Chapter - 3 MATRICES

STUDY NOTES

- **Matrix :** A matrix is an ordered rectangular array of numbers or functions. The numbers or functions are called elements of the matrix.
- **Order of a matrix :** A matrix having 'm' rows and 'n' columns is called matrix of order $m \times n$.
  $A = [a_{ij}]_{m \times n}$ denotes the order of matrix is $m \times n$.
- **Types of Matrices :**
  - **Column Matrix :** If a matrix has only one column no row, then it is called column matrix.
    $X = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ is column matrix.
  - **Row matrix :** If a matrix has only one row no column, then it is called row matrix. X = [1 8 3] is a row matrix.
  - **Square matrix :** When the number of rows are equal to the number of columns in a matrix, then it is called square matrix.
    $X = \begin{bmatrix} 2 & 5 \\ 0 & 8 \end{bmatrix}$ square matrix of order 2 × 2. And $Y = \begin{bmatrix} 2 & 5 & 0 \\ 8 & 6 & 8 \\ 1 & 0 & 9 \end{bmatrix}$ square matrix of order 3 × 3.
  - **Diagonal Matrix :** A square matrix is said to be diagonal matrix if all its non-diagonal elements are zero.
    $X = \begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix}$ is a diagonal matrix of order 2 × 2. And $Y = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ is a diagonal matrix of order 3 × 3.
  - **Scalar Matrix :** A diagonal matrix is said to be a scalar matrix it its all the diagonal elements are equal.
    $X = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ scalar matrix of order 2 × 2. And $Y = \begin{bmatrix} \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$ scalar matrix of order 3 × 3.
  - **Identity matrix :** A square matrix in which elements in the diagonal are all 1 and rest are all zero is called identity matrix.
    $X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ identity matrix of order 2. And $Y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ identity matrix of order 3.
  - **Zero matrix or Null matrix :** A matrix is said to be zero matrix whose all the elements are zero. It is denoted by O.

$$[0], \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- **Operations on Matrices :**
  - **Equality of matrices :** Two or more matrices are equal if
    (i) they are of same order. (ii) each element of matrices are equal such that $a_{ij} = b_{ij}$ for all i and j.
  - **Addition of matrices :** The sum of two or more matrices are possible when they have same order. The sum of two matrices are obtain by adding the corresponding elements of the given matrices.

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix},\ Y = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \end{bmatrix};\ X + Y = \begin{bmatrix} x_{11}+y_{11} & x_{12}+y_{12} & x_{13}+y_{13} \\ x_{21}+y_{21} & x_{22}+y_{22} & x_{23}+y_{23} \end{bmatrix}$$

  - **Difference of matrices :** If $X = [x_{ij}]$ and $Y = [y_{ij}]$ are two matrices of the same order, then difference A – B is defined as $D = x_{ij} - y_{jk}$
  - **Multiplication of matrices by a scalar :**
    Let X be a $m \times n$ matrix and 'k' be any scalar.
    Then the Matrix obtained by multiplying every element of X by 'k' is called scalar multiple of A and is denoted by kX.
    Properties of scalar multiplication :
    (a) $k(X + Y) = kX + kY$ (b) $(k + m)X = kX + mX$ (c) $(k.m)X = k(m.X) = m(kX)$
    (d) $(-k)X = -(kX) = k(-X)$ (e) $-1(X) = -X$
  - **Properties of matrix addition :**
    (i) Matrix addition is commutative : X + Y = Y + X, where X, Y are matrices of same order.
    (ii) Matrix addition is associative : (X + Z) + Y = X + (Z + Y), where X, Y, Z are matrices of same order.
    (iii) Existence of additive identity : A + O = O + A = A, where A is any matrix and O is the identity matrix.
    (iv) Existence of additive inverse : $X = [x_{ij}]_{m \times n}$. Then the negative of the matrix A is defined as the matrix $[-x_{ij}]_{m \times n}$. It is denoted by –X.
  - **Properties of matrix multiplication :**
    (i) Matrix multiplication is associative (XY)Z = Z(XY).
    (ii) Multiplication of matrices is distributive over addition of matrices X (Y + Z) = XY + XZ.
    (iii) The multiplication of matrices is not always commutative.
    (iv) Whenever XY and YX both exist and are matrices of the same order, it is not necessary that XY = YX.
    (v) The product of two matrices can be a zero matrix.
    (vi) If XY = YX → X and Y are square matrices.
    (vii) If A is a matrix of order $n \times m$, then $AI_n = A = I_nA$.
  - **Transpose of a Matrix :** If $X = [x_{ij}]$ be a $m \times n$ matrix, then the matrix obtained by interchanging the rows and columns of A is called transpose of X. Transpose of the matrix A is denoted by X^T.
    $X = [x_{ij}]_{m \times n}$; $X^T = [x_{ji}]_{n \times m}$
    Properties of transpose of a Matrices :
    (i) $(X^T)^T = X$ (ii) $(KA)' = KA'$ (iii) $(A + B)' = A' + B'$ (iv) $(A B)' = B'A'$
  - **Symmetric and skew symmetric Matrices :**
    (i) **Symmetric Matrix :** A square matrix $A = [a_{ij}]$ is called symmetric matrix if $a_{ij} = a_{ji}$ for all i, j or $A^T = A$.
    (ii) **Skew Symmetric matrix :** A square matrix $A = [a_{ij}]$ is called skew symmetric if $a_{ij} = a_{ji}$ for all i, j. $A^T = -A$.

