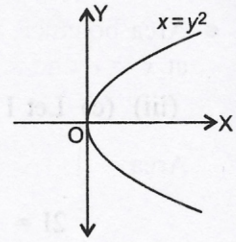
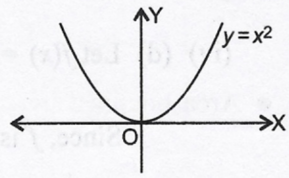
Chapter - 8 APPLICATION OF INTEGRALS

STUDY NOTES

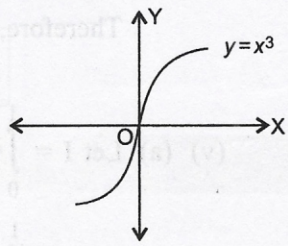
- **Symmetry about x-axis :** A graph is symmetric about x-axis if its equation is unchanged when y is replaced by $-y$. For example, the graph of the equation $x = y^2$ is symmetrical about x-axis.



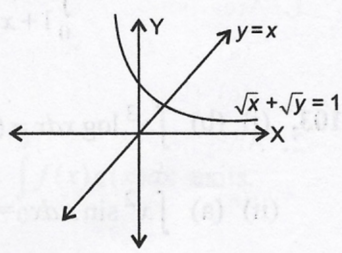
- **Symmetry about y-axis :** A graph is symmetric about y-axis if its equation is unchanged when x is replaced by $-x$. For example, the graph of the equation $y = x^2$ is symmetrical about y-axis.



- **Symmetry about origin :** A graph is symmetric about origin if its equation remains unchanged when x and y are replaced by $-x$ and $-y$ respectively.

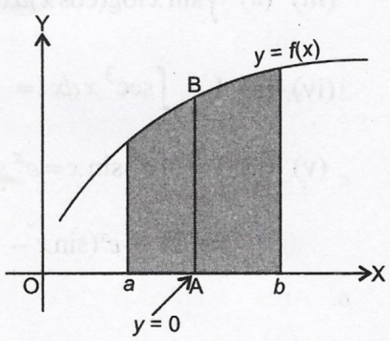


- **Symmetry about line $y = x$:** If on interchanging x and y, the equation remains same, the graph will be symmetrical about line $y = x$. For eg., the graph of equation $\sqrt{x}+\sqrt{y} = 1$ is symmetrical about $y = x$.

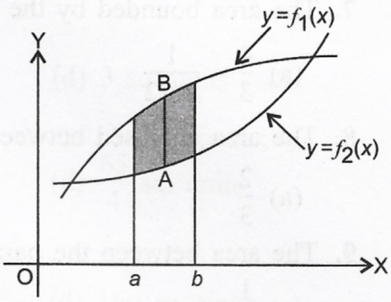


- Area between a curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$.
  In the given figure, BA is a vertical line segment running across the area.
  The height BA is (the value of y at B – the value of y at A), i.e., $(y_B - y_A)$.

  $\therefore$ Area $= \int_a^b (y_B - y_A)\,dx = \int_a^b (y_{\text{upper curve}} - y_{\text{lower curve}})\,dx$

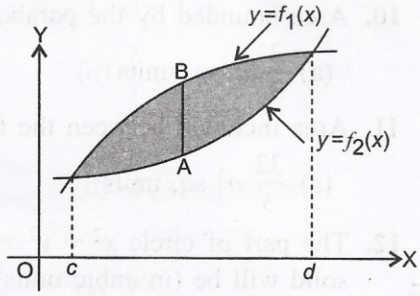


- Area between curves $y = f_1(x)$ and $y = f_2(x)$ and ordinates $x = a,\ x = b$



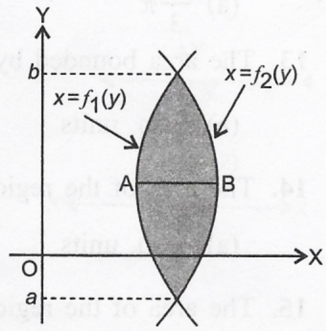
$$\text{Area} = \int_a^b (y_B - y_A)\,dx = \int_a^b [f_1(x) - f_2(x)]\,dx$$

- Area bounded by the curves $y = f_1(x)$ and $y = f_2(x)$. The two curves intersect at $x = c$ and $x = d$



$$\text{Area} = \int_c^d (y_B - y_A)\,dy = \int_c^d [f_1(x) - f_2(x)]\,dx.$$

- Area bounded by curves $x = f_1(y)$ and $x = f_2(y)$



$$\text{Area} = \int_a^b (x_B - x_A)\,dy = \int_a^b [f_2(y) - f_1(y)]\,dy.$$

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. Area bounded by the curve $y = \log x$, x-axis and the ordinates $x = 1$, $x = 2$ is :

(a) $\log 4$ sq. units (b) $(\log 4 + 1)$ sq. units
(c) $(\log 4 - 1)$ sq. units (d) $\log 4^2$ sq. units

2. Area bounded by parabola $y^2 = x$ and straight line $2y = x$ is :

(a) $\frac{4}{3}$ sq. units (b) 1 sq. units (c) $\frac{2}{3}$ sq. units (d) $\frac{3}{4}$ sq. units

3. If area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $\frac{a^2}{3}$, then the value of m is :

(a) 2 (b) –2 (c) 3 (d) $\frac{1}{2}$

4. Area bounded by the curve $xy - 3x - 2y - 10 = 0$, x-axis and the lines $x = 3$, $x = 4$ is (in sq. units)

(a) $16 \log 2 - 13$ (b) $16 \log 2 - 3$ (c) $16 \log 2 + 3$ (d) $16 \log 2 + 13$

5. The area of the region bounded by $y = |x - 1|$ and $y = 1$ is :

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) –1

6. Area bounded by lines $y = 2 + x$, $y = 2 - x$ and $x = 2$ is (in sq. units)

(a) 3 (b) 4 (c) 8 (d) 16

7. The area bounded by the straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$ is (in sq. units)

(a) $\frac{4}{3} - \frac{1}{\log 2}$ (b) $\frac{3}{\log 2} + \frac{4}{3}$ (c) $\frac{4}{\log 2} - 1$ (d) $\frac{3}{\log 2} - \frac{4}{3}$

8. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is (in sq. units)

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

9. The area between the parabola $y = x^2$ and the line $y = x$ is :

(a) $\frac{1}{6}$ sq. units (b) $\frac{1}{3}$ sq. units (c) $\frac{1}{2}$ sq. units (d) $\frac{1}{4}$ sq. units

10. Area bounded by the parabola $y^2 = 4ax$ and its latus rectum is :

(a) $\frac{2}{3}a^2$ sq. units (b) $\frac{4}{3}a^2$ sq. units (c) $\frac{8}{3}a^2$ sq. units (d) $\frac{3}{8}a^2$ sq. units

11. Area included between the two curves $y^2 = 4ax$ and $x^2 = 4ay$, is :

(a) $\frac{32}{3}a^2$ sq. units (b) $\frac{16}{3}$ sq. units (c) $\frac{32}{3}$ sq. units (d) $\frac{16}{3}a^2$ sq. units

12. The part of circle $x^2 + y^2 = 9$ in between $y = 0$ and $y = 2$ is revolved about y-axis. The volume of generating solid will be (in cubic units)

(a) $\frac{46}{3}\pi$ (b) 12π (c) 16π (d) 28π

13. The area bounded by $y = x^2$, the x-axis and the lines $x = -1$ and $x = 1$ is :

(a) $\frac{2}{3}$ sq. units (b) $\frac{1}{3}$ sq. units (c) $\frac{3}{4}$ sq. units (d) $\frac{2}{5}$ sq. units

14. The area of the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$ is :

(a) $\frac{8}{3}$ sq. units (b) $\frac{16}{3}$ sq. units (c) $\frac{32}{3}$ sq. units (d) $\frac{32}{5}$ sq. units

15. The area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is :

(a) $\frac{3}{4}$ sq. units (b) $\frac{9}{4}$ sq. units (c) $\frac{2}{3}$ sq. units (d) $\frac{5}{3}$ sq. units

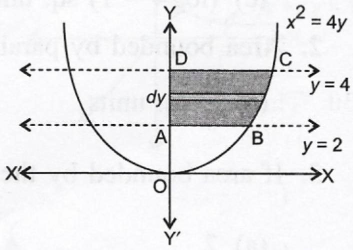
16. The area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis is:

(a) $\frac{14}{3}$ sq. units (b) $\frac{15}{7}$ sq. units (c) $\frac{8}{3}$ sq. units (d) $\frac{7}{3}$ sq. units

17. The area of the region bounded by the parabola $x^2 = 4y$ and the lines $y = 2$, $y = 4$ and y-axis in the first quadrant is :

(a) $\frac{7}{3}(8 - 2\sqrt{2})$ sq. units (b) $\frac{16}{3}$ sq. units

(c) $\frac{32\sqrt{2}}{3}$ sq. units (d) $\frac{4}{3}(8 - 2\sqrt{2})$ sq. units



18. The area bounded by the curve $y = x^2 + 1$ and the straight line $x + y = 3$ is:

(a) $\frac{9}{2}$ sq. units (b) 4 sq. units (c) $\frac{7\sqrt{17}}{6}$ sq. units (d) $\frac{17\sqrt{17}}{6}$ sq. units

19. The area bounded by the curve $x^2 = 4y + 4$ and line $3x + 4y = 0$ is:

(a) $\frac{25}{4}$ sq. units (b) $\frac{125}{8}$ sq. units (c) $\frac{125}{16}$ sq. units (d) $\frac{124}{24}$ sq. units

20. The area of the region bounded by line $y = 2x$, x-axis and ordinate $x = 2$ is :

(a) 4 sq. units (b) 5 sq. units (c) 6 sq. units (d) 3 sq. units

21. The area enclosed between the graph of $y = x^3$ and the lines $x = 0$, $y = 1$, $y = 8$ is:

(a) 7 sq. units (b) 14 sq. units (c) $\frac{45}{4}$ sq. units (d) $\frac{65}{4}$ sq. units

22. The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis is :

(a) 3π sq. units (b) 8π sq. units (c) 4π sq. units (d) 16π sq. units

23. The area of the region bounded by the line $y = x$, the x-axis and the ordinates $x = -1$, $x = 2$ is :

(a) $\frac{5}{2}$ sq. units (b) $\frac{3}{2}$ sq. units (c) $\frac{7}{2}$ sq. units (d) $\frac{7}{3}$ sq. units

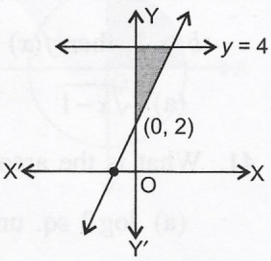
24. The area enclosed by the curve $y^2 = 4x$ and the line $x = 3$ is :

(a) $3\sqrt{2}$ sq. units (b) $8\sqrt{3}$ sq. units (c) $2\sqrt{3}$ sq. units (d) $4\sqrt{3}$ sq. units

25. The area bounded by the curves $y^2 = 4x$, and $y = x$ is :

(a) $\frac{8}{3}$ sq. units (b) $\frac{14}{3}$ sq. units (c) $\frac{7}{3}$ sq. units (d) $\frac{16}{3}$ sq. units

26. The area bounded by the lines $y - 2x = 2$, $y = 4$ and the y-axis is equal to (in sq. units)



(a) 2 (b) 3

(c) 4 (d) 1

27. The value of the integral $\int_{-1}^{2} [x]\, dx$ is :

(a) $\frac{5}{2}$ (b) 0 (c) 3 (d) 1

28. The area above the x-axis and under the curve $y = \sqrt{\frac{1}{x} - 1}$, for $\frac{1}{2} \le x \le 1$ is (in sq. units) :

(a) $\frac{\pi}{4} - \frac{1}{2}$ (b) $\frac{\pi}{4} + \frac{1}{2}$ (c) $\frac{\pi}{4} + 1$ (d) $\frac{\pi}{4} - 1$

29. The area of the region bounded by the line $y = 4$ and the curve $y = x^2$ is :

(a) $\frac{32}{3}$ sq. units (b) $\frac{1}{3}$ sq. units (c) 32 sq. units (d) $\frac{26}{3}$ sq. units

