Chapter - 12 LINEAR PROGRAMMING

STUDY NOTES

- **Linear Programming Problem :** Linear Programming problem that is concerned with finding the optimal value (maximum or minimum) of a linear function called objective function of variables x and y, called decision variables, subject to the conditions that the variable are non-negative. A linear programming problem is a special type of optimisation problem.
- **Objective Functions :** A linear function $Z = cx + dy$ (c and d are constants) which has to be maximised or minimised is called an objective functions.
- **Constraints :** The linear inequalities or equations or restrictions on the variables of the linear programming problem are called constraints. The conditions $x \geq 0$, $y \geq 0$ are called non-negative restrictions.
- **Optimal Value :** The maximum or minimum value of an objective function is known as its optimal value.
- **Solution of a Linear Programming Problem**
  - (i) **Feasible Solution :** Any solution satisfying all the constraints of a L.P.P. is called a feasible solution.
  - (ii) **Optimal Feasible Solution :** Any feasible solution which maximises or minimizes the objective function is called an optimal feasible solution.
  - (iii) **Bounded and Unbounded Region :** A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed with in a circle. Otherwise, it is called unbounded.
- **Graphical Method of Solving LPP :**

  Step 1 : Find the feasible region of the linear programming problem and determine its corner points or by solving two equations of the lines.

  Step 2 : Find the value of Z from each corner points.

  Step 3 : If Z is maximum then $ax + by > Z_{max}$. If Z is minimum $ax + by < Z_{min}$
- **Corner Point Method for Solving a LPP :**

  The method comprises of the following steps :
  - (i) Find the feasible region of the LPP and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
  - (ii) Evaluate the objective function $Z = ax + by$ at each corner point.
  - (iii) **When the feasible region is bounded :** M and N are respectively, the maximum and minimum values of Z.

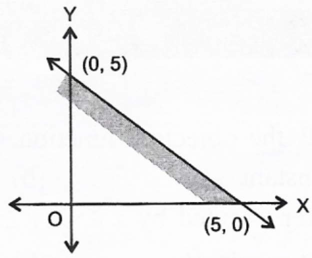
  When the feasible region is unbounded :
  - (a) M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no maximum value.
  - (b) Similarly, N is the minimum of Z, if the open half plane determined by $ax + by < N$ has no point in common with the feasible region, otherwise, Z has no minimum value.

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. In a LPP, the objective function is always
   (a) Constant (b) Linear (c) Variable (d) none of these
2. Region represented by $x \geq 0, y \geq 0$ is :
   (a) first quadrant (b) second quadrant (c) third quadrant (d) fourth quadrant
3. The optimal value of the objective function is attained at the points :
   (a) on x-axis (b) on y-axis
   (c) corner points of the feasible region (d) none of these
4. An optimisation problem may involve finding:
   (a) maximum profit (b) minimum cost
   (c) minimum use of resources (d) all of these
5. The conditions $x \geq 0, y \geq 0$ are called :
   (a) restrictions only (b) negative restrictions
   (c) non-negative restrictions (d) none of the above
6. The variables x and y in a linear programming problem are called :
   (a) decision variable (b) linear variable (c) optimal variable (d) none of these
7. The linear inequalities or equations or restrictions on the variable of a linear programming problem are called
   (a) linear relations (b) constraints (c) function (d) objective functions
8. Which of the term is not used in a linear programming problem?
   (a) optimal solution (b) feasible solution (c) concave region (d) objective function
9. The objective function of a LPP is:
   (a) a constraint (b) function to be optimised
   (c) a relation between the variable (d) none of these
10. A corner point of a feasible region is a point in the region which is the __________ of two boundary lines.
    (a) intersection (b) parallel (c) coincidence (d) perpendicular
11. In a LPP if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then every point on the line segment joining these two points give the same __________ value.
    (a) maximum (b) minimum (c) zero (d) none of these
12. The linear programming problem minimize
    $Z = 3x + 2y$ subject to the constraints $x + y \geq 8$,
    $3x + 5y \leq 15$, $x \geq 0$ and $y \geq 0$ has :
    (a) one solution (b) no feasible solution
    (c) two solutions (d) infinitely many solutions
13. If $Z = 11x + 7y$ be the objective function and the minimum value of $Z = 21$, the minimum value occurs at point:
    (a) (0, 5) (b) (0, 3) (c) (3, 2) (d) (2, 3)
14. The maximum value of $Z = 4x + 3y$ is 112. The maximum value occurs at point :
    (a) (0, –24) (b) (0, – 40) (c) (16, 16) (d) (48, 0)
15. Two persons P and Q earn ₹300 and ₹400 per day respectively. P can stitch 6 shirts and 4 pairs of trousers, while Q can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce atleast 60 shirts and 32 pairs of trousers at a minimum labour cost.
    (a) P = 5 days and Q = 3 days (b) P = 3 days and Q = 5 days
    (c) P = 5 days and Q = 5 days (d) P = 3 days and Q = 3 days