16. If A is a 3 × 3 non-singular matrix such that AA′ = A′A and B = A^{-1} A′, then BB′ equals :

(a) I + B (b) B^{-1} (c) I (d) $(B^{-1})^T$

17. Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If x_1 and x_2 are column matrices such that $Ax_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Ax_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ then $x_1 + x_2$ is equal to :

(a) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

18. If A = $\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of 3×3 matrix A, and |A| = 4, then α is equal to :

(a) 11 (b) 0 (c) 5 (d) 4

19. If $w \neq 1$ is the complex cube root of using and matrix H = $\begin{bmatrix} w & 0 \\ 0 & w \end{bmatrix}$, then H^{70} is equal to :

(a) H^2 (b) O (c) H (d) –H

20. The number of 3×3 non-singular matrices, with four entries as 1 and all other entries as 0 is :

(a) 5 (b) at least 7 (c) 6 (d) less than 4

21. Let A = $\begin{bmatrix} 2 & 2\alpha & \alpha \\ 0 & \alpha & 2\alpha \\ 0 & 0 & 2 \end{bmatrix}$. If $|A|^2 = 4$, then |α| is :

(a) $\frac{1}{5}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

22. If A and B are square matrix such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following always true.

(a) A = B (b) AB = BA

(c) either of A or B is zero matrix (d) either of A or B is identity matrix

23. If $A^2 - A + I = 0$, then the inverse of A is :

(a) A + I (b) A – I (c) A (d) I – A

24. Let A = $\begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, then which of the following is correct about matrix A ?

(a) A is a zero matrix (b) A^{-1} does not exists (c) $A^2 = I$ (d) A = (–1) I

25. If A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then :

(a) $\alpha = a^2 + b^2, \beta = ab$ (b) $\alpha = a^2 + b^2, \beta = 2ab$ (c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$ (d) $\alpha = 2ab, \beta = a^2 + b^2$

26. If A = $\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ such that $A^2 = B$. Then the value of α is :

(a) –1 (b) 2 (c) 1 (d) 4

27. Consider the simultaneous equation :

$x + y + z = 6$

$x + 2y + 3z = 10$
$x + 2y + az = b$

Then for what value of a and b, the given equation posses a unique solution :

(a) $a \neq 3$ (b) $a = 2$ (c) $a = 3$ (d) $a \neq 3$

28. For a given matrix $A = \begin{bmatrix} 0 & 0 & y \\ 2 & 5 & 1 \\ 8 & x & x \end{bmatrix}$, which of the following condition needs to be true if A is singular ?

(a) $y = 0$ (b) $x = 0$ (c) $x = 1$ (d) $y = 3$

29. If 1, w, w^2 are cube roots of unity, for what value of m, is the matrix singular ? $\begin{bmatrix} 1 & w & m \\ w & m & 1 \\ m & 1 & w \end{bmatrix}$

(a) 0 (b) 1 (c) w (d) w^2

30. The inverse of $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is :

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

31. The matrix $\begin{bmatrix} 0 & 7 & 4 \\ -7 & 0 & -5 \\ -4 & 5 & 0 \end{bmatrix}$ is :

(a) Symmetric (b) Non-singular (c) Skew-symmetric (d) Orthogonal

32. The matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$, satisfies which one of the following polynomial equations ?

(a) $A^2 + 3A + 2I = O$ (b) $A^2 + 3A - 2I = O$ (c) $A^2 - 3A - 2I = O$ (d) $A^2 - 3A + 2I = O$

33. If $\begin{bmatrix} a+b & 2a+c \\ a-b & 2c+d \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$, then the values of a, b, c and d are :

(a) 2, 2, 3, 4 (b) 3, 3 0, 1 (c) 2, 3, 1, 2 (d) 2, 3, 3, 2

34. If $A = \begin{bmatrix} 2a & 0 \\ a & a \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ then the value of a is :

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) -1

35. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, then which of following is equal to A (adj A) ?

(a) $\begin{bmatrix} 0 & 10 \\ 10 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 10 \\ 10 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 1 \\ 1 & 10 \end{bmatrix}$

36. Which of the following matrix is equal to A, for the given condition ? $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ A = $\begin{bmatrix} -1 & 0 \\ 6 & 3 \end{bmatrix}$

(a) $\begin{bmatrix} -5 & 1 \\ -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & -2 \\ 1 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & -2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 2 \\ -2 & -1 \end{bmatrix}$

37. Which of the following is equal to A^{-1}, such that, A is a square matrix and $A^2 = I$?

(a) A + I (b) A (c) Null matrix (d) Transpose of A

38. If A = $\begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ and I is an identity matrix of order 2, then which of the following is equal to $A^2 - 2A + 3I$?

(a) –I (b) –2A (c) 2A (d) 4A

39. If matrix A has inverse B and C, then which one of the following is correct ?

(a) B may not be equal to C
(b) B should be equal to C
(c) B and C should be unit matrices
(d) None of these

40. If A = $\begin{bmatrix} 1 & -2 & -3 \\ 2 & 1 & -2 \\ 3 & 2 & 1 \end{bmatrix}$, then which of the following is correct ?

