Chapter - 10 VECTOR ALGEBRA

STUDY NOTES

- **Scalar and Vector Quantity :** A quantity which has only magnitude and no direction is called a scalar quantity e.g., length, distance, speed, etc. And a quantity which has magnitude as well as direction is called vector quantity e.g., velocity, force, weight etc.

- **Scalar or Dot Product :**

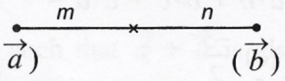
  (i) $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$, where $\theta (0 \le \theta \le 2\pi)$ is the angle between vectors $\vec{a}$ and $\vec{b}$.

  (ii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

  Then, $\vec{a} \cdot \vec{b} = a_1b_2 + a_2b_2 + a_3b_3$ and $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, $|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2}$

  (iii) $\cos\theta = \dfrac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}$

- Position vector of point dividing the joint of two points with position vectors $\vec{a}$ and $\vec{b}$ in the ratio $m : n$ is given by $\dfrac{mb + na}{m + n}$.



- **Vector or Cross Product :**

  (i) $\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\, \hat{n}$, where θ is the angle between the vectors $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to the plane of $\vec{a}$ and $\vec{b}$.

  (ii) $\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}|\,|\vec{b}|}$

  (iii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

  (iv) If $\vec{a}$ and $\vec{b}$ are any vectors then $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2\,|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$

- Area of parallelogram whose adjacent sides are represented by $\vec{a}$, $\vec{b}$ equals $|\vec{a} \times \vec{b}|$.

- Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ is $\dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

- Area of parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is $\dfrac{1}{2}|\vec{a} \times \vec{b}|$.

- Area of triangle whose adjacent sides are represented by $\vec{a}$ and $\vec{b}$ is equals $\dfrac{1}{2}|\vec{a} \times \vec{b}|$.

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. If $\vec{p} = 7\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{q} = 3\hat{i} + \hat{j} + 5\hat{k}$, then the magnitude of $\vec{p} - 2\vec{q}$ is :

(a) $\sqrt{29}$ (b) 4 (c) $\sqrt{62} - 2\sqrt{35}$ (d) $\sqrt{66}$

2. If the position vectors of the vertices of a triangle be $2\hat{i} + 4\hat{j} - \hat{k}$, $4\hat{i} + 5\hat{j} + \hat{k}$ and $3\hat{i} + 6\hat{j} - 3\hat{k}$, then the triangle is :

(a) Right angled (b) Isosceles

(c) Equilateral (d) Isosceles right angled

3. If $\vec{p} = \hat{i} + \hat{j}$, $\vec{q} = 4\hat{k} - \hat{j}$ and $\vec{r} = \hat{i} + \hat{k}$, then the unit vector in the direction of $3\vec{p} + \vec{q} - 2\vec{r}$ is :

(a) $\frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$ (b) $\frac{1}{3}(\hat{i} - 2\hat{j} - 2\hat{k})$ (c) $\frac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $\hat{i} + 2\hat{j} + 2\hat{k}$

4. If $|a| = 3$, $|b| = 4$ and $|a + b| = 5$, then $|a - b| =$

(a) 6 (b) 5 (c) 4 (d) 3

5. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x - 2)\hat{j} - \hat{k}$. If the vector $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then x equals :

(a) 0 (b) 1 (c) –4 (d) –2

6. If $\vec{a} = 2\hat{i} + 5\hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j}$, then the unit vector along $\vec{a} + \vec{b}$ will be :

(a) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$ (b) $\hat{i} + \hat{j}$ (c) $\sqrt{2}\,(\hat{i} + \hat{j})$ (d) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

7. If a, b, c are unit vectors such that $a + b + c = 0$, then $a \cdot b + b \cdot c + c \cdot a =$

(a) 1 (b) 3 (c) $\frac{-3}{2}$ (d) $\frac{3}{2}$

8. The work done by the force F = $2\hat{i} - 3\hat{j} + 2\hat{k}$ in displacing a particle from the point (3, 4, 5) to the point (1, 2, 3) is :

(a) 2 units (b) 3 units (c) 4 units (d) 5 units

9. If the vectors $3\hat{i} + 2\hat{j} - \hat{k}$ and $6\hat{i} - 4x\hat{j} + y\hat{k}$ are parallel, then the value of x and y will be :

