of Determinants :**
  (i) The determinants remains same if its rows and columns are interchanged.
  (ii) If all the elements of a row (or column) are zero, then the determinant is zero.
  (iii) The interchange of any two rows of the determinant changes its sign.
  (iv) If all the elements of a row (column) of a determinant are multiplied by a non-zero constant then the determinant gets multiplied by the same number.
  (v) If each element in any row (column) is the sum of a terms, then the determinant can be expressed as the sum of 'r' determinants.
  (vi) If the determinant $\Delta = f(x)$ and $f(a) = 0$, then $(x - a)$ is factor of the determinants.
  (vii) If in a determinant the elements in all the rows are in A.P. with same or different common difference, then the value of determinant is zero.
  (viii) The determinant of odd order skew-symmetric determinant is always zero.

Differentiation of Determinant

- Let $\Delta(x) = \begin{vmatrix} a_1(x) & b_1(x) \\ a_2(x) & b_2(x) \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} a_1'(x) & b_1'(x) \\ a_2(x) & b_2(x) \end{vmatrix} + \begin{vmatrix} a_1(x) & b_1(x) \\ a_2'(x) & b_2'(x) \end{vmatrix}$

  where dash denotes the derivate with respect to x.

- If we write $\Delta(x) = |C_1\ C_2\ C_3|$, then

  $\Delta'(x) = |C_1'C_2\,C_3| + |C_1\,C_2'| + |C_1\,C_2\,C_3'|$

- Similarly, if $\Delta(x) = \begin{vmatrix} R_1 \\ R_2 \\ R_3 \end{vmatrix}$, then $\Delta'(x) = \begin{vmatrix} R_1' \\ R_2 \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2' \\ R_3 \end{vmatrix} + \begin{vmatrix} R_1 \\ R_2 \\ R_3' \end{vmatrix}$

- **CRAMER'S RULE :** If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \neq 0$, then the solution of the system of non-homogeneous simultaneous linear equation.

$a_1x + b_1y + c_1z = d_1$
$a_2x + b_2y + c_2z = d_2$
$a_3x + b_3y + c_3z = d_3$

$x = \frac{\Delta x}{\Delta}, \; y = \frac{\Delta y}{\Delta}, \; z = \frac{\Delta z}{\Delta}$, where

$$\Delta x = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta y = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta z = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

- If $\Delta x, \Delta y, \Delta z, \in R$ and $\Delta \neq 0$, then this have unique system of solution. It is consistence independent.
- If $\Delta x, \Delta y, \Delta z = 0$ and $\Delta = 0$, then system has infinitcly many solutions, and it is said to be consistence dependent.
- If $\Delta x, \Delta y, \Delta z =$ is non zero, then the system of equation will have no solution is said to be consistent.

Linear Equation

- The system of homogeneous simultaneous linear equation

$a_1x + b_1y + c_1z = 0$
$a_2x + b_2y + c_2z = 0$
$a_3x + b_3y + c_3z = 0$

Here, $\Delta_1 = \Delta_2 = \Delta_3$ has a non-trival solution. If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$

If $\Delta \neq 0$, then the only solution is $x = y = z = 0$.

- Determinant of unit matrix is 1
- Determinant of null matrix is 0.
- If A is non-singular matrix, than $\det(A^{-1}) = \frac{1}{\det(A)} \Rightarrow |A^{-1}| = \frac{1}{|A|}$.

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. The solution of the simultaneous linear equations $2x+y = 6$ and $3y = 8 + 4x$ will also be satisfied by which one of the following linear equation ?

(a) $x+y = 5$ (b) $2x - 3y = 10$ (c) $2x+y = 9$ (d) $2x + 3y = 6$

2. The roots of the equation $\begin{vmatrix} x & \alpha & 1 \\ \beta & x & 1 \\ \beta & \gamma & 1 \end{vmatrix} = 0$ is :

(a) α, β (b) β, γ (c) α, γ (d) α, β and γ

3. If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$, such that, $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = \lambda$ then the value of λ is :

(a) $-abc$ (b) 0 (c) abc (d) 1

4. The value of determinant : $\begin{vmatrix} a+1 & a+2 & a+4 \\ a+3 & a+5 & a+8 \\ a+7 & a+10 & a+14 \end{vmatrix}$ is :

(a) -2 (b) 2 (c) $x + 2$ (d) $(x + 2)^2$

5. The given equation has 3 roots, if 5 and 7 are two roots of the equation then $\begin{vmatrix} x & 4 & 5 \\ 7 & x & 7 \\ 5 & 8 & x \end{vmatrix} = 0$ then what is the third root ?

(a) 13 (b) -12 (c) 9 (d) 14

6. If $\begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = 2$, what is the value of the determinant : $\begin{vmatrix} 6a & 3b & 15c \\ 2l & m & 5n \\ 2p & q & 5r \end{vmatrix}$?

(a) 30 (b) 20 (c) 60 (d) 0

7. If $\begin{vmatrix} a+2 & a+2 & a+3 \\ a+2 & a+3 & a+4 \\ a+l & a+m & a+n \end{vmatrix} = 0$, then l, m and n are in :

(a) AP (b) HP (c) GP (d) None of these

8. The value of determinant of $\begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+w^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix}$; where, ω is a cube root of unity.

(a) $x^3 +1$ (b) $x^3 + w$ (c) $x^3 + w^2$ (d) x^3

9. Find the value of p if $\begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 16 \\ 0 & 0 & 1+p \end{vmatrix} = 20$

(a) 0 (b) 2 (c) 1 (d) 5

10. If $a^2 + b^2 + c^2 = 1$, then the value of $\begin{vmatrix} 1 & c & -b \\ -c & 1 & a \\ b & -a & 1 \end{vmatrix}$ is :

(a) 0 (b) 1 (c) 2 (d) $2 - 2abc$

11. The following system of equations

$$kx + y + z = k - 1$$
$$x + ky + z = k - 1$$
$$x + y + kz = k - 1$$

What are the values of 'k' the system is inconsistent ?

(a) 1, -2 (b) 2, 3 (c) 3 or 4 (d) 2 or -2

12. The determinant : $\begin{vmatrix} 1 & \omega^2 & 1+2\omega^{100}+\omega^{200} \\ \omega & 1+\omega^{100}+2\omega^{200} & 1 \\ 2+\omega^{100}+2\omega^{200} & \omega^2 & \omega \end{vmatrix}$

(a) 1 (b) ω (c) ω^2 (d) none of these

13. Let A be a square matrix of order $n \times n$, where $n \geq 2$. Let B be a matrix obtained from A with first and second rows interchanged. Then which of the following is correct ?