(a) A is symmetric matrix
(b) A is anti-symmetric matrix
(c) A is singular matrix
(d) A is non-singular matrix

41. Matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is *a* :

(a) Skew-symmetric (b) Orthogonal (c) Symmetric (d) Singular

42. Given that 2A – 3B = $\begin{bmatrix} -7 & 0 \\ 7 & -13 \end{bmatrix}$, 3A + 2B = $\begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix}$, then B is equal to :

(a) $\begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 2 \\ 1 & -5 \end{bmatrix}$

43. What is the order of product of given matrices ? $[a\ b\ c] \begin{bmatrix} l & 0 & r \\ m & p & s \\ n & q & t \end{bmatrix} \begin{bmatrix} a \\ b \\ z \end{bmatrix}$

(a) 3 × 1 (b) 1 × 3 (c) 3 × 3 (d) 1 × 1

44. A and B are two matrices such that AB = A and BA = B, then the value of B^2 is :

(a) B (b) A (c) 1 (d) –1

45. Which of the following is correct for the given matrix A = $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ such that $A^2 = I$?

(a) $a = 0, b = 1$ or $a = 1, b = 0$
(b) $a = 0, b \neq 1$ or $a = 1, b = 1$
(c) $a = 1, b \neq 0$ or $a \neq 1, b = 1$
(d) $a \neq 0, b \neq 0$

46. If $l + m + n = 0$, then the system of equations

$-2x + y + z = 1$
$x - 2y + z = m$
$x + y - 2z = n$ has

(a) no solutions (b) unique solution (c) infinitely many solutions (d) a trivial solution

47. If A = $\begin{bmatrix} 3-2i & 3+5i \\ 2 & 3-2i \end{bmatrix}$ then adj(A) is :

(a) $\begin{bmatrix} 3-2i & -3-5i \\ -2 & 3-2i \end{bmatrix}$ (b) $\begin{bmatrix} 3-2i & 2 \\ 3+5i & 3-2i \end{bmatrix}$ (c) $\begin{bmatrix} 3-2i & 3+5i \\ -2 & -3+2i \end{bmatrix}$ (d) $\begin{bmatrix} 3-2i & -2 \\ -3-5i & 3-2i \end{bmatrix}$

48. Given that matrix A is singular then the solution set, A = $\begin{bmatrix} 2-x & 1 & 1 \\ 1 & 3-x & 0 \\ -1 & -3 & -x \end{bmatrix}$

(a) S = {0, 1, 2} (b) S = {0, 2, 3} (c) S = {3, 4, 0} (d) S = {1, 2, 3}

49. If A is a square matrix of order 'n' and 'K' is a scalar, then *adj* (K A) is equal to which of the following ?

(a) K^{n-1} *adj* (A) (b) K^n *adj* (A) (c) K^2 *adj* (A) (d) K^{n+1} *adj* (A)

50. A matrix A has $(a + b)$ rows and $(a + 2)$ columns and a matrix B has $(b + 1)$ rows and $(a + 3)$ columns. If both AB and BA exists, then what are the values a, b respectively ?

(a) 3, 2 (b) 2, 3 (c) 2, 4 (d) 4, 3

51. If $2\begin{bmatrix} 2 & 2 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, then $x - y$ is :

(a) 2 (b) 4 (c) 6 (d) –2

52. The value of x is : $[x\ 1\ 3] \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} = 0$

(a) 0 (b) 1 (c) –1 (d) 2

53. If A is a square matrix such that $A^2 = A$, then the value of $7A - (1 + A)^3$ is :

(a) I (b) O (c) –I (d) 2A

54. If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, then the value of $x + y$ is :

(a) 1 (b) 2 (c) 3 (d) –1

55. The value of $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ is :

(a) I (b) O (c) $-\sin\theta\cos\theta$ (d) $\sin\theta + \cos\theta$

56. Sum of two skew-symmetric is always .... matrix.

(a) Symmetric (b) Skew (c) Scalar (d) Singular

57. If A = $\begin{bmatrix} 0 & -\tan\frac{x}{2} \\ \tan\frac{x}{2} & 0 \end{bmatrix}$, then (I – A) $\begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ is :

(a) I–A (b) I (c) A (d) I+A

58. The value of x is : If $[1\ 2\ 1] \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

(a) 1 (b) –1 (c) 0 (d) 2

59. For what value of θ, the given matrix will be identity matrix. $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

(a) 0 (b) 90° (c) 60° (d) 180°

60. Let $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$. Then the value of $a + b$ is :

(a) $\frac{7}{20}$ (b) $\frac{3}{20}$ (c) $\frac{11}{20}$ (d) $\frac{19}{60}$

61. The eigen values of skew-symmetric matrix are :

(a) always zero (b) always pure imaginary

(c) Either zero or pure imaginary (d) always real

62. Given that $[A]_{3\times1}$, $[B]_{3\times3}$, $[C]_{3\times5}$, $[D]_{5\times3}$, $[E]_{5\times5}$ and $[F]_{5\times1}$ are matrices. Matrix [B] and [E] are symmetric. Then which of the following is correct statement.

(a) Matrix product $[F]^T [C]^T [B] [C] [F]$ is scalar (b) Matrix product $[D]^T [F] [D]$ is always symmetric

(c) Matrix product [C] [D] [E] is identity matrix (d) all of the above

63. A square matrix B is skew-symmetric if :

(a) $B = B^T$ (b) $B^T = -B$ (c) $B^{-1} = B$ (d) $BB^T = I$

64. The product of matrices $(PQ)^{-1} P$ is :

(a) P^{-1} (b) Q^{-1} (c) $P^{-1} Q^{-1} P$ (d) $PQ P^{-1}$

65. If $X_{4\times3}$, $Y_{4\times3}$, $P_{2\times3}$ are three matrices. Then what will be the order of $[P(X^TY)^{-1} P^T]^T$?