(a) –1, –2 (b) 1, –2 (c) –1, 2 (d) 1, 2

10. If θ be the angle between the unit vectors a and b, then $\cos\frac{\theta}{2} =$

(a) $\frac{1}{2}|a - b|$ (b) $\frac{1}{2}|a + b|$ (c) $\frac{|a-b|}{|a+b|}$ (d) $\frac{|a+b|}{|a-b|}$

11. The value of b such that scalar product of the vectors $(\hat{i} + \hat{j} + \hat{k})$ with the unit vector parallel to the sum of the vectors $(2\hat{i} + 4\hat{j} + 5\hat{k})$ and $(b\hat{i} + 2\hat{j} + 3\hat{k})$ is 1, is :

(a) –2 (b) –1 (c) 0 (d) 1

12. The position vector of the point which divides the join of points with position vectors $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ in the ratio 1 : 2 is :

(a) $\frac{3\vec{a} + 2\vec{b}}{3}$ (b) $\vec{a}$ (c) $\frac{5\vec{a} - \vec{b}}{3}$ (d) $\frac{4\vec{a} + \vec{b}}{3}$

13. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then value of $\vec{a} \cdot \vec{b}$ is :

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$ (c) $12\sqrt{3}$ (d) $4\sqrt{3}$

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$, then the value of λ such that $\vec{a}$ is perpendicular to $\lambda\vec{b} + \vec{c}$ is :

(a) 2 (b) –2 (c) 3 (d) 0

15. The projection of vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ along $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is :

(a) $\frac{1}{3}$ (b) 2 (c) $\frac{2}{3}$ (d) $\frac{4}{3}$

16. The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4 respectively and $\vec{a}\cdot\vec{b} = 2\sqrt{3}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{5\pi}{2}$

17. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is :

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$

18. The vectors $\lambda\hat{i} + \hat{j} + 2\hat{k}$, $\hat{i} + \lambda\hat{j} - \hat{k}$ and $2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if

(a) $\lambda = -2$ (b) $\lambda = 0$ (c) $\lambda = 1$ (d) $\lambda = -1$

19. If θ is the angle between the vectors $\hat{i} + 3\hat{j} + 7\hat{k}$ and $\hat{i} - 3\hat{j} + 7\hat{k}$, then $\cos\theta$ is equal to

(a) $\frac{41}{59}$ (b) $\frac{40}{59}$ (c) $\frac{42}{59}$ (d) $\frac{39}{59}$

20. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{c}| = \sqrt{24}$ and sum of any two vectors is orthogonal to the third vector, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

(a) 7 (b) $5\sqrt{2}$ (c) $7\sqrt{2}$ (d) $6\sqrt{2}$

21. Area of the parallelogram whose adjacent sides are given by the vectors $\hat{i} - \hat{j} + 3\hat{k}$ and $2\hat{i} - 7\hat{j} + \hat{k}$ is :

(a) $\sqrt{225}$ sq. units (b) $\sqrt{450}$ sq. units (c) $\sqrt{650}$ sq. units (d) $\sqrt{425}$ sq. units

22. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $\vec{a} + \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) 60° (b) 90° (c) 120° (d) 150°

23. The vector $\vec{c}$ such that $|\vec{c}| = 100$ and $\vec{c}\cdot\hat{i} = \vec{c}\cdot\hat{j} = \vec{c}\cdot\hat{k}$ is :

(a) $\frac{50}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{125}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (c) $\frac{100}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (d) $\frac{200}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

24. If $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to :

(a) 1 (b) 2 (c) 3 (d) 4

25. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is :

(a) one (b) two (c) three (d) infinite

26. If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B respectively then the position vector of a point C in BA produced such that BC = 1.5 BA is :

(a) $\frac{\vec{a} - \vec{b}}{2}$ (b) $\frac{3\vec{a} - \vec{b}}{2}$ (c) $\frac{3\vec{a} + \vec{b}}{2}$ (d) $\frac{\vec{a} - 3\vec{b}}{2}$