(a) det (A) = det (B) (b) det (A) = – det (B) (c) A = B (d) A = –B

14. The value of determinant of $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ is :

(a) $4abc$ (b) $4a^2b^2c^2$ (c) $2a^2b^2c^2$ (d) $2abc$

15. If $|A_{n\times n}| = 3$ and $|adj\ A| = 243$, what is the value of n ?

(a) 4 (b) 5 (c) 6 (d) 7

16. For positive numbers x, y, z, the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z x & 1 \end{vmatrix}$ is :

(a) 0 (b) $\log_e xyz$ (c) 1 (d) none of these

17. If A is a matrix of order 3×2 and B is a matrix of order 2×3, then $|kAB|$ equal to (where k is any scalar quantity)

(a) $k\ |AB|$ (b) $k^3\ |AB|$ (c) $k^2\ |AB|$ (d) $|AB|$

18. What is the value of $\begin{vmatrix} \sin 10° & \cos 10° \\ \sin 80° & \cos 80° \end{vmatrix}$?

(a) 0 (b) 1 (c) – 1 (d) $\frac{-1}{2}$

19. The value of determinant $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$ is :

(a) 2(11! 12! 13!) (b) 2(10! 13!) (c) 2(10! 12! 13!) (d) 2(10! 11! 12!)

20. The value of determinant $\begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$ is :

(a) $a^3+b^3+c^3$ (b) $3\ bc$ (c) $a^3+b^3+c^3 - 3abc$ (d) 0

21. Let A be $n \times n$ matrix. If det $(\lambda A) = \lambda^S$ det (A) what is the value of S ?

(a) 0 (b) 1 (c) – 1 (d) n

22. If a, b, c are in GP, then what is the value of : $\begin{vmatrix} a & b & a+b \\ b & c & b+c \\ a+b & b+c & 0 \end{vmatrix}$?

(a) 0 (b) 1 (c) – 1 (d) 2

23. If ω is the cube root of unity, then what is one root of the equation $\begin{vmatrix} x^2 & -2x & -2\omega^2 \\ 2 & \omega & -\omega \\ 0 & \omega & 1 \end{vmatrix} = 0$?

(a) 1 (b) –2 (c) 2 (d) w

24. The value of x and y are, if $\begin{vmatrix} x & -3i & 1 \\ y & 1 & i \\ 0 & 2i & -i \end{vmatrix} = 6+11i$

(a) –3, 4 (b) 3, 4 (c) 3, –4 (d) –3, –4

25. The determinant of AB is, where A = $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and B = $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$.

(a) 0 (b) 1 (c) 10 (d) 20

26. If A and B are square matrices or order 3, such that |A| = – 1, |B| = 3, than |3AB | is equal to :

(a) – 9 (b) – 81 (c) – 27 (d) 81

27. If $D_r = \begin{vmatrix} 2^{r-1} & 2.3^{r-1} & 4.5^{r-1} \\ x & y & z \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=1}^{n} D_r$ is equal to :

(a) 0 (b) 1 (c) – 1 (d) 2

28. If two triangles with vertices (x_1, y_1) (x_2, y_2) (x_3, y_3) and (a_1, b_1) (a_2, b_2) (a_3, b_3) satisfy the relation.

$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix}$, then triangle are

(a) Congruent (b) Unequal area (c) Similar (d) Unequal perimeter

29. If $\begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = ka^2b^2c^2$ and $a^2 + b^2 + c^2 = 0$, then the value of k is :

(a) 1 (b) 0 (c) – 1 (d) 4

30. If $A = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix}$ and $B = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$ then which of the following is correct?

(a) $|A| = 3|B|^2$ (b) $\frac{d}{dx}|A| = 3|B|$ (c) $\frac{d}{dx}|A| = 2|B|^2$ (d) $|A| = 3|B|^{3/2}$

31. If $\begin{vmatrix} a & -b & 0 \\ 0 & a & b \\ b & 0 & a \end{vmatrix} = 0$, then which of the following is true ?

(a) a is one of the cube roots of unity (b) b is one of the cube roots of unity

(c) $\frac{a}{b}$ is one of the cube roots of unity (d) None of these

32. The determinant $\begin{vmatrix} 1+i & 1-i & i \\ 1-i & i & 1+i \\ i & 1+i & 1-i \end{vmatrix}$ is :

(a) $-4-7i$ (b) $3+7i$ (c) $8+11i$ (d) $7+4i$

33. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$; are

(a) – 1, – 2 (b) –1, 2 (c) 1, –2 (d) 1, 2

34. If a, b, c are real numbers such that $a^2+b^2+c^2 = 1$, then $\begin{vmatrix} ax-by-c & bx+acy & cx+a \\ bx+ay & -ax+by-c & cy+b \\ cx+a & cy+b & -ax-by+c \end{vmatrix} = 0$ represents

(a) parabola (b) circle (c) hyperbola (d) straight line

35. Express $\Delta = \begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$ as the product of two determinants.

(a) $\begin{vmatrix} a^2 & -2a & 1 \\ b^2 & -2b & 1 \\ c^2 & -2c & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$ (b) $\begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} \times \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \times \begin{vmatrix} 1 & -2x & x^2 \\ 1 & -2y & y^2 \\ 1 & -2z & z^2 \end{vmatrix}$ (d) none of these

36. Let $\Delta n = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4x-2 \\ (a-1)^3 & 3n^3 & 13n^2-3n \end{vmatrix}$ then which of the following is equal to $\sum_{a=1}^{n} \Delta n$?

(a) 0 (b) 1 (c) – 1 (d) 2

37. If $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x + \cot x \operatorname{cosec} x \\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 x \end{vmatrix}$ then the value of $\int_0^{\pi/2} f(x)dx$ is :

(a) $\frac{\pi}{4}$ (b) $\left(\frac{\pi}{4} - \frac{8}{15}\right)$ (c) $-\left(\frac{\pi}{4} + \frac{8}{15}\right)$ (d) $\frac{\pi}{4} + \frac{8}{15}$

38. For what value of p and q, the system of equation $2x + py + 6z = 8$, $x + 2y + qz = 5$, $x + y + 3z = 4$ has a unique solution

(a) $p = 2$, $q = 3$ (b) $p \neq 2$, $q \neq 3$ (c) $p = 0$, $q = 2$ (d) $p = 2$, $q = 3$

39. If α, β and γ are the roots of the equation $x^2 (px + q) = r(x + 1)$. Then the value of determinant $\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix}$ is :

(a) α, β, π (b) $1 + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ (c) 0 (d) none of these

40. One factor of the equation $\begin{vmatrix} a^2+\lambda & ab & ac \\ ab & b^2+\lambda & cb \\ ca & cb & c^2+\lambda \end{vmatrix}$ is :

(a) λ^2 (b) $\frac{1}{\lambda}$

(c) $(a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda)$ (d) None of these

41. If $\begin{vmatrix} x^2 & 1 & x+1 \\ 2x^2-1 & 1 & x+2 \\ 3x^2-2 & 1 & x+3 \end{vmatrix} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots\ldots$ then a, is equal to :

(a) 1 (b) 3 (c) 2 (d) 0

42. The system of equation $6x+5y+\lambda z = 0$, $3x - y + 4z = 0$, $x + 2y - 3z = 0$, has non-trival solution for,

(a) $\lambda = 0$ (b) $\lambda = -5$ (c) $\lambda = 1$ (d) none of these

43. Consider an identity matrix $(n \times n)$ In; $\lambda \in R^+$ then $|\text{adj}(\lambda\, I_n)|$ is :

(a) λ^{n-1} (b) λ^n (c) $\lambda^{n(n-1)}$ (d) $\lambda^{2(n-1)}$

44. If $\Delta(x) = \begin{vmatrix} e^x & \sin x & \tan x \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} = A + Bx + Cx^2 + \ldots\ldots\ldots\ldots$. Then the value of B is :

(a) 0 (b) 1 (c) 2 (d) none of these

45. If the maximum and minimum values of the determinant $\begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$ are α and β then which of the following is incorrect.