30. The volume of solid generated by revolution parabola $y^2 = 4ax$, cut of by latus rectum, about tangent at vertex is (in sq. units) :

(a) $\frac{4\pi}{5} a^3$ (b) $4\pi a^3$ (c) $\frac{4\pi}{5} a^2$ (d) $\frac{4\pi}{5}$

31. The area of the region bounded by the y-axis, $y = \cos x$ and $y = \sin x$, where $0 \le x \le \frac{\pi}{2}$ is :

(a) $\sqrt{2}$ sq. units (b) $(\sqrt{2} + 1)$ sq. units (c) $(\sqrt{2} - 1)$ sq. units (d) $(2\sqrt{2} - 1)$ sq. units

32. Area of the region in the first quadrant enclosed by the x-axis the line $y = x$ and the circle $x^2 + y^2 = 32$ is :

(a) 16π sq. units (b) 4π sq. units (c) 32π sq. units (d) 24π sq. units

33. The area of the region bounded by the curve $y = \sin x$, between the ordinates $x = 0$ and $x = \frac{\pi}{2}$ and the x-axis is:

(a) 2 sq. units (b) 4 sq. units (c) 3 sq. units (d) 1 sq. unit

34. The area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$ and $y = -1$ is :

(a) 4 sq. units (b) $\frac{3}{2}$ sq. units (c) 6 sq. units (d) 8 sq. units

35. Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is :

(a) 2 sq. units (b) 4 sq. units (c) 3 sq. units (d) 1 sq. unit

36. The area of the region bounded by the curves $y^2 = 9x$, $y = 3x$ is (sq. units) :

(a) 1 sq. unit (b) $\frac{1}{2}$ sq. units (c) $\frac{3}{2}$ sq. units (d) $\frac{1}{4}$ sq. units

37. The area of the region enclosed by the parabola $x^2 = y$ and the lines $y = x + 2$ is :

(a) $\frac{3}{2}$ sq. units (b) $\frac{9}{2}$ sq. units (c) $\frac{7}{2}$ sq. units (d) $\frac{5}{2}$ sq. units

38. The area of the region bounded by the line $2y = 5x + 7$, x-axis and the lines $x = 2$ and $x = 8$ is:

(a) 32 sq. units (b) 64 sq. units (c) 96 sq. units (d) 72 sq. units

39. The area under the curve $y = |\cos x - \sin x|$, $0 \le x \le \frac{\pi}{2}$, and above x-axis is (in sq. units) :

(a) $2\sqrt{2}$ (b) $2\sqrt{2}-2$ (c) $2\sqrt{2}+2$ (d) 0

40. The area bounded by the x-axis, the curve $y = f(x)$ and the lines $x = 1$, $x = b$, is equal to $\sqrt{b^2+1}-\sqrt{2}$ for all $b > 1$, then $f(x)$ is :

(a) $\sqrt{x-1}$ (b) $\sqrt{x+1}$ (c) $\sqrt{x^2+1}$ (d) $\frac{x}{\sqrt{1+x^2}}$

41. What is the area bounded by $y = \tan x$, $y = 0$ and $x = \frac{\pi}{4}$?

(a) $\log 2$ sq. units (b) $\frac{\log 2}{2}$ sq. units (c) $2(\log 2)$ sq. units (d) none of these

42. The area of the figure bounded by $y = e^x$, $y = e^{-x}$ and $x = 1$ is :

(a) $2(e - 1)$ (b) $e + \frac{1}{e} - 2$ (c) $e - \frac{1}{e} + 2$ (d) $e + \frac{1}{e}$

43. Area of the triangle formed by the line $x + y = 3$ and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is :

(a) 2 sq. units (b) 4 sq. units (c) 6 sq. units (d) 8 sq. units

44. The area of the region R = $\{(x, y) : |x| \le |y| \text{ and } x^2 + y^2 \le 1\}$ is :

(a) $\frac{3\pi}{8}$ sq. units (b) $\frac{5\pi}{8}$ sq. units (c) $\frac{\pi}{2}$ sq. units (d) $\frac{\pi}{8}$ sq. units

45. The area of the region bounded by the curve $y = x|x|$, x-axis and the ordinates $x = 1$, $x = -1$ is given by :

(a) zero (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

46. The area between the curve $y = 1 - |x|$ and the x-axis is equal to :

(a) 1 sq. unit (b) $\frac{1}{2}$ sq. unit (c) $\frac{1}{3}$ sq. unit (d) 2 sq. units

47. Area intercepted by the curves $y = \cos x$, $x \in [0, \pi]$ and $y = \cos 2x$, $x \in [0, \pi]$, is (in sq. units) :

(a) $\frac{3\pi}{2}$ (b) $\frac{3\sqrt{3}}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\sqrt{3}}{4}$

48. The area enclosed by the curve $y = -x^2$ and the straight line $x + y + 2 = 0$ is :

(a) $\frac{9}{2}$ sq. units (b) $\frac{7}{2}$ sq. units (c) $\frac{5}{2}$ sq. units (d) $\frac{9}{4}$ sq. units

49. The area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$ is :

(a) $\left(\pi - \frac{4}{3}\right)$ sq. units (b) $\left(\pi - \frac{8}{3}\right)$ sq. units (c) $2\left(\pi - \frac{4}{3}\right)$ sq. units (d) $2\left(\pi - \frac{8}{3}\right)$ sq. units

50. The area of the region bounded by the triangle whose vertices are (–1, 1), (0, 5) and (3, 2) using integration is:

(a) $\frac{9}{2}$ sq. units (b) $\frac{15}{2}$ sq. units (c) $\frac{7}{2}$ sq. units (d) $\frac{11}{2}$ sq. units

INPUT TEXT BASED MCQ's

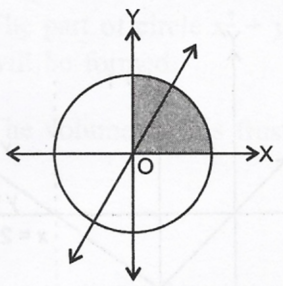
51. A child cut a cake with knife. The cake is circular in shape which is represented by $x^2 + y^2 = 16$ and edge of knife represents a straight line by $y = x$.