27. The vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, then the value of $\vec{r}$ is :

(a) $2(\hat{i} + \hat{j} + \hat{k})$ (b) $3(\hat{i} + \hat{j} + \hat{k})$ (c) $2(\hat{i} - \hat{j} - \hat{k})$ (d) $2(\hat{i} + \hat{j} - \hat{k})$

28. The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ is :

(a) $\frac{1}{\sqrt{7}}$ (b) $\frac{2}{\sqrt{7}}$ (c) $\frac{3}{\sqrt{7}}$ (d) $\frac{4}{\sqrt{7}}$

29. The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is :

(a) $\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (c) $3(\hat{i} - 2\hat{j} + 2\hat{k})$ (d) $9(\hat{i} - 2\hat{j} + 2\hat{k})$

30. The value of λ such that the vectors the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal is :

(a) 0 (b) 1 (c) $\frac{3}{2}$ (d) $\frac{-5}{2}$

31. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is :

(a) $\vec{a}^2$ (b) $3\vec{a}^2$ (c) $4\vec{a}^2$ (d) $2\vec{a}^2$

32. The dot product of a vector with the vectors $\hat{i} + \hat{j} - 3\hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$ are 0, 5 and 8 respectively. The vector is :

(a) $\hat{i} + 2\hat{j} + \hat{k}$ (b) $-\hat{i} + 3\hat{j} - 2\hat{k}$ (c) $\hat{i} + 2\hat{j} + 3\hat{k}$ (d) $\hat{i} - 3\hat{j} - 3\hat{k}$

33. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, then the value of $\frac{\vec{a}\cdot(\vec{b} \times \vec{c})}{(\vec{c} \times \vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a} \times \vec{c})}{\vec{c}\cdot(\vec{a} \times \vec{b})}$ is :

(a) 0 (b) 2 (c) 1 (d) None of these

34. If the middle points of sides BC, CA and AB of triangle ABC are respectively D, E, F then position vector of centre of triangle DEF, when position vector of A, B, C are respectively $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ is :

(a) $\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$ (b) $(\hat{i} + \hat{j} + \hat{k})$ (c) $2(\hat{i} + \hat{j} + \hat{k})$ (d) $\frac{2}{3}(\hat{i} + \hat{j} + \hat{k})$

35. The angle between any two diagonal of a cube is :

(a) 45° (b) 60° (c) 30° (d) $\tan^{-1}(2\sqrt{2})$

36. Which one of the following is the unit vector perpendicular to both $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} - \hat{k}$?

(a) $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (b) $\hat{k}$ (c) $\frac{\hat{j}+\hat{k}}{\sqrt{2}}$ (d) $\frac{\hat{i}-\hat{j}}{\sqrt{2}}$

37. If $a \cdot b = a \cdot c$ and $a \times b = a \times c$, then correct statement is :

(a) $a \parallel (b - c)$ (b) $a \perp (b - c)$ (c) $a = 0$ or $b = c$ (d) None of these

38. The unit vector perpendicular to the vectors $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 6\hat{j} - 2\hat{k}$ is :

(a) $\frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$ (b) $\frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$ (c) $\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$ (d) $\frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7}$

39. Let a, b and c be three vectors satisfying $a \times b = (a \times c)$, $|a| = |c| = 1$, $|b| = 4$ and $|b \times c| = \sqrt{15}$
If $b - 2c = \lambda a$, then λ equals :

(a) 1 (b) −1 (c) 2 (d) −4

40. With respect to a rectangular cartesian coordinate system, three vectors are expressed as :
$\vec{a} = 4\hat{i} - \hat{j}$, $\vec{b} = -3\hat{i} + 2\hat{j}$ and $\vec{c} = -\hat{k}$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors, along the X, Y and Z-axis respectively. The unit vector $\hat{r}$ along the direction of sum of these vector is :

(a) $\hat{r} = \frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (b) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j}-\hat{k})$ (c) $\hat{r} = \frac{1}{3}(\hat{i}-\hat{j}+\hat{k})$ (d) $\hat{r} = \frac{1}{\sqrt{2}}(\hat{i}+\hat{j}+\hat{k})$

41. Two vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$. The angle between the two vectors will be:

(a) 60° (b) 90° (c) 180° (d) 0°

42. If vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar, then the value of a is :

(a) 2 (b) −2 (c) −1 (d) −4

43. If vectors $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 676$ and $|\vec{b}| = 2$ then $|\vec{a}|$ is equal to :