(a) $\alpha + \beta^{99} = 4$ (b) $\alpha^3 - \beta^{17} = 26$

(c) $(\alpha^{2n} - \beta^{2n})$ is always an even integer $n \in N$ (d) A triangle can be constructed of sides α, β and $\alpha - \beta$.

46. If a_i ; $i = 1, 2, \ldots\ldots\ldots\ldots 9$ are perfect odd squares, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is always a multiple of

(a) 4 (b) 7 (c) 6 (d) 5

47. If $g(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then $g(2x) - g(x)$ is not divisible by :

(a) x (b) $2a + 3x$ (c) a (d) x^2

48. Consider the determinant $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$ when M_{ij} denotes minor and C_{ij} denotes cofactors of an element, then the value of $pC_{21} + qC_{22} + rC_{23}$ is:

(a) 0 (b) $-\Delta$ (c) Δ (d) Δ^2

49. For the matrix $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$, where M_{ij} denotes Minor and C_{ij} denotes cofactor. Find : Then, $qM_{12} - yM_{22} + mM_{32}$ is :

(a) 0 (b) Δ (c) $-\Delta$ (d) Δ^2

50. The number of possible dimensions of a matrix containing 32 elements is :

(a) 3 (b) 6 (c) 10 (d) 14

51. If $\begin{vmatrix} x & 2 & x \\ x^2+1 & x & 5 \\ 2x-1 & 1 & x \end{vmatrix} = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$, then the value of $5A + 4B + 3C + 2D + E$ is k, then $\frac{k}{5}$ is equal to :

(a) 0 (b) 1 (c) -12 (d) 13

52. The set of all values of λ for which the system of linear equations :

$$2x_1 - 2x_2 + x_3 = \lambda x_1$$
$$2x_1 - 3x_2 + 2x_3 = \lambda x_2$$
$$-x_1 + 2x_2 = \lambda x_3$$

has a non-trival solution,

(a) is a singleton (b) Contains two elements

(c) Contains more than two element (d) is an empty set

53. If $\alpha, \beta \neq 0$ and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then k is equal to

(a) α, β (b) $\frac{1}{\alpha\beta}$ (c) 1 (d) -1

54. The value of k, for which the system of equations $(k+1)\, x + 8y = 4k$; $kx + (k+3)\, y = 3k - 1$ has no solution, is :

(a) 1 (b) 2 (c) 3 (d) infinite

55. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to:

(a) 0 (b) -1 (c) -2 (d) 1

56. If the trivial solution is the only solution of the equation

$$x - ky + z = 0$$
$$ky + 3y - kz = 0$$
$$3x + y - z = 0$$

(a) $R - \{-3\}$ (b) $\{2, -3\}$ (c) $R - \{2, -3\}$ (d) $R - \{2\}$

57. System of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has :

(a) exactly 3 solutions (b) no solution

(c) a unique solution (d) infinite number of solutions

58. Let a, b, c be such that $b(a + c) \neq 0$.

If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^n c \end{vmatrix} = 0$; then the values of 'n' is

(a) zero (b) any odd integer (c) any even integer (d) any integer

59. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is :

(a) divisible by neither x nor y (b) divisible by both x and y

(c) divisible by x but not y (d) divisible by 'y' but not x

60. l, m, n are pth, qth and rth terms of a G.P. all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals :

(a) -1 (b) 1 (c) 2 (d) 0

61. Determinant of a skew symmetric matrix of order 3 is :

(a) 0 (b) 1 (c) –1 (d) 2

62. If $a > 0$ and discriminant of $ax^2 + 2bx + c$ is negative then $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & 0 \end{vmatrix}$ is :

(a) positive (b) negative

(c) $(ac - b^2)(ax^2 + 2bx + c)$ (d) 0

63. If 1, ω, ω^2 are cube root of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to :

(a) 0 (b) ω (c) 1 (d) ω^2

64. If the system of linear equations $x + 2ay + az = 0$; $x + 3by + bz = 0$; $x + 4cy + cz = 0$ has a non zero solution, then a, b, c

(a) are in AP (b) are in GP (c) are in HP (d) satisfy $a+2b+3c = 0$

65. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $(1, a, a^2)$ $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals:

(a) 2 (b) 1 (c) –1 (d) 0

66. If $a_1, a_2, a_3, \ldots\ldots a_n$ are in GP, then determinant $\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to

(a) 1 (b) 0 (c) 4 (d) 2

67. If $\begin{vmatrix} 6i & -3i & 1 \\ 4 & 3i & -1 \\ 20 & 3 & i \end{vmatrix} = x + iy$, then

(a) $x = 3, y = 1$ (b) $x = 1, y = 3$ (c) $x = 0, y = 3$ (d) $x = 0, y = 0$

68. Let $\omega = \dfrac{-1}{2} + \dfrac{i\sqrt{3}}{2}$, then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is :

(a) 3ω (b) $3\omega(\omega-1)$ (c) $3\omega^2$ (d) $3\omega(1-\omega)$

69. Which of the following values of α satisfy the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$?

(a) –4 (b) –8 (c) –9 (d) 4

70. Let $f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, p is a constant. Then $\dfrac{d^3}{dx^3} f(x)$ at $x = 0$ is :

(a) p (b) $p + p^3$ (c) $p + p^2$ (d) independent of p

71. The number of values of 'k' for which the system of equation $(k+1)x + 8y = 4k$ and $kx + (k+3)y = 3k - 1$ has infinitely many solution.

(a) 0 (b) 1 (c) 2 (d) infinite

72. If $a \neq p$, $b \neq q$ and $c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$. Then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ is :

(a) 1 (b) 2 (c) 3 (d) 4

73. The determinant : $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ equal to :

(a) 0 (b) 1 (c) –1 (d) 2

74. If x, y, z are three different numbers and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then which of the following is incorrect ?

(a) $xyz = -1$ (b) $x = y = z$ (c) $1 + xyz = 0$ (d) all are incorrect

75. Which of the following is a root of the $\begin{vmatrix} 3-x & -6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix} = 0$?

(a) 6 (b) 0 (c) 3 (d) none of these

76. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x-1)(x+1) \end{vmatrix}$. Then f (2010) is:

(a) 1 (b) 2010 (c) 2009 (d) 0

77. The system of equation $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non-zero solution. Then possible values of 'k' are :

(a) –1, 2 (b) 1, 2 (c) 0, 1 (d) –1, 1

78. Let $f(x) = \begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix}$, if $x = -9$ is a root of $f(x) = 0$, then the other roots are

(a) 2,7 (b) 3,5 (c) 7,5 (d) 6,2

79. If $A = \begin{vmatrix} \sin(\theta+\alpha) & \cos(\theta+\alpha) & 1 \\ \sin(\theta+\beta) & \cos(\theta+\beta) & 1 \\ \sin(\theta+\gamma) & \cos(\theta+\gamma) & 1 \end{vmatrix}$, then

(a) A = 0 for all θ (b) A = 0 for all $\theta = \alpha + \beta + \gamma$
(c) A is an odd function (d) A is independent of θ.