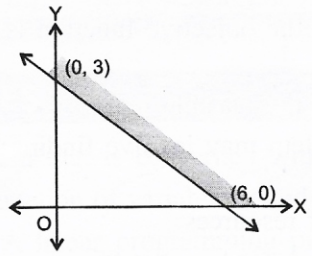
16. Shaded region is represented by



(a) $x + y \leq 5$ (b) $x + y \leq -5$

(c) $x + y \geq 5$ (d) $x + y \geq -5$

17. Shaded region is represented by



(a) $x + 2y \leq 6$ (b) $x + 2y \leq -6$

(c) $x + 2y \geq 6$ (d) $x + 2y \geq -6$

18. If the objective function for a LPP is Z = $x + y$ and the corner points of the bounded feasible region are (0, 6), (2, 3) and (11, 0), then the minimum value of Z occurs at:

(a) (0, 6) (b) (2, 3) (c) (11, 0) (d) none of these

19. If the objective function for a LPP is Z = $4x + 3y$, and the corner points of the bounded feasible region are (0, 0), (0, 24), (25, 0) and (16, 16), then the maximum value of Z occurs at:

(a) (0, 0) (b) (0, 24) (c) (25, 6) (d) (16, 16)

20. Maximize Z = 8000 x + 12000 y, subject to the constraints

$3x + 4y \leq 60$

$x + 3y \leq 30$ and $x \geq 0, y \geq 0$

(a) Maximum Z = 0 at (0, 0) (b) Maximum Z = 1,60,000 at (20, 0)

(c) Maximum Z = 1,68,000 at (12, 6) (d) Maximum Z = 1,20,000 at (0, 10)

21. Minimize Z = $6x + 3y$, subject to the constraints $4x + y \geq 80$, $x + 5y \geq 115$, $3x + 2y \leq 150$

$x \geq 0, y \geq 0$

(a) Minimum Z = 285 at (40, 15) (b) Minimum Z = 150 at (15, 20)

(c) Minimum Z = 228 at (2, 72) (d) Minimum Z = 0 at (0, 0)

22. Maximize Z = $3x + 2y$, subject to the constraints $x + 2y \leq 10$, $3x + y \leq 15$; $x \geq 0, y \geq 0$

The area of the feasible region is:

(a) 15.5 sq units (b) 17.5 sq units (c) 18.5 sq units (d) 16.5 sq units

23. Maximize Z = $x + y$ subject to the constraints $x - y \leq -1$, $-x + y \leq 0$; $x \geq 0, y \geq 0$ is :

(a) Maximum Z = 150 (b) Maximum Z = 250

(c) Maximum Z = 180 (d) Maximum value of Z does not exist

24. The corner points of the feasible region determined by the following system of linear inequalities $2x + y \leq 10$, $x + 3y \leq 15$, $x, y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5).

Let Z = $ax + by$, where $a, b > 0$

Condition on a and b, so that the maximum of Z occurs at both (3, 4) and (0, 5) is:

(a) $a = b$ (b) $a = 2b$ (c) $a = 3b$ (d) $b = 3a$

25. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is $Z = 4x + 3y$.
Compare the quantity in column A and column B

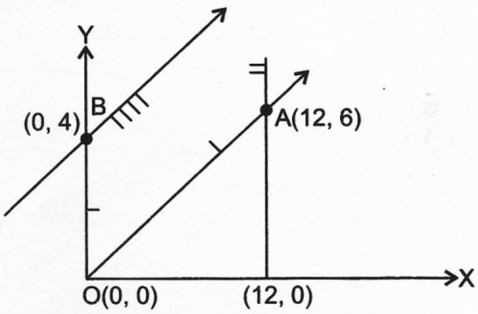
Column A	Column B
Maximum of Z	325

(a) The quantity in column A is greater
(b) The quantity in column B is greater
(c) The two quantities are equal
(d) The relationship can not be determined on the basis of information supplied.

26. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0) let $Z = px + qy$, where $p, q > 0$, condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is :

(a) $p = 2q$ (b) $p = \frac{q}{2}$ (c) $p = 3q$ (d) $p = q$

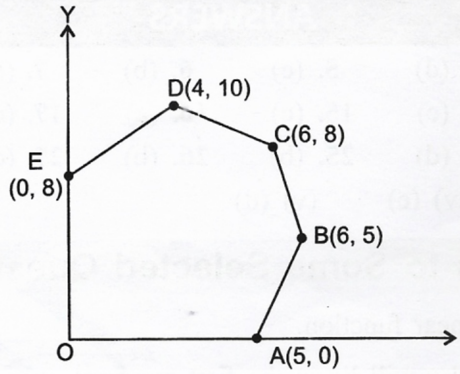
27. The feasible region for an LPP is shown in the figure.
Let $F = 3x - 4y$ be the objective function. Maximum value of F is :
(a) 0 (b) 8
(c) 12 (d) –18



28. For the following LPP :
Maximum $Z = -0.1x_1 + 0.5x_2$
$2x_1 + 5x_2 \leq 80$
$x_1 + x_2 \leq 20$
$x_1, x_2 \geq 0$
to get the optimum solution, the values of x_1, x_2 are :

(a) (20, 0) (b) $\left(\frac{20}{3}, \frac{40}{3}\right)$ (c) (0, 16) (d) (8, 0)

29. The feasible solution for a LPP is shown in figure.
Let $Z = 3x - 4y$ be the objective function. Minimum of Z occurs at :



(a) (0, 0) (b) (0, 8) (c) (5, 0) (d) (4, 10)

30. A farmer mixes two brands P and Q of cattle feed. Brand P costing ₹250 per bag, contains 3 units of nutritional element A, 2.5 units of element B and 2 units of element C. Brand Q costing ₹200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B, and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. What is the minimum cost of the mixture per bag?

(a) ₹2650 (b) ₹1950 (c) ₹4500 (d) ₹2400

INPUT TEXT BASED MCQ's

31. Corner point of the feasible region for an LPP are (0, 2), (4, 0), (3, 5), (0, 6). Let Z = $2x - 3y$ be the objective function.

Answer the following questions :

(i) The maximum value of Z occurs at

(a) (0, 2) (b) (5, 3) (c) (0, 6) (d) (4, 0)

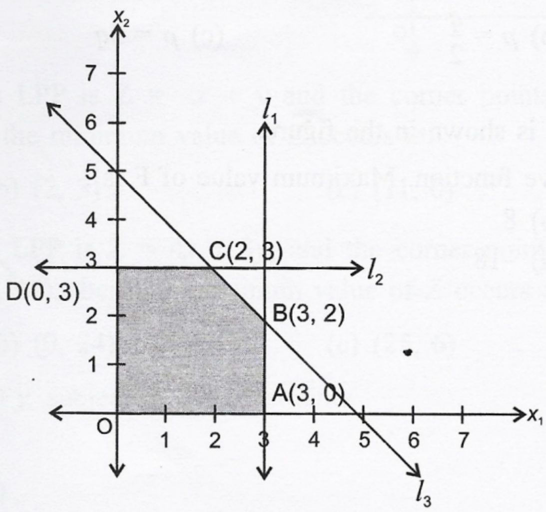
(ii) Minimum value of Z occurs at

(a) (0, 6) (b) (0, 2) (c) (4, 0) (d) (3, 5)

(iii) Maximum of Z. Minimum of Z =

(a) 28 (b) 26 (c) 30 (d) 42

(iv) The corner points of the feasible region determined by the system of linear inequalities are



(a) (0, 0), (–3, 0), (3, 2), (2, 3) (b) (3, 0), (3, 2), (2, 3), (0, –3)

(c) (0, 0), (3, 0), (3, 2), (2, 3), (0, 3) (d) None of these

(v) The feasible solution of LPP belongs to

(a) first and second quadrant (b) first and third quadrant

(c) only second quadrant (d) only first quadrant

ANSWERS

1. (b)	**2.** (a)	**3.** (c)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (a)
11. (d)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (b)	**19.** (d)	**20.** (c)
21. (b)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (b)	**26.** (b)	**27.** (c)	**28.** (c)	**29.** (b)	**30.** (b)

31. (i) (d) (ii) (a) (iii) (b) (iv) (c) (v) (d)

Hints to Some Selected Questions

1. (b) Objective function is always linear function.

2. (a) All the positive values of x and y will lie in the first quadrant.

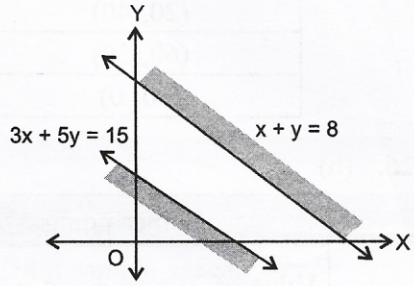
3. (c) Any point in the feasible region that gives the optimal value (maximum or minimum) of the objective function is called an optimal solution.