(a) 13 (b) 26 (c) 39 (d) None of these

44. What is the vector joining the points (3, 1, 14) and (–2, –1, –6)?

(a) $-\hat{i} + 2\hat{j} + 12\hat{k}$ (b) $-\hat{i} - 2\hat{j} + 12\hat{k}$ (c) $-\hat{i} - 2\hat{j} - 12\hat{k}$ (d) $5\hat{i} + 2\hat{j} + 20\hat{k}$

45. For vectors b and c and any non-zero vector a, the value of $\{(a + b) \times (a + c)\} \times (b + c)\} \cdot (b + c)$ is :

(a) $|a|^2$ (b) $2|a|^2$ (c) $3|a|^2$ (d) None of these

46. If the middle points of sides BC, CA and AB of triangle ABC are respectively D, E, F then position vector of centre of triangle DEF, when position vector of A, B, C are respectively $\hat{i} + \hat{j}, \hat{j} + \hat{k}, \hat{k} + \hat{i}$ is :

(a) $\frac{1}{3}(\hat{i} + \hat{j} + \hat{k})$ (b) $(\hat{i} + \hat{j} + \hat{k})$ (c) $2(\hat{i} + \hat{j} + \hat{k})$ (d) $\frac{2}{3}(\hat{i} + \hat{j} + \hat{k})$

47. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{b}$ and $(2\lambda - 1)\vec{c}$ are non coplanar for

(a) no value of λ (b) all except one value of λ

(c) all except two values of λ (d) all values of λ

48. If the position vectors of the vertices A, B, C of a triangle ABC are $7\hat{j} + 10\hat{k}$, $-\hat{i} + 6\hat{j} + 6\hat{k}$ and $-4\hat{i} + 9\hat{j} + 6\hat{k}$ respectively, the triangles is :

(a) equilateral (b) isosceles

(c) scalene (d) right angled and isosceles also

49. If θ be the angle between vectors $a = \hat{i} + 2\hat{j} + 3\hat{k}$ and $b = 3\hat{i} + 2\hat{j} + \hat{k}$, then $\cos\theta$ equals :

(a) $\frac{5}{7}$ (b) $\frac{6}{7}$ (c) $\frac{4}{7}$ (d) $\frac{1}{2}$

50. The angle between the vectors $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{B} = -\hat{i} + 2\hat{j} - \hat{k}$ is :

(a) 15° (b) 45° (c) 35° (d) 60°

51. A vector of magnitude 5 and perpendicular to $(\hat{i} - 2\hat{j} + \hat{k})$ and $(2\hat{i} + \hat{j} - 3\hat{k})$ is :

(a) $\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} - \hat{k})$ (c) $\frac{5\sqrt{3}}{3}(\hat{i} - \hat{j} + \hat{k})$ (d) $\frac{5\sqrt{3}}{3}(-\hat{i} + \hat{j} + \hat{k})$

52. $\hat{i} \times (\hat{j} \times \hat{k}) + \hat{j} \times (\hat{k} \times \hat{i}) + \hat{k} \times (\hat{i} \times \hat{j})$ equals :

(a) $\hat{i}$ (b) $\hat{j}$ (c) $\hat{k}$ (d) 0

53. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} =$

(a) –1 (b) $\frac{-1}{2}$ (c) $\frac{1}{2}$ (d) 1

54. Let $a = 2\hat{i} - \hat{j} + \hat{k}$, b $= \hat{i} + 2\hat{j} - \hat{k}$ and $c = \hat{i} + \hat{j} - 2\hat{k}$ be three vectors. A vector in the plane of b and c whose projection on a is of magnitude $\sqrt{\frac{2}{3}}$ is :

(a) $2\hat{i} + 3\hat{j} - 3\hat{k}$ (b) $2\hat{i} + 3\hat{j} + 3\hat{k}$ (c) $2\hat{i} - \hat{j} + 5\hat{k}$ (d) $2\hat{i} + \hat{j} + 5\hat{k}$

55. The shortest distance between the lines $r = (3\hat{i} - 2\hat{j} - 2\hat{k}) + \hat{i}t$ and $r = (\hat{i} - \hat{j} + 2\hat{k}) + \hat{j}s$ (t and s being parameters) is :