80. The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$.

(a) 0 (b) 2 (c) 1 (d) 3

81. If 3, –2 are the eigen values of non-singular matrix A and |A| = 4, then eigen values of adj (A) are :

(a) $\frac{3}{4}, -\frac{1}{2}$ (b) $\frac{4}{3}, -2$ (c) 12, – 8 (d) – 12, 8

82. How many values of x in the closed interval [–4, –1] is the matrix $\begin{vmatrix} 3 & -1+x & 2 \\ 3 & -1 & x+2 \\ x-3 & -1 & 2 \end{vmatrix}$ singular ?

(a) 2 (b) 3 (c) 0 (d) 1

83. The determinant $\begin{vmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{vmatrix}$, the ratio of cofactor to its minor of element -3 is :

(a) -1 (b) 0 (c) 1 (d) 2

84. If the value of the determinant $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ is positive, then

(a) $abc > 1$ (b) $abc > -8$ (c) $abc < -8$ (d) $abc > -2$

85. The value of determinant : $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$?

(a) 213 (b) –231 (c) 231 (d) 39

86. Which of the following is equal to $\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$?

(a) $\begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$ (b) $\begin{vmatrix} 1 & 1 & c \\ x & y & z \\ p & q & r \end{vmatrix}$ (c) $\begin{vmatrix} a-p & b-q & c \\ x & y & z \\ p & q & r \end{vmatrix}$ (d) $\begin{vmatrix} x & a & p \\ y & b & q \\ z & c & r \end{vmatrix}$

87. The value of determinant $\begin{vmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{vmatrix}$ is :

(a) 1 (b) 0 (c) –1 (d) 2

88. If is equal to $D = \begin{vmatrix} \alpha & \beta \\ r & \delta \end{vmatrix}$ then $\begin{vmatrix} 2\alpha & 2\beta \\ 2r & 2\delta \end{vmatrix}$ is equal to

(a) D (b) 2D (c) 4D (d) 16D

89. The value of k, $\begin{vmatrix} 3^2+k & 4^2 & 3^2+3+k \\ 4^2+k & 5^2 & 4^2+4+k \\ 5^2+k & 6^2 & 5^2+5+k \end{vmatrix} = 0$ is :

(a) 0 (b) 1 (c) -1 (d) 2

90. The value of $\begin{vmatrix} 13 & 16 & 19 \\ 14 & 17 & 20 \\ 15 & 18 & 21 \end{vmatrix}$

(a) 96 (b) -39 (c) 57 (d) 0

91. If $\Delta = \begin{vmatrix} 3 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & x & 5 \end{vmatrix}$, than $\begin{vmatrix} x & 10 & 5 \\ 5 & 3 & 6 \\ 8 & 7 & 7 \end{vmatrix}$ is equal to :

(a) $-\Delta$ (b) Δx (c) Δ (d) 0

92. If $p + q + r = 0 = a + b + c$, then the value of determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is :

(a) 0 (b) 1 (c) $pa + qb + rc$ (d) none of these

93. If $x \neq y \neq z$ and $\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = 0$, then which of the following is equal to xyz ?

(a) 0 (b) 1 (c) -1 (d) $x + y + z$

94. If each element of third order determinant of value Δ is multiplied by 4, then the value of new determinant will be :

(a) Δ (b) 128Δ (c) 21Δ (d) 64Δ

95. The determinant : $\begin{vmatrix} 1 & 5 & \pi \\ \log_e e & 5 & \sqrt{5} \\ \log_{10} 10 & 5 & e \end{vmatrix}$ is :

(a) $\sqrt{\pi}$ (b) e (c) 1 (d) 0

96. The roots of the equation : $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+x \end{vmatrix}$ are :

(a) 0, -3 (b) 0, 3 (c) 0, 0, 3 (d) 0, 0, -3

97. The value of : $\begin{vmatrix} \frac{1}{a} & a^2 & bc \\ \frac{1}{b} & b^2 & ca \\ \frac{1}{c} & c^2 & ab \end{vmatrix}$ is :

(a) abc (b) $ab + bc + ca$ (c) $2abc$ (d) 0

98. The value of the determinant : $\begin{vmatrix} 31 & 37 & 92 \\ 31 & 58 & 71 \\ 31 & 105 & 24 \end{vmatrix}$ is :

(a) -2 (b) 0 (c) 81 (d) 27

99. The determinant value of $\begin{vmatrix} \sin^2 x & \cos^2 x & 1 \\ \cos^2 x & \sin^2 x & 1 \\ -10 & 12 & 2 \end{vmatrix}$ is :

(a) 0 (b) $12 \cos^2 x - 10 \sin^2 x$

(c) $12 \sin^2 x - 10 \cos^2 x - 2$ (d) $10 \sin 2x$

INPUT TEXT BASED MCQ's

100. A factory produces three types of bulb B_1, B_2, B_3 every day. Their production in certain day is 45. On this day the product of B_3 is exceeds by B_1 by 8, while the total production of B_1 and B_3 is twice the production of B_2

Answer the following questions :

(i) If the present equation the express in equation them which of the following is incorrect?

(a) $B_1 + B_2 + B_3 = 45$ (b) $B_1 - 2B_2 + B_3 = 0$

(c) $B_1 - 2B_2 + B_3 = 0$ (d) $B_1 - 2B_2 + 3B_3 = 0$

(ii) If $\begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & -2 \\ 1 & -1 & 1 \end{pmatrix}^{-1} = \frac{1}{6}\begin{pmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{pmatrix}$, then the inverse of $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{pmatrix}$ is :

(a) $\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{-1}{2} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{2} & 0 & \frac{-1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{6} & \frac{-1}{3} & \frac{1}{6} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & 0 & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{2} & \frac{1}{6} \end{pmatrix}$ (d) none of these

(iii) $B_1 : B_2 : B_3$ is equal to :

(a) 12 : 13 : 20 (b) 11 : 15 : 19 (c) 15 : 19 : 11 (d) 13 : 12 : 20

(iv) Which of the following is not true?

(a) $|A| = |A'|$ (b) $(A^1)^{-1} = (A^{-1})^1$

(c) matrix of odd order, $|A| = 0$ (d) $|AB| = |A| + |B|$

(v) Which of the given statement is incorrect for given matrix $A = [aij]_{3\times3}$?

(a) Order of minor is less than order of the det(A)

(b) Minor of an element can never be equal to cofactor of the same element.

(c) Value of a determinant is obtained by multiplying elements of a row or column by corresponding cofactors.

(d) Order of minors and cofactors of same element of A is same.

101. Consider the determinant, $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$

M_{ij} denotes the minor of an element in i^{th} row and j^{th} column.

C_{ij} denotes the cofactor of an element in i^{th} row and j^{th} column.