(a) 2 × 2 (b) 3 × 3 (c) 4 × 4 (d) 3 × 4

66. If a, b, h are real numbers, then the roots of the equation $\begin{vmatrix} a-x & h \\ h & b-x \end{vmatrix} = 0$ are :

(a) All real (b) All imaginary

(c) Equal (d) One real and the other imaginary

67. Let $A = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$ be expressed as P + Q, where P is symmetric and Q is skew symmetric matrix. Which one of the following is correct ?

(a) $Q = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ \frac{2}{3} & 0 \end{bmatrix}$ (b) $Q = \begin{bmatrix} 0 & \frac{3}{2} \\ \frac{3}{2} & 0 \end{bmatrix}$ (c) $Q = \frac{1}{2}\begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix}$ (d) $Q = \frac{1}{2}\begin{pmatrix} 2 & 3 \\ 0 & 5 \end{pmatrix}$

68. The matrix addition is :

(a) Associative and commutative (b) Commutative but not associative

(c) Neither associative nor commutative (d) Associative but not commutative

69. Which of following is not correct ?

(a) The transpose of a symmetric matrix need not be symmetric matrix

(b) If A and B are symmetric matrix of same order, then AB + BA must be symmetric matrix

(c) If A is symmetric matrix, then all positive integral power of A are symmetric matrices

(d) If A is any square matrix, then A + A′ is always symmetric.

70. Which of the following statement is/are correct ?

(a) If A is orthogonal matrix, then A is non-singular and $A^{-1} = A^1$

(b) If A is orthogonal matrix, then $|A| = \pm 1$
(c) Transpose of an orthogonal matrix is orthogonal
(d) All of the above

71. $\begin{bmatrix} a \\ b \\ c \end{bmatrix} \times [x\ y\ z] \times \begin{bmatrix} p \\ q \\ r \end{bmatrix} =$

(a) $xyz, \dfrac{pqr}{abc}$ (b) $pqr. \dfrac{xyz}{abc}$ (c) $pqr. \dfrac{abc}{xyz}$ (d) none of these

72. If $B = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, then B^{4n}, where 'n' is a natural number equals :

(a) –B (b) –I (c) B (d) I

73. If A, B and C are square matrices of the same order, then which of the following is true ?
(a) AB = AC (b) $AB = I \Rightarrow AB = BA$
(c) $AB = 0 \Rightarrow A = 0$ or $B = 0$ (d) $(AB)^2 = A^2B^2$

74. If $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$, then X = ?

(a) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$

75. If $A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix}$, then :

(a) $A^2 = A$ (b) AB = BA (c) $AB \neq BA$ (d) $B^2 = B$

76. The order of the given matrix $[a_1x_1 + a_2x_2 + a_3x_3]$ is :
(a) 1×1 (b) 1×2 (c) 1×3 (d) 2×1

77. If $\begin{bmatrix} a+b+c \\ a+b \\ b+c \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$, then the value of (a, b, c) is :

(a) (3, 2, 4) (b) (4, 3, 2) (c) (2, 3, 4) (d) (2, 3, 3)

78. If $A = \begin{pmatrix} 1 & x+3 \\ 2x+1 & x-1 \end{pmatrix}$ is symmetric matrix, then x is equal to :
(a) 3 (b) 7 (c) 5 (d) 2

79. If $A = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$, then $A^2 = B^2 = C^2$ is equal to :

(a) 2I (b) –I (c) I (d) I^2

80. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then A^5 is equal to :

(a) 9A (b) 81A (c) 273A (d) 5A

81. If $\begin{bmatrix} m+3 & 2n+m \\ p-1 & 4p-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2q \end{bmatrix}$, then the value of m, n, p and q is :

(a) –3, –2, 4, 5 (b) –4, 2, 3, –3 (c) 4, 2, 3, –3 (d) 3, –4, 2, –3

82. If $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, then which of the following is correct?

(a) $x = z$ (b) $x + y + z = 2$ (c) $y = 0$ (d) $y + z = -2$

83. If A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ then which of the following is incorrect ?

(a) $A^3 = 9A$ (b) A^{-1} does not exist (c) $A^2 = 3A$ (d) $A^5 = 27A$

84. Let A and B are two matrices of same order 3 × 3, where A = $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$, B = $\begin{bmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{bmatrix}$

If A is singular matrix, then $(A + B)^T$ is :

(a) $\begin{bmatrix} 4 & 5 & 5 \\ 5 & 6 & 6 \\ 10 & 13 & 14 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 5 & 10 \\ 5 & 6 & 13 \\ 5 & 6 & 14 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 5 & 10 \\ 5 & 6 & 13 \\ 4 & 5 & 10 \end{bmatrix}$ (d) none of these

85. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, (where $bc \neq 0$) satisfies equation $x^2 + k = 0$, then :

(a) $a + d = 0$ (b) $k = -|A|$ (c) $k = -|A^2|$ (d) $ab = bc$

86. If A is symmetric matrix and $n \in N$, then A^n is :

(a) Skew-symmetric (b) Symmetric (c) Diagonal matrix (d) Singular

87. If A = K $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$ is an orthogonal matrix then the value of K is :

(a) $\pm\frac{1}{3}$ (b) ±2 (c) ±3 (d) $\pm\frac{1}{2}$

88. If A = $\begin{pmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{pmatrix}$, B = $\begin{pmatrix} 3 & 2 & 4 \\ 3 & 2 & 5 \\ 2 & 1 & 4 \end{pmatrix}$ and 2A + 3B is singular. Then the value of 2λ is :

(a) $\frac{13}{2}$ (b) $\frac{15}{2}$ (c) $\frac{17}{2}$ (d) $\frac{11}{2}$

89. If $f(x) = x^2 + 4x - 5$ and A = $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ then the value of $f(A)$ is :

(a) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$

90. If A = $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, then $(A^{-1})^3$ is equal to :

(a) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & 27 \end{bmatrix}$ (b) $\frac{1}{27}\begin{bmatrix} 1 & -26 \\ 0 & -27 \end{bmatrix}$ (c) $\frac{1}{27}\begin{bmatrix} -1 & -26 \\ 0 & 27 \end{bmatrix}$ (d) $\frac{1}{27}\begin{bmatrix} -1 & -26 \\ 0 & -27 \end{bmatrix}$

91. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$ AB = 0, then which of the following is correct.