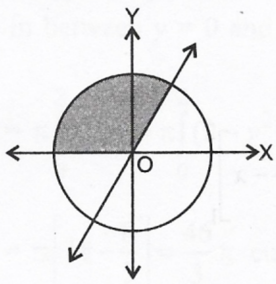
Answer the following questions :

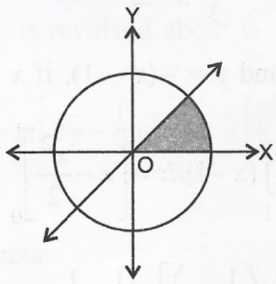
(i) Point of intersection of the given curve and line is :

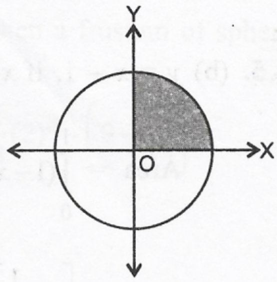
(a) (4, 0) (b) $(2\sqrt{2}, 2\sqrt{2})$ (c) $(2\sqrt{2}, 0)$ (d) $(0, 2\sqrt{2})$

(ii) Which of the following shaded portion represent the area bounded by given two curves?

(a) 

(b) 

(c) 

(d) 

(iii) The value of integral $\int_{2\sqrt{2}}^{4} \sqrt{16 - x^2}\,dx$ is :

(a) $2(\pi - 2)$ (b) $2(\pi - 8)$ (c) $4(\pi - 2)$ (d) $4(\pi + 2)$

(iv) Area bounded by the circular cake and knife is :

(a) 3π sq. units (b) $\frac{\pi}{2}$ sq. units (c) π sq. units (d) 2π sq. units

(v) Area of whole cake is :

(a) 2π sq. units (b) 4π sq. units (c) 16π sq. units (d) 20 sq. units

ANSWERS

1. (c)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (c)	**9.** (a)	**10.** (c)
11. (d)	**12.** (a)	**13.** (a)	**14.** (c)	**15.** (b)	**16.** (a)	**17.** (d)	**18.** (a)	**19.** (d)	**20.** (a)
21. (c)	**22.** (b)	**23.** (a)	**24.** (b)	**25.** (a)	**26.** (d)	**27.** (b)	**28.** (a)	**29.** (a)	**30.** (a)
31. (c)	**32.** (b)	**33.** (d)	**34.** (c)	**35.** (a)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (b)	**40.** (d)
41. (b)	**42.** (b)	**43.** (a)	**44.** (c)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (a)	**49.** (d)	**50.** (b)

51. (i) (b) (ii) (c) (iii) (a) (iv) (d) (v) (c)

Hints to Some Selected Questions

1. (c) We have, $y = \log x$ and $x = 1$ and $x = 2$

Hence, required area $= \int_1^2 \log x\, dx = [x\log x - x]_1^2$

$= 2\log 2 - 2 + 1 = (\log 4 - 1)$ sq. units

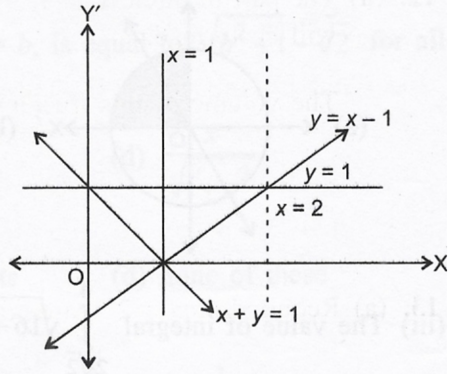
2. (a) $y^2 = x$ and $2y = x \Rightarrow y^2 = 2y \Rightarrow y = 0, 2$

$\therefore$ Required area $= \int_0^2 (y^2 - 2y)dy = \left(\frac{y^3}{3} - y^2\right)_0^2 = \frac{4}{3}$ sq. units

4. (c) Given curve is $y(x-2) = 3x + 10 \Rightarrow y = \frac{3x+10}{x-2}$

Required area $= \int_3^4 y dx = \int_3^4 \frac{3x+10}{x-2} dx = [3x + 16\log(x-2)]_3^4 = 3 + 16 \log 2$ sq. units

5. (b) $y = x - 1$, if $x > 1$ and $y = -(x-1)$, if $x < 1$



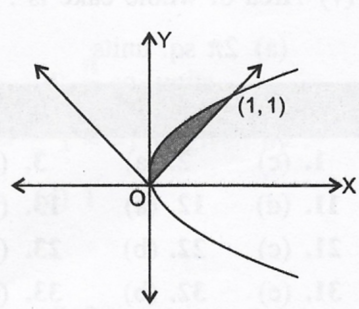
Area $= \int_0^1 (1-x)dx + \int_1^2 (x-1)dx = \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^2$

$= \left[1 - \frac{1}{2}\right] + \left[0 - \left(\frac{1}{2} - 1\right)\right] = \frac{1}{2} + \frac{1}{2} = 1.$

7. (d) Required area $= \int_0^2 [(2^x - (2x - x^2)]dx = \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^2$

$= \frac{4}{\log 2} - 4 + \frac{8}{3} - \frac{1}{\log 2} = \frac{3}{\log 2} - \frac{4}{3}$

8. (c) Required area $= \int_0^1 \left(\sqrt{x} - x\right) dx$

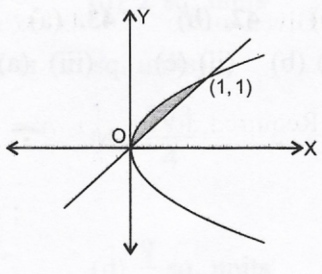


$= \left[\frac{2}{3}x^{3/2} - \frac{x^2}{2}\right]_0^1$

$= \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$ sq. units

9. (a) Given curves are $y = x^2$ and $y = x$

On solving, we get $x = 0$, $x = 1$.



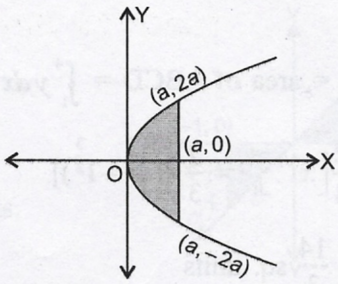
Therefore, required area A $= \int_0^1 (x^2 - x)dx = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$

$= \frac{1}{3} - \frac{1}{2} = \frac{1}{6}$ sq. units.