Answer the following questions :

(i) The value of $p \cdot C_{21} + qC_{22} + rC_{23}$ is equal to :

(a) 0 (b) $-\Delta$ (c) Δ (d) Δ^2

(ii) The value of $xC_{21} + yC_{22} + zC_{23}$ is equal to :

(a) 0 (b) $-\Delta$ (c) Δ (d) Δ^2

(iii) The value of $qM_{12} - y.\ M_{22} + mM_{32}$ is equal to :

(a) 0 (b) $-\Delta$ (c) Δ (d) Δ^2

(iv) What would be the determinant of $\begin{vmatrix} 3p & eq & 3r \\ 3x & 3y & 3z \\ 3l & 3m & 3n \end{vmatrix}$ d

(a) 3Δ (b) Δ^3 (c) 9Δ (d) 27Δ

(v) Which of the following is equal to $\begin{vmatrix} p & x & l \\ q & y & m \\ r & z & n \end{vmatrix}$?

(a) $-\Delta$ (b) Δ (c) Δ^2 (d) 2Δ

ANSWERS

1. (a)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (a)	**8.** (d)	**9.** (c)	**10.** (c)
11. (a)	**12.** (d)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (c)
21. (d)	**22.** (a)	**23.** (c)	**24.** (a)	**25.** (a)	**26.** (b)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (b)
31. (c)	**32.** (c)	**33.** (b)	**34.** (d)	**35.** (a)	**36.** (a)	**37.** (c)	**38.** (b)	**39.** (c)	**40.** (a)
41. (d)	**42.** (b)	**43.** (c)	**44.** (a)	**45.** (c)	**46.** (a)	**47.** (c)	**48.** (a)	**49.** (c)	**50.** (b)
51. (d)	**52.** (c)	**53.** (c)	**54.** (c)	**55.** (c)	**56.** (c)	**57.** (b)	**58.** (b)	**59.** (b)	**60.** (d)
61. (a)	**62.** (b)	**63.** (a)	**64.** (c)	**65.** (c)	**66.** (b)	**67.** (d)	**68.** (b)	**69.** (c)	**70.** (d)
71. (b)	**72.** (b)	**73.** (a)	**74.** (b)	**75.** (a)	**76.** (d)	**77.** (d)	**78.** (a)	**79.** (d)	**80.** (c)
81. (b)	**82.** (d)	**83.** (a)	**84.** (b)	**85.** (c)	**86.** (a)	**87.** (b)	**88.** (c)	**89.** (b)	**90.** (d)
91. (a)	**92.** (a)	**93.** (a)	**94.** (d)	**95.** (d)	**96.** (d)	**97.** (d)	**98.** (b)	**99.** (a)	

100. (i) (d) (ii) (c) (iii) (b) (iv) (d) (v) (b)

101. (i) (a) (ii) (c) (iii) (b) (iv) (d) (v) (b)

Hints to Some Selected Questions

1. (a) By solving the given equation we get, $x = 1,\ \ y = 4$
This solution is clearly satisfy by the equation $x + y = 5$.

2. (b) On applying, $R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} x & \alpha & 1 \\ \beta - x & x-\alpha & 1 \\ \beta - x & \gamma-\alpha & 1 \end{vmatrix} = 0 \Rightarrow (\beta - x)(\gamma - \alpha) - (x - \alpha)(\beta - x) = 0$$

$$\Rightarrow (\beta - x)[\gamma - \alpha - x + \alpha] = 0 \Rightarrow (\beta - x)(\gamma - x) = 0 \Rightarrow x = \beta, \gamma$$

3. (c) $(1 + a)[(1 + b)(1 + c) - 1] - 1[1 + c - 1] + 1[1 - 1 - b] = \lambda$.

$$\Rightarrow (1+a)\{b + c + bc\} - c - b = \lambda \Rightarrow bc + ab + ac + abc = \lambda$$

$$abc\left[\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right] + abc = \lambda \Rightarrow abc = \lambda$$

5. (b) $x(x^2 - 56) - 4(7x - 35) + 5(56 - 5x) = 0 \Rightarrow x^3 - 56x - 28x + 140 + 280 - 25x = 0$

$$\Rightarrow x^3 - 109x - 420 = 0 \Rightarrow (x-5)(x-7)(x-12) = 0 \Rightarrow x = -12$$

6. (c) $2 \times 5 \begin{vmatrix} 3a & 3b & 3c \\ l & m & 5n \\ p & q & r \end{vmatrix} = 3\times 10 \begin{vmatrix} a & b & c \\ l & m & n \\ p & q & r \end{vmatrix} = 30 \times 2 = 60$

8. (d) $2 \times 5 \begin{vmatrix} x+1+\omega+\omega^2 & \omega & \omega^2 \\ x+\omega+1+\omega^2 & x+\omega^2 & 1 \\ 1+\omega^2+x+\omega & 1 & x+\omega \end{vmatrix} \Rightarrow x \begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & x+\omega^2 & 1 \\ 1 & 1 & x+\omega \end{vmatrix}$

$\Rightarrow x\,[(x+\omega^2)(x+\omega)+\omega(1-x-\omega)+\omega^2(1-x-\omega^2)]$

$= x\,[x^2+x\omega+x\omega^2+\omega^3+\omega-\omega x-\omega^2+\omega^2+x\omega^2-\omega^4) = x^3$

9. (c) $2[5+5p-0]-4\,[0]+0=20 \Rightarrow 10+10p=20 \Rightarrow p=\frac{10}{10}=1$

10. (c) $1[1-a^2]-c\,[-c-ab]+b\,[ac-b]=1+a^2+c^2+abc-abc+b^2=1+1=2$

11. (a) $A=\begin{vmatrix}k&1&1\\1&k&1\\1&1&k\end{vmatrix}$ $B=\begin{vmatrix}k-1\\k-1\\k-1\end{vmatrix}$ $X=\begin{vmatrix}x\\y\\z\end{vmatrix}$ $\Rightarrow |A|=k^3-3k+2$

Inconsistent, $k^3-3k+2=0 \Rightarrow (k-1)^2(k+2)=0 \Rightarrow k=1$ or -2

13. (b) Let $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$, $B=\begin{bmatrix}c&d\\a&b\end{bmatrix} \Rightarrow |B|=bc-ad \Rightarrow |B|=-(ab-bc) \Rightarrow |B|=-|A|$

14. (b) $abc\begin{vmatrix}-a&a&a\\b&-b&b\\c&c&-c\end{vmatrix}=a^2b^2c^2\begin{vmatrix}-1&1&1\\1&-1&1\\1&1&-1\end{vmatrix}=a^2b^2c^2\,\{-1\,(1-1)-1\,(-1-1)+1\,(1+1)\}=4a^2b^2c^2$

15. (c) $|A_{n\times n}|=3$, $|adj\ A|=243$

$|adj\ (A)|=|A_{n\times n}|^{n-1} \Rightarrow 243=3^{n-1} \Rightarrow 3^5=3^{n-1} \Rightarrow n-1=5 \Rightarrow n=6$

17. (b) $|kAB|=k^3\,|AB|$. Therefore, order of AB is 3×3.