(a) $\sqrt{21}$ (b) $\sqrt{102}$ (c) 4 (d) 3

56. The image of the point with position vector $\hat{i} + 3\hat{k}$ in the plane $r \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ is :

(a) $\hat{i} + 2\hat{j} + \hat{k}$ (b) $\hat{i} - 2\hat{j} + \hat{k}$ (c) $-\hat{i} - 2\hat{j} + \hat{k}$ (d) $\hat{i} + 2\hat{j} - \hat{k}$

57. If $\vec{A} = \hat{i} - 2\hat{j} - 3\hat{k}$, $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{C} = \hat{i} + 3\hat{j} - 2\hat{k}$, then $(\vec{A} \times \vec{B}) \times \vec{C}$ is :

(a) $5(-\hat{i} + 3\hat{j} + 4\hat{k})$ (b) $4(-\hat{i} + 3\hat{j} + 4\hat{k})$ (c) $5(-\hat{i} - 3\hat{j} - 4\hat{k})$ (d) $4(\hat{i} + 3\hat{j} + 4\hat{k})$

58. $a[(b + c) \times (a + b + c)]$ is equal to :

(a) $[abc]$ (b) $2[abc]$ (c) $3[abc]$ (d) 0

59. If $u = \hat{i} \times (a \times \hat{i}) + \hat{j} \times (a \times \hat{j}) + \hat{k} \times (a \times \hat{k})$, then

(a) $u = 0$ (b) $u = \hat{i} + \hat{j} + \hat{k}$ (c) $u = 2a$ (d) $u = a$

60. The value of $[a - b \quad b - c \quad c - a]$ where $|a| = 1$, $|b| = 5$ and $|c| = 3$ is :

(a) 0 (b) 1 (c) 2 (d) 4

61. If a, b, c are non-coplanar unit vectors such that $a \times (b \times c) = \dfrac{b+c}{\sqrt{2}}$, then the angle between a and b is :

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{3\pi}{4}$ (d) π

62. If the points $P(\vec{a} + \vec{b} + \vec{c})$, $Q(2\vec{a} + 3\vec{b})$, $R(\vec{b} + t\vec{c})$ are collinear, where $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vector, the value of t is :

(a) 2 (b) -2 (c) $\dfrac{1}{2}$ (d) $\dfrac{-1}{2}$

63. If $|\vec{a} - \vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{\pi}{4}$ (d) 0

64. If $\vec{a}$, $\vec{b}$ are unit vectors such that $\vec{a} - \vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is :

(a) $\dfrac{\pi}{6}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{2}$

65. If $\vec{a}$, $\vec{b}$ are unit vectors such that $|\vec{a} + \vec{b}| = 1$ and $|\vec{a} - \vec{b}| = \sqrt{3}$, then $|3\vec{a} + 2\vec{b}| =$

(a) 7 (b) 4 (c) $\sqrt{7}$ (d) $\sqrt{19}$

66. A vector of magnitude 4 which is equally inclined to the vectors $\hat{i} + \hat{j}$, $\hat{j} + \hat{k}$ and $\hat{k} + \hat{i}$ is :

(a) $\dfrac{4}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$ (b) $\dfrac{4}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (c) $\dfrac{4}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$ (d) $\dfrac{1}{\sqrt{3}}(-\hat{i} - \hat{j} - \hat{k})$

67. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is :

(a) 50 units (b) 20 units (c) 30 units (d) 40 units

68. A unit vector perpendicular to the plane of $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\vec{b} = 4\hat{i} + 3\hat{j} - \hat{k}$ is :

(a) $\dfrac{1}{\sqrt{26}}(4\hat{i} + 3\hat{j} - \hat{k})$ (b) $\dfrac{1}{7}(2\hat{i} - 6\hat{j} - 3\hat{k})$ (c) $\dfrac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$ (d) $\dfrac{1}{7}(2\hat{i} - 3\hat{j} - 6\hat{k})$

69. A non-zero vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}$, $\hat{i} + \hat{k}$. The angle between $\vec{a}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$ is :