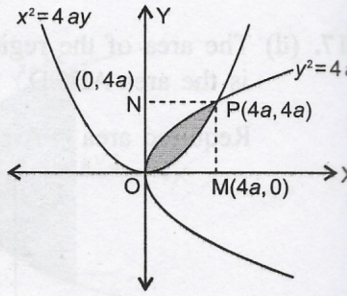
10. (c) Area $= 2\int_0^a y dx = 2\int_0^a \sqrt{4ax}\, dx$

$= 2 \times 2\sqrt{a} \times \frac{2}{3}\left[x^{3/2}\right]_0^a$

$= \frac{8}{3}a^2$ sq. units



11. (d) Solving the two equations, we have $x^4 = 64a^3x \Rightarrow x = 0, 4a$



Required area $= \int_0^{4a} 2a^{1/2}x^{1/2}dx - \int_0^{4a} \frac{x^2}{4a}dx$

$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2$ sq. units

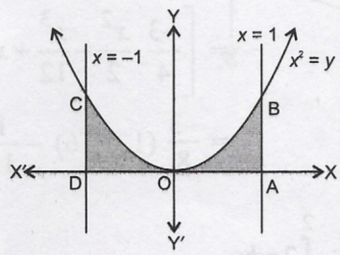
12. (a) The part of circle $x^2 + y^2 = 9$ in between $y = 0$ and $y = 2$ is revolved about y-axis. Then a frustum of sphe will be formed.

The volume of this frustum $= \pi\int_0^2 x^2 dy = \pi\int_0^2 (9 - y^2)dy = \pi\left[9y - \frac{1}{3}y^3\right]_0^2 = \pi\left[9\times 2 - \frac{1}{3}(2)^3 - \left(9 - \frac{1}{3}\right)\right]$

$= \pi\left[18 - \frac{8}{3}\right] = \frac{46}{3}\pi$ cubic units.

13. (a) Required area = 2 × area of OAB $= 2\int_0^1 y dx$

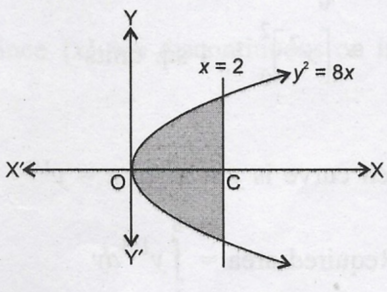
$= 2\int_0^1 x^2\, dx = \frac{2}{3}\left[x^3\right]_0^1 = \frac{2}{3}$ sq. units



14. (c) Required area $= 2\int_0^2 y dx = 2\int_0^2 \sqrt{8x}\, dx$

$= 2 \times 2\sqrt{2}\int_0^2 x^{\frac{1}{2}}\, dx = 4\sqrt{2}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^2$

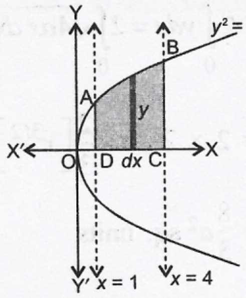
$= \frac{8}{3}\sqrt{2}\left[2^{\frac{3}{2}} - 0\right] = \frac{8\sqrt{2}}{3} \times 2\sqrt{2} = \frac{32}{3}$ sq units



15. (b) Given curve: $y^2 = 4x \Rightarrow x = \frac{y^2}{4} \Rightarrow y = 2\sqrt{x}$

Required area $= \int_0^3 x\, dy = \int_0^3 \frac{y^2}{4}\, dy = \frac{1}{4}\left[\frac{y^3}{3}\right]_0^3 = \frac{1}{3} \times \frac{27}{4} = \frac{9}{4}$ sq units.

16. (a) Required area = area of ABCD = $\int_1^4 y\,dx$



$= \int_1^4 \sqrt{x}\,dx = \frac{2}{3}\left[x^{3/2}\right]_1^4 = \frac{2}{3}\,[(4^{\frac{3}{2}} - 1^{\frac{3}{2}})]$

$= \frac{2}{3}\,(8 - 1) = \frac{14}{3}$ sq. units

17. (d) The area of the region bounded by the curve, $x^2 = 4y$, $y = 2$ and $y = 4$, and the y-axis in the first quadrant is the area ABCD.

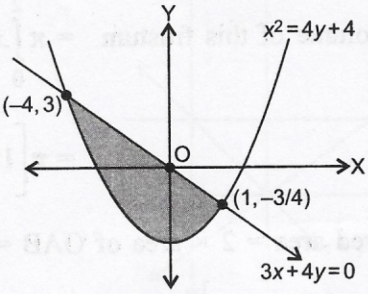
Required area = Area of ABCD = $\int_2^4 x dy = \int_2^4 2\sqrt{y}\,dy = 2\int_2^4 \sqrt{y}\,dy$

$= 2\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_2^4 = \frac{4}{3}\left[(4)^{\frac{3}{2}} - (2)^{\frac{3}{2}}\right] = \frac{4}{3}[8 - 2\sqrt{2}]$ sq. units.

19. (d) We have, $x^2 = 4y + 4$ and $3x + 4y = 0$

On solving, we get, $x = -4, 1$

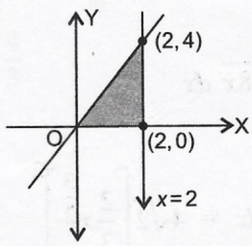
Required area = $\int_{-4}^{1}\left[\frac{-3x}{4} - \left(\frac{x^2}{4} - 1\right)\right]dx$



$= \left[\frac{-3}{4}\cdot\frac{x^2}{2} - \frac{x^3}{12} + x\right]_{-4}^{1}$

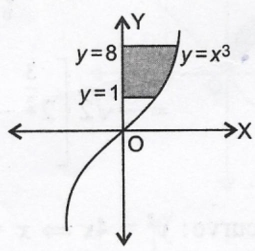
$= \frac{-3}{8}\,(1 - 16) - \frac{1}{12}\,(1 + 64) + 5 = \frac{125}{24}$ sq. units.