(a) $(\alpha-\beta)$ is an odd multiple of $\frac{\pi}{2}$ (b) $(\alpha-\beta)$ is an even multiple of $\frac{\pi}{2}$

(c) $(\alpha-\beta)$ is multiple of $\frac{\pi}{2}$ (d) $(\alpha-\beta)$ is multiple of $\frac{\pi}{3}$

92. If $B = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$. Then the value of A^2 is :

(a) $\begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$ (b) $\begin{bmatrix} -\cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$ (c) $\begin{bmatrix} \sin 4\theta & -\cos 4\theta \\ \cos 4\theta & -\sin 4\theta \end{bmatrix}$ (d) $\begin{bmatrix} \sin 4\theta & -\cos 4\theta \\ -\cos 4\theta & \sin 4\theta \end{bmatrix}$

93. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then the value of $A^2 - 4A$ is :

(a) 3I (b) 2I (c) 5I (d) 4I

94. Which of the following matrix satisfy the given equation ? $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$

95. If A is a skew-symmetric matrix of order 'n'. C is a column matrix of order $n \times 1$. Then which of the following is correct for C^TAC ?

(a) A zero matrix of order 1 (b) An identity matrix of order n

(c) an identity matrix of order 1 (d) A matrix of order n

96. If a square matrix satisfies $AA^T = I = A^TA$, then $|A|$ is equal to :

(a) 0 (b) –1 (c) ±2 (d) ±3

97. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ then which of the following is incorrect for the given equation ?

(a) $a = \cos\theta$ (b) $a = 1$ (c) $b = \sin 2\theta$ (d) $b = 1$

98. Which of the following statement is true about the diagonal matrix ?

(a) diagonal elements are different (b) diagonal elements are zero

(c) diagonal elements are same (d) none of these

99. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then A^n is :

(a) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} n & n \\ 0 & n \end{bmatrix}$ (c) $\begin{bmatrix} n & 1 \\ 0 & n \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 0 & n \end{bmatrix}$

100. If $[m, n] \begin{bmatrix} m \\ n \end{bmatrix} = 841$, $m < n$, then $(m, n) =$

(a) (21, 20) (b) (20, 21) (c) (22, 20) (d) 21, 22)

INPUT TEXT BASED MCQ's

101. Due to the lockdown imposed by the Government to combat the current Covid situation, it was observed that the sales of two companies in the automobile sector fell simultaneously. The companies were Mercedes and BMW. Akshay decided to collect the sales data of the two companies for the months of July and August to analyse it for a research project.

Mercedes-Benz

The data showed that in the month of July, Mercedes (M) sold 7 'C-Class' models, 5 'E- Class' and 6 GLC cars, while BMW (B) sold 4 'Five-Series', 3 'Seven-Series' and 5 'Three-Series' cars.

In August, Mercedes (M) sold 3 'C-Class', 4 'E-Class' and 5 GLC cars; BMW (B) sold 5 'Five-Series', 4 'Seven-Series' and 3 'Three-Series' cars.

Answer the following questions :

(i) The matrices M and B of order 2 × 3 respectively are :

(a) M. $= \begin{bmatrix} 7 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 5 \\ 3 & 4 & 3 \end{bmatrix}$ (b) M. $= \begin{bmatrix} 3 & 5 & 6 \\ 7 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 3 \end{bmatrix}$

(c) M. $= \begin{bmatrix} 7 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 & 5 \\ 5 & 4 & 3 \end{bmatrix}$ (d) M. $= \begin{bmatrix} 5 & 7 & 6 \\ 4 & 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \end{bmatrix}$

(ii) Matrix M + B is :

(a) $\begin{bmatrix} 11 & 8 & 11 \\ 8 & 8 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 8 & 8 \\ 11 & 8 & 11 \end{bmatrix}$ (c) $\begin{bmatrix} 11 & 11 & 8 \\ 8 & 8 & 11 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 8 & 8 \\ 11 & 8 & 11 \end{bmatrix}$

(iii) Matrix M − B is :

(a) $\begin{bmatrix} 3 & 2 & -1 \\ -2 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 & 1 \\ -2 & 2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 2 & 1 \\ 2 & 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 2 & 1 \\ -2 & 0 & 2 \end{bmatrix}$

(iv) MB^T is equal to :

(a) $\begin{bmatrix} 73 & 70 \\ 49 & 46 \end{bmatrix}$ (b) $\begin{bmatrix} 73 & 73 \\ 49 & 46 \end{bmatrix}$ (c) $\begin{bmatrix} 70 & 73 \\ 46 & 49 \end{bmatrix}$ (d) $\begin{bmatrix} 73 & 72 \\ 46 & 49 \end{bmatrix}$

(v) $(M + B)^T + (M - B)^T$ is equal to

(a) $\begin{bmatrix} 14 & 6 \\ 10 & 8 \\ 12 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 49 & 25 & 36 \\ 9 & 16 & 26 \end{bmatrix}$ (c) $\begin{bmatrix} 6 & 14 \\ 8 & 10 \\ 10 & 12 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & 5 & 6 \\ 3 & 4 & 5 \end{bmatrix}$