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{6}$ (d) $\dfrac{\pi}{2}$

70. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero non collinear vectors such that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$, then $\vec{a} + \vec{b} + \vec{c} =$

(a) abc (b) 0 (c) 1 (d) 2

71. The number of distinct values of λ for which the vectors $-\lambda^2\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \lambda^2\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} - \lambda^2\hat{k}$ are coplanar, is :

(a) 0 (b) 1 (c) $\pm\sqrt{2}$ (d) $\pm\sqrt{3}$

72. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}\cdot\vec{b} = 1$ and $\vec{a}\times\vec{b} = \hat{j} - \hat{k}$, then $\vec{b}$ is :

(a) $\hat{i} - \hat{j} + \hat{k}$ (b) $2\hat{j} - \hat{k}$ (c) $\hat{i}$ (d) $2\hat{i}$

INPUT TEXT BASED MCQ's

73. If $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along positive directions of the coordinate axes, then every vector $\vec{r}$ can be represented as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ uniquely.

Answer the following questions :

(i) If $x(\hat{j} + \hat{k}) + y(-\hat{j} + 2\hat{k}) + z\hat{k} = \hat{i}$, then

(a) $x = 0, y = 1, z = -1$ (b) $x = -1, y = 1, z = 1$

(c) no real values of x, y, z exist (d) all real values of x, y, z

(ii) If $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = m\hat{i} - 2\hat{j} - 3\hat{k}$, then the value of m for which $\vec{a}$ and $\vec{b}$ are perpendicular is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{-4}{3}$ (d) $\frac{-3}{4}$

(iii) If $x(\hat{i} + \hat{j} + 3\hat{k}) + y(3\hat{i} - 3\hat{j} + \hat{k}) + z(-4\hat{i} + 5\hat{j}) = \lambda(x\hat{i} + y\hat{j} + z\hat{k})$ where at least one of x, y, z is not zero, then the number of distinct values of λ is:

(a) –1 (b) 2 (c) –3 (d) 0

(iv) If $x(2\hat{i} - \hat{j} + \hat{k}) + y(\hat{i} + 2\hat{j} - 3\hat{k}) + z(3\hat{i} + a\hat{j} + 5\hat{k}) = 0$ where x, y, z are scalars such that $(x, y, z) \neq (0, 0, 0)$ then the value of a is :

(a) –2 (b) –3 (c) –4 (d) 2

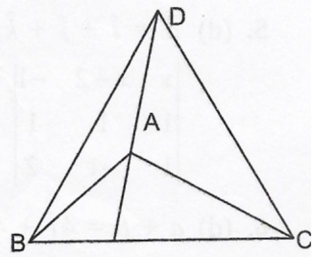
(v) The value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector is :

(a) $\pm\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) $\pm\frac{1}{\sqrt{2}}$ (d) $\pm\sqrt{2}$

74. Nishi bought an air plant holder which is shaped like a tetrahedron.

Let A, B, C and D are the coordinates of the air plant holder.

where A = (1, 2, 1), B = (2, 1, 3), C = (3, 2, 3) and D = (4, 3, 2).



Answer the following questions :

(i) The position vector of $\overrightarrow{AB}$ is :

(a) $\hat{i} - 2\hat{j} - 3\hat{k}$ (b) $\hat{i} - \hat{j} + 2\hat{k}$ (c) $\hat{i} + \hat{j} + \hat{k}$ (d) $\hat{i} - \hat{j} - \hat{k}$

(ii) The position vector of $\overrightarrow{AC}$ is :

(a) $2\hat{i} + 2\hat{k}$ (b) $2\hat{i} - 2\hat{j} - 2\hat{k}$ (c) $2\hat{i} - 2\hat{j}$ (d) $2\hat{j} + 2\hat{k}$

(iii) What is the area of ΔABC?