20. (a) Area = $\int_0^2 2x dx$



$= \left[x^2\right]_0^2 = 4$ sq. units

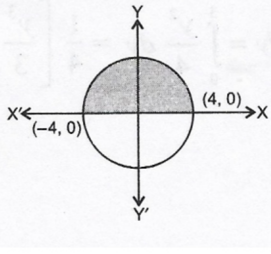
21. (c) Given curve is $y = x^3$ or $x = y^{1/3}$



$\therefore$ Required area = $\int_1^8 y^{1/3} dy$

$= \frac{3}{4}\left[y^{4/3}\right]_1^8 = \frac{3}{4} \times 15 = \frac{45}{4}$ sq. units

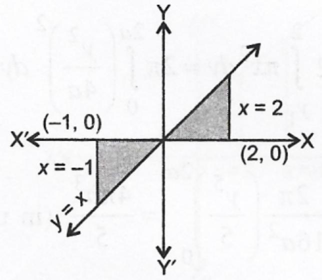
22. (b) Area = $2\int_0^4 \sqrt{16 - x^2}\,dx$



$= 2\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4 = 8\pi$ sq. units.

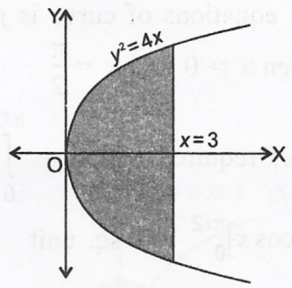
23. (a) Required area $= \int_0^2 y\,dx + \int_{-1}^0 (-y)\,dx = \int_0^2 x\,dx + \int_{-1}^0 (-x)\,dx$

$$= \left[\frac{x^2}{2}\right]_0^2 + \left[-\frac{x^2}{2}\right]_{-1}^0 = 2 + \frac{1}{2} = \frac{5}{2} \text{ sq. units.}$$



24. (b) Required area $= 2\int_0^3 y\,dx = 2\int_0^3 2\sqrt{x}\,dx$

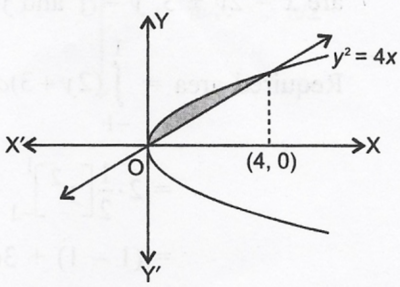
$$= 4\cdot\left[\frac{x^{3/2}}{\frac{3}{2}}\right]_0^3 = \frac{8}{3}[3^{2/3} - 0] = \frac{8}{3}3\cdot\sqrt{3} = 8\sqrt{3} \text{ sq units.}$$



25. (a) Required area $= \int_0^4 2\sqrt{x}\,dx - \int_0^4 x\,dx$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 - \left[\frac{x^2}{2}\right]_0^4 = \frac{2\times 2}{3}[8] - \left[\frac{16}{2}\right]$$

$$= \frac{32}{3} - 8 = \frac{32 - 24}{3} = \frac{8}{3} \text{ sq. units}$$



26. (d) Area bounded by the lines $= \int_2^4 \frac{y-2}{2}\,dy = \frac{1}{2}\left[\frac{y^2}{2} - 2y\right]_2^4 = 1$

27. (b) $\int_{-1}^2 [x]dx = \int_{-1}^0 [x]dx + \int_0^1 [x]dx + \int_1^2 [x]dx$ [Since $\{x\}$ are discontinuous on integral values]

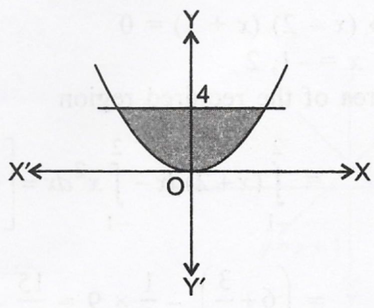
$$= \int_{-1}^0 -1\,dx + \int_0^1 0\,dx + \int_1^2 1\,dx = -[x]_{-1}^0 + 0 + [x]_1^2$$

$= -(0 + 1) + 0 + [2 - 1] = -1 + 0 + 1 = 0$

29. (a) $y = x^2 \Rightarrow x = \sqrt{y}$

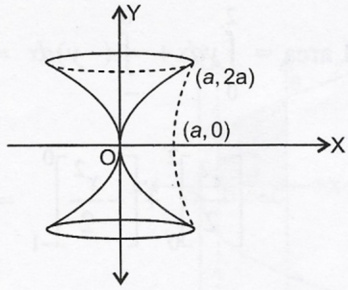
$\therefore$ Area $= 2\int_0^4 \sqrt{y}\,dy$

$$= 2\left[\frac{2}{3}y^{3/2}\right]_0^4 = \frac{4}{3}[8 - 0] = \frac{32}{3} \text{ sq. units.}$$



30. (a) $V = 2\int_{y_1}^{2} \pi x^2 dy = 2\pi \int_0^{2a} \left(\frac{y^2}{4a}\right)^2 dy$

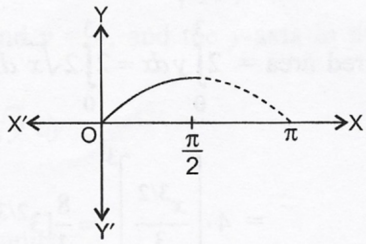
$= \frac{2\pi}{16a^2}\left(\frac{y^5}{5}\right)_0^{2a} = \frac{4\pi a^3}{5}$ (in units)



33. (d) Given equations of curve is $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{2}$