18. (b) $\sin 10° \cos 80° + \sin 80° \cos 10° \Rightarrow \sin(10°+80°)=\sin 90°=1$

19. (d) $10!\ 11!\ 12!\begin{vmatrix}1&11&11\times12\\1&12&12\times13\\1&13&13\times14\end{vmatrix} \Rightarrow 10!\ \ 11!\ \ 12!\ \ (50-48)=2(10!\ \ 11!\ \ 12!).$

20. (c) Use $C_3 \to C_3 + C_2$

$$\begin{vmatrix}a-b&b+c&a+b+c\\b-c&c+a&a+b+c\\c-a&a+b&a+b+c\end{vmatrix}=(a+b+c)\begin{vmatrix}a-b&b+c&1\\b-c&c+a&1\\c-a&a+b&1\end{vmatrix}=a^3+b^3+c^3-3abc.$$

21. (d) If A is on $n\times n$ matrix, then $\det(\lambda A)=\lambda^n \det(A)$

$\Rightarrow \det(\lambda A)=\lambda^S \det(A)$. Thus, on comparing $= S = n$

22. (a) $a[-(b+c)^2]-b[(a+b)(b+c)]+(a+b)\,[b^2+bc-ac-bc]$

$=-ab^2-ac^2-2abc+ab^2+2abc+b^2c=-ab^2+ac^2=0$

23. (c) $C_2 \to C_2 + C_3$

$$\begin{vmatrix}x^2&-2x-2\omega^2&-2\omega^2\\2&0&-\omega\\0&1+\omega&1\end{vmatrix}=0 \Rightarrow \begin{vmatrix}x^2&-2x-2\omega^2&-2\omega^2\\2&0&-\omega\\0&-\omega^2&1\end{vmatrix}=0\ \text{(Expand to } R_3)$$

$\Rightarrow \omega^2(-\omega x^2+4\omega^2)+(4x+4\omega^2)=0 \Rightarrow -x^2+4\omega+4x+4\omega^2=0$

$\Rightarrow -x^2+4x-4=0 \Rightarrow (x-2)^2=0,\ x=2$

24. (a) $x\,(-i-2i^2)+3i\,(-iy-0)+1\,(2yi-0)=6+11i$

$x\,(-i+2)+3y+2yi=6+11i \Rightarrow (2x+3y)+i\,(-x+2y)=6+11i$

On comparing, $2x+3y=6$ and $-x+2y=11$. Thus, we get $x=-3,\ y=4$.

25. (a) $AB=\begin{pmatrix}1&2\\2&3\end{pmatrix}\begin{pmatrix}1&0\\1&0\end{pmatrix}=\begin{pmatrix}3&0\\5&0\end{pmatrix} \Rightarrow |AB|=\begin{pmatrix}3&0\\5&0\end{pmatrix}=0.$

26. (b) $|AB| = |A|\,|B| = -1 \times 3 = -3$

$|3AB| = (3)^3.(-3) = -81$

30. (b) $\frac{d}{dx}|A| = 3(x^2 - ab), |B| = x^2 - ab \Rightarrow \frac{d}{dx}|A| = 3|B|$

31. (c) $a[a^2] + b[-b^2] + 0] = 0 \Rightarrow a^3 - b^3 = 0$

$\Rightarrow (a-b)(a^2 + b^2 + ab) = 0 \Rightarrow a - b\ \frac{a^2}{b^2} + 1 + \frac{ab}{b^2} = 0$

$\frac{a}{b} = 1$ and $\left(\frac{a}{b}\right)^2 + \left(\frac{a}{b}\right) + 1 = 0$ Thus, $\frac{a}{b}$ is one cube roots of unity

32. (c) $C_1 \to C_1 + C_2$, $C_2 \to C_2 + C_3$

$$\begin{vmatrix} 2 & 1 & i \\ 1 & 1+2i & 1+i \\ 1+2i & 1 & 1-i \end{vmatrix} \Rightarrow 2(3+i) - 1(-4i + 2) + i(5-4i) = 8 + 11i$$

33. (b) $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} 0 & 6 & 15 \\ 0 & -2-2x & 5(1-x)^2 \\ 1 & 2x & 5x \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 0 & 6 & 1 \\ 0 & -(1+x) & 1-x^2 \\ 1 & x & x^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+x)\begin{vmatrix} 0 & 1 & 1 \\ 0 & -1 & 1-x \\ 1 & x & x^2 \end{vmatrix} = 0 \Rightarrow (1+x)(2-x) = 0\ ;\ x = -1, 2.$$

35. (a) $$\begin{vmatrix} (a^2 - 2ax + x^2) & (a^2 + y^2 - 2ay) & (a^2 + z^2 - 2az) \\ (b^2 - 2bx + x^2) & (b^2 + y^2 - 2by) & (b^2 + z^2 - 2bz) \\ (c^2 - 2cx + x^2) & (c^2 + y^2 - 2cy) & (c^2 + z^2 - 2cz) \end{vmatrix} = \begin{vmatrix} a^2 & -2a & 1 \\ b^2 & -2b & 1 \\ c^2 & -2c & 1 \end{vmatrix} \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

36. (a) $$\begin{vmatrix} \frac{(n-1)n}{2} & n & 6 \\ \frac{n(n-1)(2n-1)}{6} & 2n^2 & 4x-2 \\ \frac{(n-1)^2 n^2}{4} & 3n^3 & 3n^2 - 3n \end{vmatrix}$$

$\frac{n(n-1)}{2}$ common from C_1 and 6 from C_3, we get

$$= 3n(n-1)\begin{vmatrix} 1 & n & 6 \\ \frac{2n-1}{3} & 2n^2 & \frac{2n-1}{3} \\ \frac{n(n-1)}{2} & 3n^3 & \frac{n(n-1)}{2} \end{vmatrix}$$. Thus, C_1 and C_2 are identical so $\Delta = 0$.

38. (b) $\Delta = \begin{vmatrix} 2 & p & 6 \\ 1 & 2 & q \\ 1 & 1 & 3 \end{vmatrix} = 2(6-q) - p(3-q) + 6(1-2) = (p-2)(q-3)$

when $\Delta \neq 0$, i.e., $p \neq 2$ and $q \neq 3$.

39. (c) $px^3 + qx^2 - rx - r = 0$, roots are α, β, γ

$$\alpha + \beta + \gamma = \frac{-q}{p};\ \alpha\beta + \beta\gamma + \gamma\alpha = \frac{-r}{p},\ \alpha\beta\gamma = \frac{r}{p}$$

$$D = \alpha\beta\gamma\left(1 + \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right) = \alpha\beta\gamma\left(\frac{\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma}\right) = 0$$

41. (d) $a_1 = f'(0)$, $f'(x) = \begin{vmatrix} 2x & 1 & x+1 \\ 4x & 1 & x+2 \\ 6x & 1 & x+3 \end{vmatrix} + 0 + 0 = 0$. Thus, $f'(0) = 0$. Therefore, $a_1 = 0$

42. (b) For non-trival solution $\Delta = 0$

$$\begin{vmatrix} 6 & 5 & \lambda \\ 3 & -1 & 4 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow -30 + 65 + 7\lambda = 0 \Rightarrow \lambda = -5$$

43. (c) λ (In) Adj (λ In) = | λ In | I

Adj (λ In) = λ^{n-1} In $\Rightarrow$ |Adj (λ In)| = $\lambda^{n(n-1)}$

44. (a) $$\Delta'(x) = \begin{vmatrix} e^x & 2\cos 2x & 2x\sec^2 x^2 \\ \ln(1+x) & \cos x & \sin x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \frac{1}{1+x} & -\sin 2x & \cos x \\ \cos x^2 & e^x - 1 & \sin x^2 \end{vmatrix} + \begin{vmatrix} e^x & \sin 2x & \tan x^2 \\ \ln(1+x) & \cos x & \sin x \\ -2x\sin x^2 & e^x - 1 & 2x\cos x^2 \end{vmatrix}$$

$$B = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{vmatrix} = 0 \quad \text{[Put } x = 0]$$

45. (c) Applying $C_1 \rightarrow C_1 + C_1$, we get $\begin{vmatrix} 2 & \cos^2 x & \sin 2x \\ 2 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - R_1$

$$\begin{vmatrix} 2 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 2 + \sin^2 x.$$ Maximum value is 3 Minimum value is 1.