(a) $\frac{\sqrt{3}}{2}$ sq. units (b) $\frac{2}{\sqrt{3}}$ sq. units (c) $\sqrt{3}$ sq. units (d) $\frac{1}{2}\sqrt{2}$ sq. units

(iv) The position vector of $\overrightarrow{AD}$ is :

(a) $3\hat{i} - 2\hat{j} + 2\hat{k}$ (b) $3\hat{i} - \hat{j} - \hat{k}$ (c) $\hat{i} - \hat{j} + \hat{k}$ (d) $3\hat{i} + \hat{j} + \hat{k}$

(v) The unit vector along $\overrightarrow{AD}$ is :

(a) $\frac{1}{\sqrt{11}}(3\hat{i} + \hat{j} + \hat{k})$ (b) $\frac{1}{\sqrt{13}}(3\hat{i} + \hat{j} + \hat{k})$ (c) $\frac{1}{2\sqrt{3}}(3\hat{i} + 2\hat{j} + \hat{k})$ (d) $\frac{1}{3\sqrt{2}}(2\hat{i} + \hat{j} - \hat{k})$

ANSWERS

1. (d)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (a)	**9.** (a)	**10.** (b)
11. (d)	**12.** (d)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (a)	**19.** (a)	**20.** (a)
21. (b)	**22.** (c)	**23.** (c)	**24.** (c)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (c)	**30.** (d)
31. (d)	**32.** (a)	**33.** (a)	**34.** (d)	**35.** (d)	**36.** (a)	**37.** (c)	**38.** (c)	**39.** (d)	**40.** (a)
41. (b)	**42.** (d)	**43.** (a)	**44.** (d)	**45.** (d)	**46.** (d)	**47.** (c)	**48.** (d)	**49.** (a)	**50.** (d)
51. (a)	**52.** (d)	**53.** (d)	**54.** (a)	**55.** (c)	**56.** (c)	**57.** (a)	**58.** (d)	**59.** (c)	**60.** (a)
61. (c)	**62.** (a)	**63.** (a)	**64.** (b)	**65.** (c)	**66.** (c)	**67.** (d)	**68.** (c)	**69.** (a)	**70.** (b)
71. (c)	**72.** (c)								

73. (i) (c) (ii) (b) (iii) (a) (iv) (c) (v) (a)

74. (i) (b) (ii) (a) (iii) (c) (iv) (d) (v) (a)

Hints to Some Selected Questions

1. (d) $p - 2q = \hat{i} - 4\hat{j} - 7\hat{k}$

$|p - 2q| = \sqrt{1+16+49} = \sqrt{66}$

3. (a) $\vec{p} = \hat{i} + \hat{j}$, $\vec{q} = 4\hat{k} - \hat{j}$ and $\vec{r} = \hat{i} + \hat{k}$

Vector in the direction of $3\vec{p} + \vec{q} - 2\vec{r} = 3(\hat{i} + \hat{j}) + (4\hat{k} - \hat{j}) - 2(\hat{i} + \hat{k})$

$= 3\hat{i} + 3\hat{j} + 4\hat{k} - \hat{j} - 2\hat{i} - 2\hat{k} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\Rightarrow$ Unit vector is $\frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$

4. (b) We know that, $|a+b|^2 + |a - b|^2 = 2(|a|^2 + |b|^2)$

$\therefore\ 25 + |a - b|^2 = 2(9 + 16) \Rightarrow |a - b| = 5$

5. (d) $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$; and $\vec{c} = x\hat{i} + (x - 2)\hat{j} + \hat{k}$

$$\begin{vmatrix} x & x-2 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix} = 0 \Rightarrow 3x + 2 - x + 2 = 0 \Rightarrow x = -2$$

6. (d) $a + b = 4\hat{i} + 4\hat{j}$, therefore, unit vector $\frac{4(\hat{i}+\hat{j})}{\sqrt{32}} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$.

7. (c) Squaring $(a + b + c) = 0$

We get $a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 0$

$\Rightarrow |a|^2 + |b|^2 + |c|^2 + 2(ab + bc + ca) = 0 \Rightarrow ab + bc + ca = -\frac{3}{2}$.

8. (a) Here, $F = 2\hat{i} - 3\hat{j} + 2\hat{k}$, $d = -2\hat{i} - 2\hat{j} - 2\hat{k}$

Work done $= F.d. = -4 + 6 - 4 = -2$ or 2 units.

9. (a) Obviously, $\frac{3}{6} = \frac{2}{-4x} = \frac{-1}{y} \Rightarrow x = -1$ and $y = -2$.