Area of required region $= \int_0^{\pi/2} \sin x\, dx$

$= -\left[\cos x\right]_0^{\pi/2} = 1$ sq. unit

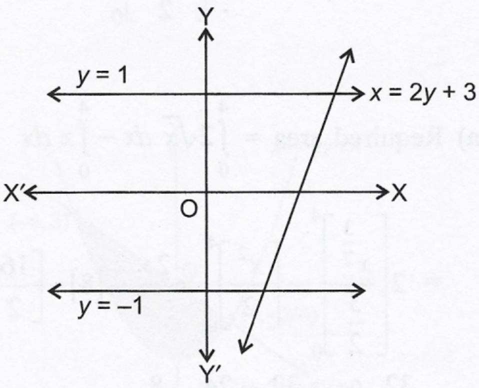


34. (c) Given equation of lines are $x = 2y + 3$, $y = 1$ and $y = -1$

Required area $= \int_{-1}^{1} (2y+3)dy$

$= 2\cdot\frac{1}{2}\left[y^2\right]_{-1}^{1} + 3\left[y\right]_{-1}^{1}$

$= (1 - 1) + 3(1 + 1) = 6$ sq. units



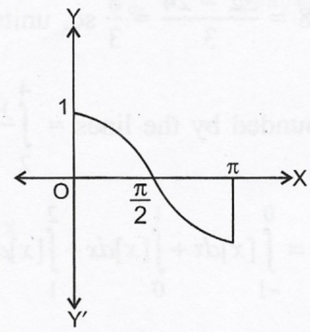
35. (a) Given that : $y = \cos x$, $x = 0$, $x = \pi$

Required area $= \int_0^{\pi/2} \cos x dx + \left|\int_{\pi/2}^{\pi} \cos x\, dx\right|$

$= \left[\sin x\right]_0^{\pi/2} + \left|\left(\sin x\right)_{\pi/2}^{\pi}\right|$

$= \left[\sin\frac{\pi}{2} - \sin 0\right] + \left|\left[\sin \pi - \sin\frac{\pi}{2}\right]\right|$

$= (1 - 0) + |(0 - 1)| = 2$ sq. units



37. (b) Here, $x^2 = y$ and $y = x + 2$

$\therefore\ x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0 \Rightarrow x^2 - 2x + x - 2 = 0$

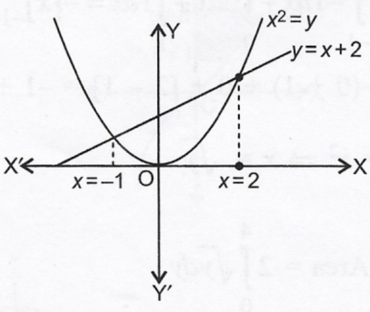
$\Rightarrow (x - 2)(x + 1) = 0$

$\therefore\ x = -1, 2$

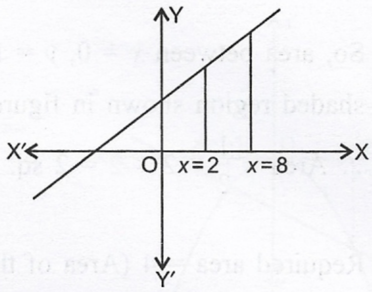
Area of the required region

$= \int_{-1}^{2} (x+2)dx - \int_{-1}^{2} x^2 dx = \left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \left(6 + \frac{3}{2}\right) - \frac{1}{3} \times 9 = \frac{15}{2} - 3 = \frac{9}{2}$ sq. units



38. (c) We have, $2y = 5x + 7$, x-axis, $x = 2$ and $x = 8$



$y = \frac{5x+7}{2}$

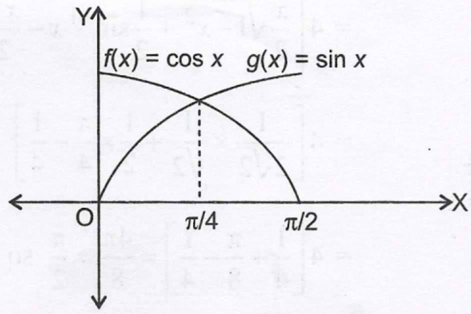
Area of the required shaded region

$= \int_2^8 \left(\frac{5x+7}{2}\right) dx = \frac{1}{2}\left[\frac{5}{2}x^2 + 7x\right]_2^8$

$= \frac{1}{2}\left[\frac{5}{2}(64-4) + 7(8-2)\right]$

$= \frac{1}{2}[150 + 42] = \frac{1}{2} \times 192 = 96$ sq. units

39. (b) $y = |\cos x - \sin x|$



Required area $= 2\int_0^{\pi/4} (\cos x - \sin x)dx = [\sin x + \cos x]_0^{\pi/4}$

$= 2\left[\frac{2}{\sqrt{2}} - 1\right] = (2\sqrt{2} - 2)$ sq. units

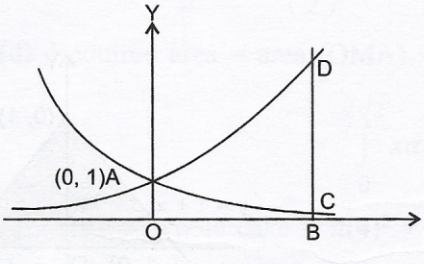
40. (d) Given $\int_1^b f(x)dx = \sqrt{b^2+1} - \sqrt{2}$

Differentiating with respect to b

$f(b) = \frac{b}{\sqrt{b^2+1}} \Rightarrow f(x) = \frac{x}{\sqrt{x^2+1}}$

41. (b) Required area $= \int_0^{\pi/4} \tan x \, dx = [\log \sec x]_0^{\pi/4} = \log\sqrt{2} - 0 = \frac{1}{2}\log 2.$

42. (b)

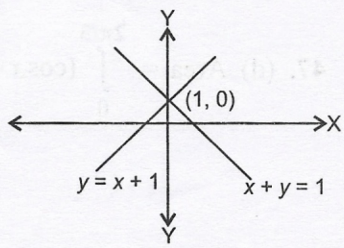


Required area $= \int_0^1 (e^x - e^{-x})dx = \left[e^x + e^{-x}\right]_0^1$

$= (e^1 + e^{-1}) - (1 + 1) = e + \frac{1}{e} - 2$

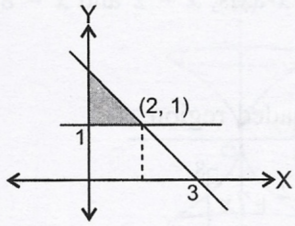
43. (a) $x^2 - y^2 + 2y = 1 \Rightarrow x = \pm(y - 1)$

Bisectors of above line are $x = 0$ and $y = 1$



So, area between $x = 0$, $y = 1$ and $x + y = 3$ is shaded region shown in figure.