4. (d) An optimisation problem involve maximum profit; minimum cost and minimum use of resources.

5. (c) non negative restrictions

6. (b) The variables x and y in a LPP are called linear variable

7. (b) The linear inequalities or restrictions on the variables of a LPP called constraints
8. (c) concave region is not used in a LPP.
9. (b) The objective function of a LPP is function to be optimised
10. (a) intersection
11. (d) Maximum
12. (b) There is no point, which can satisfy all the constraints.



13. (b) $Z = 11x + 7y$

For point (0, 3)

$Z = 11 \times 0 + 7 \times 3 = 0 + 21 = 21$

14. (c) Max $Z = 4x + 3y$. For point (16, 16); $x = 16$ and $y = 16$

Max. $Z = 4 \times 16 + 3 \times 16 = 64 + 48 = 112$

16. (a) $x + y \leq 5$

At point (0, 0)

$0 + 0 \leq 5$

$0 \leq 5$. It is true.

17. (c) $x + 2y \geq 6$

At point (0, 0) ; $0 + 0 \geq 6$

$0 \geq 6$. It is false.

19. (d) $Z = 4x + 3y$ *at point (16, 16)*

$Z_{max} = 4 \times 16 + 3 \times 16 = 64 + 48 = 112$

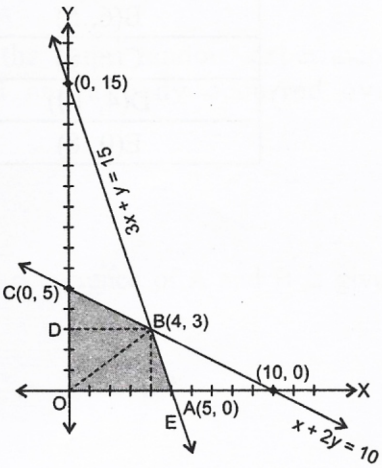
20. (c) Maximum $Z = 168000$ at (12, 6)
21. (b) Minimum $Z = 150$ at (15, 20)
22. (b) Area of feasible region = Area of ΔBOC + Area of ΔOAB

$= \frac{1}{2} \times OC \times BD + \frac{1}{2} \times OA \times BE$

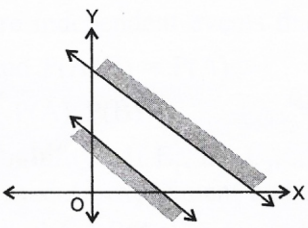
$= \frac{1}{2} \times 5 \times 4 + \frac{1}{2} \times 5 \times 3$

$= 10 + \frac{15}{2} = 10 + 7.5$

$= 17.5$ sq units



23. (d) does not exist



24. (d) $a \times 3 + b \times 4 = a \times 0 = b \times 5$

$3a + 4b = 5b$

$3a = b \Rightarrow b = 3a$

25. (b)

Corner points	Value of Z = 4x + 3y	
(0, 0)	Z = 0	
(0, 40)	Z = 0 + 3(40) = 120	
(20, 40)	Z = 4(20) + 3(40) = 200	
(60, 20)	Z = 4(60) + 3(20) = 300	Maximum
(60, 0)	Z = 4(60) + 3(0) = 240	

26. (b)

Corner points	(0, 3)	(1, 1)	(3, 0)
Value of Z = $px + qy$; $p, q > 0$	$Z = p(0) + q(3) = 3q$	$Z = p(1) + q(1) = p + q$	$Z = p(3) + q(0) = 3p$

So, condition of p and q that the minimum of Z occurs at (3, 0) and (1, 1) is :

$$p + q = 3p \Rightarrow p - 3p + q = 0 \Rightarrow p = \frac{q}{2}$$

27. (c) The feasible region is shown in the figure for which the objective function $F = 3x - 4y$

Corner points	O(0, 0)	A(12, 6)	B(0, 4)
Value of $F = 3x - 4y$	F = 0	F = 3 × 12 – 4 × 6 = 12	F = 0 – 4 × 4 = –16
		Maximum	Minimum

29. (b)

Corner points	Value of Z = 3x – 4y	
O(0, 0)	Z = 0	
A(5, 0)	Z = 3(5) – 0 = 15	
B(6, 5)	Z = 3(6) – 4(5) = –2	
C(6, 8)	Z = 3(6) – 4(8) = –14	
D(4, 10)	Z = 3(4) – 4(10) = –28	
E(0, 8)	Z = 3(0) – 4(8) = –32	Minimum