ANSWERS

1. (d)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (b)
21. (c)	**22.** (b)	**23.** (d)	**24.** (c)	**25.** (b)	**26.** (c)	**27.** (a)	**28.** (a)	**29.** (d)	**30.** (a)
31. (c)	**32.** (c)	**33.** (a)	**34.** (a)	**35.** (c)	**36.** (c)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (d)
41. (b)	**42.** (c)	**43.** (d)	**44.** (a)	**45.** (a)	**46.** (c)	**47.** (a)	**48.** (b)	**49.** (a)	**50.** (b)
51. (c)	**52.** (c)	**53.** (c)	**54.** (c)	**55.** (a)	**56.** (a)	**57.** (d)	**58.** (b)	**59.** (a)	**60.** (a)
61. (c)	**62.** (a)	**63.** (b)	**64.** (b)	**65.** (a)	**66.** (c)	**67.** (c)	**68.** (a)	**69.** (a)	**70.** (d)
71. (d)	**72.** (d)	**73.** (b)	**74.** (b)	**75.** (c)	**76.** (d)	**77.** (c)	**78.** (d)	**79.** (b)	**80.** (b)
81. (a)	**82.** (b)	**83.** (d)	**84.** (a)	**85.** (a)	**86.** (b)	**87.** (b)	**88.** (c)	**89.** (a)	**90.** (a)
91. (a)	**92.** (a)	**93.** (d)	**94.** (d)	**95.** (a)	**96.** (b)	**97.** (c)	**98.** (b)	**99.** (a)	**100.** (b)

101. (i) (c) (ii) (a) (iii) (d) (iv) (b) (v) (a)

Hints to Some Selected Questions

1. (d) $A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$= \alpha^2 = 1 \Rightarrow \alpha = \pm 1$ and $\alpha + 1 = 5 \Rightarrow \alpha = 4$.

Which is not possible at the same time thus no real values exits.

4. (c) $A^2 = \begin{pmatrix} 0 & 0 & 0 \\ 3 & 3 & 9 \\ -1 & -1 & -3 \end{pmatrix}$, $A^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$; $A^3 = 0$, i.e., $A^k = 0$. Thus, order of matrix is 3

6. (d) $A = \begin{bmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{bmatrix} \Rightarrow |A| = x^3 - 2x = f(x)$

8. (c) Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ $A' = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \Rightarrow A + A' = \begin{bmatrix} 2a & b+d & g+c \\ b+d & 2e & f+h \\ g+c & h+f & 2i \end{bmatrix} \rightarrow$ which is a symmetric

9. (b) $A(A^2 - 1) - 2(A^2 - I) = 0 \Rightarrow (A - 2I)(A^2 - I) = 0$

10. (b) $|D| = d_1 d_2 d_3 \ldots d_n$ and $adj\ D = [d_1 d_2 d_n, d_1 d_3 \ldots d_n, d_1 d_2 \ldots d_{n-1}] \Rightarrow D^{-1} = \text{diag}\ [d_1^{-1}, d_2^{-1}, \ldots d_n^{-1}]$

11. (b) $[f(x)\, g(y)]^{-1} = [g(y)]^{-1}\, [f(x)]^{-1}$

$\therefore [f(x)]^{-1} = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-x)$

$\Rightarrow g(y)^{-1} = g(-y)$, Similarly $[f(x)\, g(y)]^{-1} = g(-y)\, f(-x)$

12. (b) $(A^{-1} BA)^2 = (A^{-1} BA)(A^{-1} BA) = A^{-1} B^2 A$. Similarly, $(A^{-1} BA)^n = A^{-1} B^n A$.

13. (b) $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

Hence, $\det A = \sec^2 x \Rightarrow \det A^T = \sec^2 x$

$\therefore f(x) = \det(A^T A^{-1}) = \det(A^T)(\text{deA})^{-1} = \dfrac{\det A^T}{\det A} = 1$. Hence, $f(x) = 1$

14. (a) $A - A^T$ is skew symmetric of order 3 $\Rightarrow (A - A^T)^{2011}$ is skew symmetric of order 3

$\therefore |(A - A^T)^{2011}| = 0$

15. (b) $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} \Rightarrow AA^T = (b_{ij})_{3\times 3}$

$\Rightarrow b_{13} = 0 \Rightarrow a + 4 + 2b = 0$ and $b_{23} = 0 \Rightarrow 2a + 2 - 2b = 0$

$\therefore 3a + 6 = 0 \Rightarrow a = -2, b = -1$

18. (a) $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$. $|\text{Adj } A| = |A|^2 \Rightarrow |\text{Adj } A| = 16$

$\Rightarrow 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 16 \Rightarrow 2\alpha - 6 \Rightarrow 16 \Rightarrow 2\alpha = 22 \Rightarrow \alpha = 11.$

20. (b) First row with exactly one zero, total cases = 6
First row 2 zeros we get more cases
Total we get more than 7.