$\therefore$ Area $= \frac{1}{2} \times 2 \times 2 = 2$ sq. units



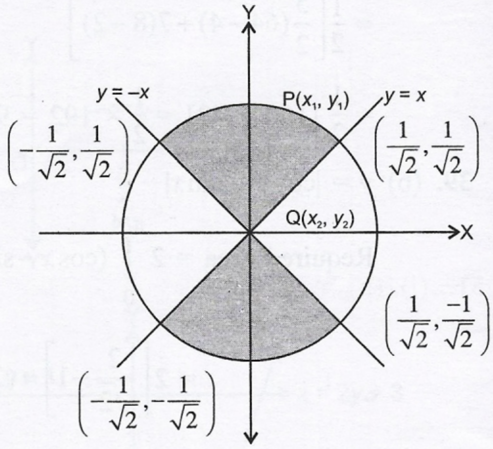
44. (c) Required area = 4 (Area of the shaded region in first quadrant)

$$= 4\int_0^{1/\sqrt{2}} (y_1 - y_2)dx = 4\int_0^{1/\sqrt{2}} \left(\sqrt{1-x^2} - x\right)dx$$

$$= 4\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x - \frac{x^2}{2}\right]_0^{1/\sqrt{2}}$$

$$= 4\left[\frac{1}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{\pi}{4} - \frac{1}{4}\right]$$

$$= 4\left[\frac{1}{4} + \frac{\pi}{8} - \frac{1}{4}\right] = \frac{4\pi}{8} = \frac{\pi}{2} \text{ sq. units}$$



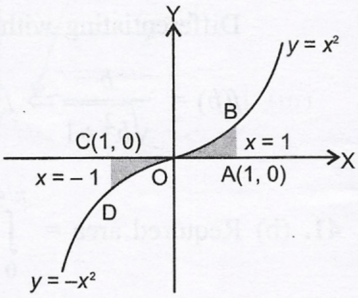
45. (c) The area of the region bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ is given by

$$\text{Area} = \left|\int_a^b y\,dx\right|$$

According to the question, $y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

$\therefore$ Required area = 2 × Area of region OAB

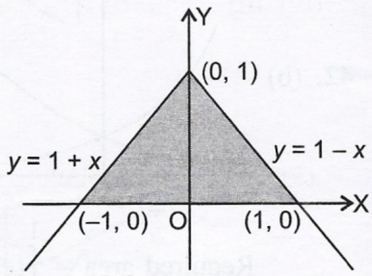
$$= 2\int_0^1 x^2 dx = \frac{2}{3} \text{ sq. units}$$



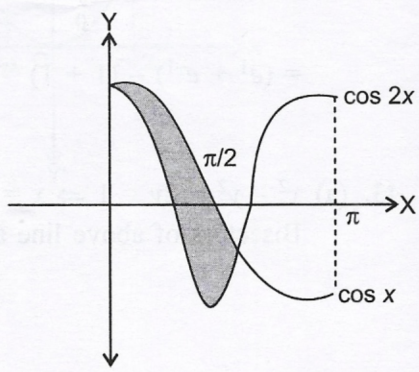
46. (a) The curves are $y = 1 - x$, $y = 1 + x$, $y = 0$

Area $= \frac{1}{2} \times$ base $\times$ height

Base = 2, height = 1 $\Rightarrow$ Area $= \frac{1}{2} \times 2 \times 1 = 1$ sq. unit



47. (d) Area $= \int_0^{2\pi/3} (\cos x - \cos 2x)dx = \frac{3\sqrt{3}}{4}$



48. (a) We have, $y = -x^2$ or $x^2 = -y$

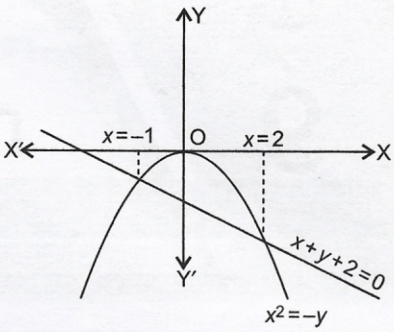
and the line $x + y + 2 = 0$

Solving the two equations, we get

$\Rightarrow x - x^2 + 2 = 0 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = -1, 2$

Area of the required shaded region



$$= \left|\int_{-1}^{2} (-x-2)dx - \int_{-1}^{2} -x^2 dx\right| = \left|-\left[\left(\frac{4}{2}+4\right)-\left(\frac{1}{2}-2\right)\right]+\frac{1}{3}(8+1)\right|$$

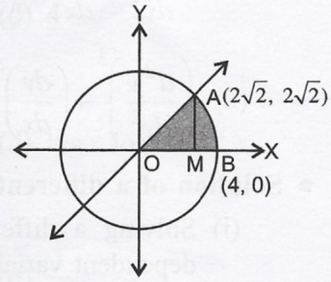
$$= \left|-\left(6+\frac{3}{2}\right)+\frac{1}{3}(9)\right| = \left|-\frac{15}{2}+3\right| = \frac{9}{2} \text{ sq. units}$$

51. (i) (b) We have, $x^2 + y^2 = 16$ ...(i)

and $y = x$ ...(ii)

From (i) and (ii), $2x^2 = 16$ $x = 2\sqrt{2}$

$\therefore$ Point of intersection is $(2\sqrt{2}, 2\sqrt{2})$.



(ii) (c) The shaded region which represent the area bounded by two circular cake and knife in first quadrant is shown here.

(iii) (a) $$\int_{2\sqrt{2}}^{4} \sqrt{16-x^2}\,dx = \left[\frac{x}{2}\sqrt{16-x^2}+\frac{16}{2}\cdot\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{2}}^{4}$$

$$= 8\sin^{-1}(1) - 4 - 8\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= 8\left(\frac{\pi}{2}\right) - 4 - 8\left(\frac{\pi}{4}\right) = 4\pi - 4 - 2\pi = 2(\pi - 2).$$

(iv) (d) Required area = area (OMA) + area (ABM)

$$= \int_{0}^{2\sqrt{2}} x\,dx + \int_{2\sqrt{2}}^{4} \sqrt{16-x^2}\,dx = 4 + 2(\pi - 2) = 2\pi \text{ sq. units}$$

(v) (c) Area of whole cake $= \pi(4)^2 = 16\pi$ sq. units.