47. (c) $g(x) = a(a + x)^2$

$\therefore g(2x) - g(x) = a(a + 2x)^2 - a(a + x)^2 = ax\,(2a + 3x)$

48. (a) p, q, r are the entries of first row and C_{21}, C_{22}, C_{23} are the co fectors of second row

$$pC_{21} + qC_{22} + rC_{23} = 0$$

49. (c) $qM_{12} - yM_{22} + mM_{32} = -qC_{12} - yC_{22} + mC_{32} = -(-qC_{12} - yC_{22} + mC_{32}) = -\Delta$.

50. (b) Possible matrix x are : 1×32, 2×16, 4×8, 32×1, 16×2, 8×4. i.e., 6

52. (c) $$\begin{vmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow (2 - \lambda)(3 - \lambda + \lambda^2 - 4) + 2(-2\lambda + 2) + 1(4 - 3 - \lambda) = 0$

$\Rightarrow (2 - \lambda)(\lambda + 4)(\lambda - 1) + 5(1 - \lambda) = 0 \Rightarrow \lambda = 1, 1, -3$

53. (c) $\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \alpha-1 & \beta-1 \\ 1 & \alpha^2-1 & \beta^2-1 \end{vmatrix}^2$

$= (\alpha-1)(\beta^2-1) - (\beta-1)(\alpha^2-1)^2 = (\alpha-1)^2(\beta-1)^2 - (\alpha-\beta)^2 \Rightarrow k = 1$

54. (c) For no solution

$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1} \Rightarrow (k+1)(k+3)\ 8k = 0 \Rightarrow k^2 = 4k + 3 = 0 \Rightarrow k = 1, 3$ ...(i)

But for $k = 1$, eq is not satisfied eq. (i). Thus, $k = 3$.

56. (c) $\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} \Rightarrow 1(-3+k) + k(-k+3k) + 1(k-9) = 0$

$\Rightarrow k - 3 + 2k^2 + k - 9 = 0 \Rightarrow 2k^2 + 2k - 12 = 0 \Rightarrow k = 2, -3$

Thus, R – {2, –3}

57. (b) $D = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{vmatrix} = 0, D_1 = \begin{vmatrix} 3 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 2 \end{vmatrix} \neq 0$

Given statement, does not have any solution therefore, no solution.

59. (b) $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$\begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy$. Hence [D] is divisible by both x and y.

62. (b) $R_3 \to R_3 - (xR_1 + R_2)$

$\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ 0 & 0 & -(ax^2+2bx+x) \end{vmatrix} = (ax^2 + abx + c)(b^2 - ac) =$ negative.

63. (a) $\Delta = \begin{vmatrix} 1+\omega^n+\omega^{2n} & \omega^n & \omega^{2n} \\ 1+\omega^n+\omega^{2n} & \omega^{2n} & 1 \\ 1+\omega^n+\omega^{2n} & 1 & \omega^n \end{vmatrix} = 0$

$1 + \omega^n + \omega^{2n} = 0$, '$n$' is not a multiple of 3. Thus, roots are identical.

64. (c) Coefficient of determinant $= \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \Rightarrow b = \frac{2ac}{a+c}$

65. (c) $\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0 \Rightarrow (1+abc)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0$ Thus, $abc = -1$

66. (b) $C_1 - C_2$, $C_2 - C_3$. Thus, two rows become identical. So $\Delta = 0$

67. (d) $6i(3i^2+3) + 3i(4i+20) + 1(12-60i) = -12 + 60i + 12 - 60i = 0$

68. (b) $R_1 \to R_1 - R_3$ $\qquad\qquad\qquad$ $R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & 1 & 1 \\ 1-\omega^2 & -1-\omega^2 & \omega^2 \\ 1-\omega^4 & \omega^2 & \omega^4 \end{vmatrix} \begin{vmatrix} 0 & 0 & 1 \\ 1-\omega^2 & -1-2\omega^2 & \omega^2 \\ 1-\omega^4 & \omega^2-\omega^4 & \omega^4 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 1-\omega^2 & \omega-\omega^2 & \omega^2 \\ 1-\omega & \omega^2-\omega & \omega^4 \end{vmatrix}$$ [Expanding = $3\omega\ (\omega - 1)$]

70. (d) $\dfrac{d}{dx}f(x) = \begin{vmatrix} 3x^2 & \cos x & -\sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, $\dfrac{d^2}{dx^2}f(x) = \begin{vmatrix} 6x & -\sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$

$$\frac{d^3}{dx^3}f(x) = \begin{vmatrix} 6 & \cos x & \sin x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}_{\text{at } x=0} = \begin{vmatrix} 6 & -1 & 0 \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix} = 0$$

71. (b) $\dfrac{k+1}{k} = \dfrac{8}{k+3} = \dfrac{4k}{3k-1} \Rightarrow \dfrac{8}{k+3} = \dfrac{4k}{3k-1} \Rightarrow 5k = 5 \Rightarrow k = 1.$

73. (a) $\begin{vmatrix} 1 & \dfrac{\log y}{\log x} & \dfrac{\log z}{\log x} \\ \dfrac{\log x}{\log y} & 1 & \dfrac{\log z}{\log y} \\ \dfrac{\log x}{\log z} & \dfrac{\log y}{\log z} & 1 \end{vmatrix}$ On dividing and multiplying R_1, R_2, R_3 by $\log x$, $\log y$ and $\log z$.

$$= \frac{1}{\log x \log y \log z}\begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

75. (a) Apply $C_2 \to C_2 + C_1 + C_3$, we get $-x\begin{vmatrix} 1 & 1 & 1 \\ -6 & 3-x & 3 \\ 3 & 3 & -6-x \end{vmatrix}$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$ we get $\begin{vmatrix} 1 & 1 & 1 \\ -6 & 9-x & 9 \\ 3 & 0 & -9-x \end{vmatrix}$ expand it.

76. (d) $C_3 \to (C_1 + C_2)$

$f(x) = \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix}$. Thus $f(x) = 0$ for all real x.