22. (b) $A^2 - B^2 = (A-B)(A+B) = A^2 + AB - BA - B^2 \Rightarrow AB = BA$

23. (d) $A^2 - A + I = 0 \Rightarrow A^{-1}A^2 - A^{-1}A + A^{-1}I = A^{-1}0 \Rightarrow A^{-1} = I - A$

25. (b) $A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix} \Rightarrow \alpha = a^2 + b^2,\ \beta^2 = 2ab$

28. (a) $\begin{bmatrix} 0 & 0 & y \\ 2 & 5 & 1 \\ 8 & x & x \end{bmatrix} \Rightarrow |A| = 0$. Thus, $y = 0$. For $x = 0 \Rightarrow |A| = -40y$. Therefore, A is not singular

29. (d) $\begin{vmatrix} 1 & w & m \\ w & m & 1 \\ m & 1 & w \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 1+w+m & w & m \\ 1+w+m & m & 1 \\ 1+w+m & 1 & w \end{vmatrix} = 0 \Rightarrow (1+w+m)\begin{vmatrix} 1 & w & m \\ 1 & m & 1 \\ 1 & 1 & w \end{vmatrix} = 0$

$\Rightarrow 1 + w + m = 0 \Rightarrow m = -(1+w) \Rightarrow -(w^2) = w^2$

30. (a) $adj\,(A) = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}\,adj\,(A) = \frac{-1}{1}\,adj\,(A) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

31. (c) $A = \begin{bmatrix} 0 & 7 & 4 \\ -7 & 0 & -5 \\ -4 & 5 & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -7 & -4 \\ 7 & 0 & 5 \\ 4 & -5 & 0 \end{bmatrix} = -A$

So, it is a skew symmetric matrix.

32. (c) $A^2 = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = \begin{vmatrix} 1+4 & 2+4 \\ 2+4 & 4+4 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 6 & 8 \end{vmatrix}$

$A^2 - 3A - 2I = \begin{bmatrix} 5 & 6 \\ 6 & 8 \end{bmatrix} - \begin{vmatrix} 3 & 6 \\ 6 & 6 \end{vmatrix} - \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 5-3-2 & 6-6-0 \\ 6-6-0 & 8-6-2 \end{vmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

34. (b) $|A| = 2a^2 - 0 = 2a^2$ and $adj\,(A) = \begin{vmatrix} a & -a \\ 0 & 2a \end{vmatrix}^T = \begin{bmatrix} a & 0 \\ -a & 2a \end{bmatrix}$

$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{2a^2}\begin{bmatrix} a & 0 \\ -a & 2a \end{bmatrix} = \begin{bmatrix} \frac{1}{2a} & 0 \\ -\frac{1}{2a} & \frac{1}{a} \end{bmatrix} \Rightarrow \frac{1}{2a} = 1 \Rightarrow a = \frac{1}{2}$

35. (c) $A\,(adj\,A) = I_2|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} 10 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

37. (b) $A^2 = I \Rightarrow A^{-1}A^2 = A^{-1}I$

$A^{-1}(A.A) = A^{-1} \Rightarrow (A^{-1}A).\,A = A^{-1} \Rightarrow A = A^{-1}$

39. (b) Every matrix has unique inverse. So, B is equal to C.

42. (c) $2A - 3B = \begin{bmatrix} -7 & 0 \\ 7 & -13 \end{bmatrix}$ ...(i) and $3A + 2B = \begin{bmatrix} 9 & 13 \\ 4 & 13 \end{bmatrix}$ ...(ii)

Multiply (i) by 3 and (ii) by 2, then subtract (i) from (ii)

$$B = \begin{bmatrix} 39 & 26 \\ -13 & 65 \end{bmatrix} \Rightarrow B = \begin{bmatrix} 3 & 2 \\ -1 & 5 \end{bmatrix}$$

45. (a) $A^2 = \begin{bmatrix} a^2+b^2 & 2ab \\ 2ab & a^2+b^2 \end{bmatrix} \Rightarrow a^2 + b^2 = 1 \Rightarrow 2ab = 0 \Rightarrow a = 0, b = 1$ or $b = 0, a = 1$

47. (a) $B = \begin{bmatrix} 3-2i & -2 \\ -6-5i & 3-2i \end{bmatrix} \Rightarrow adj\ (A) = B^T = \begin{bmatrix} 3-2i & -3-5i \\ -2 & 3-2i \end{bmatrix}$

48. (b) When the matrix is singular, $|A| = 0$

$$\begin{vmatrix} 2-x & 1 & 1 \\ 1 & 3-x & 0 \\ -1 & -3 & -x \end{vmatrix} = 0$$

$\Rightarrow (2-x)(-3x + x^2) -1(-x) + 1(-3 + 3 - x) = 0 \Rightarrow (2-x)(x^2 - 3x) + x - x = 0 \Rightarrow x = 2, 0, 3$

51. (c) $\begin{bmatrix} 4 & 4 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \Rightarrow \begin{bmatrix} 5 & 4+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$

$\Rightarrow 4 + y = 0 \Rightarrow y = -4$ and $2x + 1 = 5 \Rightarrow x = 2 \Rightarrow x - y = 2 - (-4) = 6.$

52. (c) $[x-2+3 \quad 0+1+6 \quad 0+1+6] = [0 \ \ 0 \ \ 0]$

$[x+1 \quad 7 \quad 7] = [0, 0, 0]$

$\therefore x+1 = 0 \Rightarrow x = -1$

53. (c) $7A - (I + A)^3 = 7A - [I^3 + A^3 + 3IA\,(A+I)] = 7A - [I + A + 3A + 3A] = 7A - [I + 7A] = -I$

58. (b) $[1+4+1 \quad 2+0+0 \quad 0+2+2] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

$\Rightarrow [6\ 2\ 4] \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0 \Rightarrow [0 + 4 + 4x] = 0 \Rightarrow x = -1$

59. (a) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \cos\theta = 1 \Rightarrow \theta = 0$ and $\cos\theta = 0 \Rightarrow \theta = 0$