77. (d) $\begin{vmatrix} 1 & -k & -1 \\ k & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0 \Rightarrow 1(1 + 1) + k(-k + 1) - 1(k + 1) = 0$

$\Rightarrow -k^2+1 = 0 \Rightarrow k = \pm 1.$

78. (a) $R_1 \to R_1 + R_2 + R_3$

$$f(x) = \begin{vmatrix} x-9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = (x+9)\begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = (x+9)\begin{vmatrix} 1 & 0 & 0 \\ 2 & x-2 & 0 \\ 7 & -1 & 7 \end{vmatrix} = (x+9)\ (x-2)\ (x-7)$$

79. (d) $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$\therefore$ A = {cos (θ + γ) − cos (θ + α)} {sin(θ + β)} −sin(θ + α)}

− {cos (θ + β) − cos (θ + α)}{sin (θ + γ) − sin(θ + α)} = sin (β + γ) − sin (β − α) − sin (α − γ)

$\therefore$ This is indepent of θ.

80. (c) $(2\cos x + \sin x)\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & 0 \\ 1 & \cos x & \sin x - \cos x \end{vmatrix} = 0 \Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\therefore$ $\tan x = -2, 1$. We take $\tan x = 1$, so $x = \dfrac{\pi}{4}$

81. (b) $A^{-1} = \dfrac{adj\,A}{|A|}, -2$ if λ is eign value of A then λ^{-1} is eign value of A^{-1}

$adj\,(A)\,X = (A^{-1}X)\,|A| = |A|\ \lambda^{-1} I$

Thus eigen value corresponding to $\lambda = 3$ is $\dfrac{4}{3}$ and $\lambda = -2$ is -2.

82. (d) $\therefore$ matrix is singular.

$(R_2 \to R_2 - R_1)$ $(R_3 \to R_3 - R_1)$

$\begin{vmatrix} 3 & -1+x & 2 \\ 0 & -x & x \\ x & -x & -0 \end{vmatrix} = 0$ $[C_1 \to C_1 + C_2 + C_3] \Rightarrow \begin{vmatrix} x+4 & -1+x & 2 \\ 0 & -x & x \\ 0 & -x & 0 \end{vmatrix} = 0 \Rightarrow (x+4)(x^2) = 0$. Thus $x \to -4, 0$ $4 \in [-4, -1]$

83. (a) Ratio of cofactor to its minor of the element which is in the 3rd row and 2nd column $(-1)^{3+2} = -1$.

85. (c) $1\begin{vmatrix} 3 & 6 \\ -7 & 9 \end{vmatrix} - 2\begin{vmatrix} -4 & 6 \\ 2 & 9 \end{vmatrix} + 3\begin{vmatrix} -4 & 3 \\ 2 & -7 \end{vmatrix} = 135 - (18 - 114) = 231$

87. (b) $D = abc\begin{vmatrix} a & a & a \\ b & b & b \\ c & c & c \end{vmatrix} = 0$

89. (b) $\Delta = \begin{vmatrix} 3^2+k & 4^2 & 3 \\ 4^2+k & 5^2 & 4 \\ 5^2+k & 6^2 & 5 \end{vmatrix} = 0$ $[C_1 \to C_3 - C_1]$

$\Rightarrow \begin{vmatrix} 9+k & 16 & 3 \\ 16+k & 25 & 4 \\ 25+k & 36 & 5 \end{vmatrix} = \begin{vmatrix} 9+k & 16 & 3 \\ 7 & 9 & 1 \\ 9 & 11 & 1 \end{vmatrix} = 0 \Rightarrow k - 1 = 0 \Rightarrow k = 1$

90. (d) Apply $C_3 \to C_3 - C_2$, $C_3 \to C_2 - C_1 \Rightarrow \begin{vmatrix} 13 & 3 & 3 \\ 14 & 3 & 3 \\ 15 & 3 & 3 \end{vmatrix} = 0$

92. (a) $pqr\,(a^3 + b^3 + c^3) - abc\,(p^3 + q^3 + r^3) = pqr\,(3abc) - abc\,(3pqr) = 0$.

93. (a) $xyz\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = 0$ Then, $xyz = 0$

97. (d) $\frac{1}{abc}\begin{vmatrix}1 & a^3 & abc\\ 1 & b^3 & abc\\ 1 & c^3 & abc\end{vmatrix} = \frac{abc}{abc}\begin{vmatrix}1 & a^3 & 1\\ 1 & b^3 & 1\\ 1 & c^3 & 1\end{vmatrix} = 0$

98. (b) $R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$

$$\begin{vmatrix}31 & 129 & 92\\ 0 & 0 & -21\\ 0 & 0 & 47\end{vmatrix} = 0 \ (C_2 \to C_2 + C_3)$$

99. (a) $C_1 \to C_1 + C_2$ $\begin{vmatrix}1 & \cos^2 x & 1\\ 1 & \sin^2 x & 1\\ 2 & 12 & 2\end{vmatrix} = 0$

100. (i) (d) $B_1 - 2B_2 + 3B_3 = 0$ is correct relation

(ii) (c) $A = \begin{pmatrix}1 & 1 & 1\\ 1 & 0 & -2\\ 1 & -1 & 1\end{pmatrix}$ them $A^{-1} = \frac{1}{6}\begin{pmatrix}2 & 2 & 2\\ 3 & 0 & -3\\ 1 & -2 & 1\end{pmatrix}$

$$(A^1)^{-1} = (A^{-1})^1 = \frac{1}{6}\begin{pmatrix}2 & 2 & 1\\ 3 & 0 & -2\\ 1 & -3 & 1\end{pmatrix} = \begin{pmatrix}\frac{1}{3} & \frac{1}{2} & \frac{1}{6}\\ \frac{1}{3} & 0 & \frac{-1}{3}\\ \frac{1}{3} & \frac{-1}{2} & \frac{1}{6}\end{pmatrix}$$

(iii) (b) $A'X = B$

$$A' = \begin{pmatrix}1 & 1 & 1\\ 1 & 0 & -1\\ 1 & -2 & 1\end{pmatrix} x = \begin{pmatrix}B_1\\ B_2\\ B_3\end{pmatrix} \text{ and } B = \begin{pmatrix}45\\ -8\\ 0\end{pmatrix}$$

$$X = (A')^{-1}\,B = \frac{1}{6}\begin{bmatrix}2 & 3 & -1\\ 2 & 0 & -2\\ 2 & -3 & 1\end{bmatrix}\begin{bmatrix}45\\ -8\\ 0\end{bmatrix} = \begin{bmatrix}11\\ 15\\ 19\end{bmatrix} = 11 : 15 : 19.$$

101. (i) (a) p, q, r are the entries of first row and C_{21}, C_{22}, C_{23} are the cofactor of second row.
$\therefore pC_{21} + q\cdot C_{22} + r\cdot C_{23} = 0.$

(ii) (c) x, y, z are the entries of second row. $xC_{21} + yC_{22} + zC_{23} = \Delta$

(iii) (b) $qM_{12} - cM_{22} + mM_{32} = -qC_{12} - yC_{22} - mC_{32} = -(-qC_{12} - yC_{22} - mC_{32}) = -\Delta$

(iv) (d) Take 3 common from each column or row.

(v) (b) On interchanging the rows and columns the value of determining remain same.