60. (a) $AA^{-1} = I \Rightarrow \begin{pmatrix} 2 & -0.1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & a \\ 0 & b \end{pmatrix} = \begin{pmatrix} 1 & 2a-0.1b \\ 0 & 3b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow b = \frac{1}{3}$ and $a = \frac{1}{60}$

$\therefore a + b = \frac{1}{3} + \frac{1}{60} = \frac{7}{20}$

62. (a) $I = [F]^T_{1\times5}\ [C]^T_{5\times3}\ [B]_{3\times3}\ \ [C]_{3\times5}\ \ [F]_{5\times1} \Rightarrow [I]_{1\times1} =$ scalar

64. (b) $(PQ)^{-1} P = Q^{-1}P^{-1}P = Q^{-1}$

65. (a) $\{(2\times3) \times [(3\times4)\times(4\times3)]^{-1}\ (3\times2)\}^T = \{(2\times3) \times [(3\times3) \times 3 \times 2]^T = \{(2\times3) \times (3\times2)\}^T = 2 \times 2$

66. (c) $a - x = 0 \Rightarrow x = a$ and $b - x = 0 \Rightarrow x = b$

67. (c) $P = \frac{1}{2}\ (A+A^T),\ Q = \frac{1}{2}\ (A-A^T) \Rightarrow A^T \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$ and $Q = \frac{1}{2}\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

73. (b) $AB = I \Rightarrow AB = BA$

AB = identity matrix then BA must be identity.

74. (b) $2x\begin{bmatrix}3 & 8\\7 & 2\end{bmatrix}-\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}=\begin{bmatrix}2 & 6\\4 & -2\end{bmatrix} \Rightarrow x=\begin{bmatrix}1 & 3\\2 & -1\end{bmatrix}$

75. (c) $AB=\begin{bmatrix}3 & 6\\3 & 0\end{bmatrix}$ and $BA=\begin{bmatrix}-1 & -2\\-7 & 4\end{bmatrix}$ So, $AB \neq BA$

77. (c) $a + b = 5 \Rightarrow a + 2b + c = 12$ ...(i)

$b + c = 7$ and $a + b + c = 9$ ...(ii)

Subtract (ii) from (i) $\rightarrow$ $b = 3, a = 2, c = 4.$

78. (d) Symmetric matrix $\Rightarrow A = A^T$

$\therefore \begin{bmatrix}1 & x+3\\2x+1 & x-1\end{bmatrix}=\begin{bmatrix}1 & 2x+1\\x+3 & x-1\end{bmatrix} \Rightarrow x + 3 = 2x + 1 \Rightarrow x = 2$

79. (b) $A^2=\begin{bmatrix}i & 0\\0 & -i\end{bmatrix}=\begin{bmatrix}i & 0\\0 & -i\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}=-I$ and $B^2=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}=\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}=-I$

80. (b) $A=(3)^5\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=81\begin{bmatrix}3 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=81A$

81. (a) $m + 3 = 0 \Rightarrow m = -3$ and $p - 1 = 3 \Rightarrow p = 4 \Rightarrow 2n + m = -7$

$\Rightarrow 2n - 3 = -7 \Rightarrow n = -2 \Rightarrow 4p - 6 = 2q \Rightarrow 16 - 6 = 2q \Rightarrow q = 5$

85. (a) $A^2=\begin{bmatrix}a^2+bc & b(a+d)\\c(a+d) & d^2+bc\end{bmatrix}+\begin{bmatrix}k & 0\\0 & k\end{bmatrix}=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

$\Rightarrow b(a+d) = 0$ and $c(a+d) = 0$ Hence, $a+d = 0$

87. (b) For orthogonal matrix $A^T = A^{-1}$

89. (a) $f(A) = A^2 + 4A - 5 = \begin{bmatrix}1 & 2\\4 & -3\end{bmatrix}\begin{bmatrix}1 & 2\\4 & -3\end{bmatrix}+\begin{bmatrix}4 & 8\\16 & -12\end{bmatrix}-5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}8 & 4\\8 & 0\end{bmatrix}$

90. (a) $adj\,A=\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix} \Rightarrow A^{-1}=\frac{1}{|A|}adj\,A$ and $|A| = 3$

$\Rightarrow A^{-1}=\frac{1}{3}\begin{bmatrix}1 & -2\\0 & 3\end{bmatrix} \quad \Rightarrow (A^{-1})^3=\frac{1}{27}\begin{bmatrix}1 & -26\\0 & 27\end{bmatrix}$

93. (d) $A^2=\begin{bmatrix}9 & 8 & 8\\8 & 9 & 8\\8 & 8 & 9\end{bmatrix}$, $4A=\begin{bmatrix}4 & 8 & 8\\8 & 4 & 8\\8 & 8 & 4\end{bmatrix} \Rightarrow A^2-4A=\begin{bmatrix}5 & 0 & 0\\0 & 5 & 0\\0 & 0 & 5\end{bmatrix}=4I.$

94. (d) $\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & -1\end{bmatrix}=\begin{bmatrix}1 & 4-3\\0 & -1\end{bmatrix}=\begin{bmatrix}1 & 1\\0 & -1\end{bmatrix}$

95. (a) $C^T = (1\times n)$, $C^T = (n\times 1)$

99. (a) $A^2=\begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}, A^4=\begin{bmatrix}1 & 4\\0 & 1\end{bmatrix}$

Therefore, on generalising the result we get, $A^n=\begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$

100. (b) $m^2 + n^2 = 841$

$\therefore$ The value satisfy are $m = 20$ and $n = 21$.