10. (b) $(a + b)\cdot(a + b) = |a|^2 + |b|^2 + 2ab$ or $|a + b|^2 = 2.2\cos^2\frac{\theta}{2} \Rightarrow \cos\frac{\theta}{2} = \frac{1}{2}|a + b|$.

12. (d) Applying section formulas the position vector of the required point is $\frac{2(\vec{a} + \vec{b}) + 1(2\vec{a} - \vec{b})}{2+1} = \frac{4\vec{a} + \vec{b}}{3}$.

13. (c) Using the formula $|\vec{a} \times \vec{b}| = |\vec{a}|\cdot|\vec{b}|\,|\sin\theta|$, we get $\theta = \pm\frac{\pi}{6}$

Therefore, $\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta = 8 \times 3 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}$

14. (b) We have, $\lambda\vec{b} + \vec{c} = \lambda(\hat{i} + \hat{j} - 2\hat{k}) + (\hat{i} + 3\hat{j} - \hat{k}) = (\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} - (2\lambda + 1)\hat{k}$

Since, $\vec{a} \perp (\lambda\vec{b} + \vec{c})$, $\vec{a}\cdot(\lambda\vec{b} + \vec{c}) = 0$

$\Rightarrow (2\hat{i} - \hat{j} + \hat{k})\,[(\lambda + 1)\hat{i} + (\lambda + 3)\hat{j} - (2\lambda + 1)\hat{k}] = 0$

$\Rightarrow 2(\lambda + 1) - 1(\lambda + 3) - (2\lambda + 1) = 0 \Rightarrow \lambda = -2$

15. (c) Projection of a vector $\vec{a}$ on $\vec{b}$ is

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{(2\hat{i}-\hat{j}+\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{1+4+4}} = \frac{2}{3}$$

16. (b) Here, $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2\sqrt{3}$

From scalar product, we know that

$\therefore\ \vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta \Rightarrow 2\sqrt{3} = \sqrt{3}\cdot 4\cdot\cos\theta$

$$\Rightarrow \cos\theta = \frac{2\sqrt{3}}{\sqrt{3}\cdot 4} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

17. (a) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

If $\vec{a}\,||\,\vec{b}$ $\therefore \dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3} \Rightarrow \dfrac{3}{2} = \dfrac{-6}{-4} = \dfrac{1}{\lambda} \Rightarrow \dfrac{1}{\lambda} = \dfrac{3}{2} \Rightarrow \lambda = \dfrac{2}{3}.$

19. (a) $\cos\theta = \dfrac{1-9+49}{\sqrt{59}\sqrt{59}} = \dfrac{41}{59}$

20. (a) By hypothesis,

$(\vec{a} + \vec{b}) \times \vec{c} = 0$; $(\vec{b} + \vec{c}) \times \vec{a} = 0$ and $(\vec{c} + \vec{a}) \times \vec{b} = 0$

Therefore, $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 0$

Now, $|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 9 + 16 + 24 + 2 \times 0 = 49$

$\therefore\ |\vec{a} + \vec{b} + \vec{c}| = 7$

21. (b) Area $= |\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix} = 20\hat{i} + 5\hat{j} - 5\hat{k}$

Area of ||gm $= |\vec{a} \times \vec{b}| = \sqrt{400+25+25} = \sqrt{450}$ sq. units

22. (c) We have : $|\vec{a}| = |\vec{b}| = |\vec{a}| + |\vec{b}| = 1$

Now, $|\vec{a} \times \vec{b}|^2 = 1 \Rightarrow (\vec{a} + \vec{b})^2 = 1$

$\Rightarrow (\vec{a})^2 + (\vec{b})^2 + 2(\vec{a}\cdot\vec{b}) = 1 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = 1 \Rightarrow \cos\theta = \dfrac{-1}{2} \Rightarrow \theta = 120°.$

23. (c) Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$ $\quad\therefore\ \vec{c}\cdot\hat{i} = (x\hat{i} + y\hat{j} + z\hat{k})\cdot\hat{i} = x$

Similarly $\vec{c}\cdot\hat{j} = y$ and $\vec{c}\cdot\hat{k} = z$

Given, $\vec{c}\cdot\hat{i} = \vec{c}\cdot\hat{j} = \vec{c}\cdot\hat{k}$ i.e., $x = y = z$.

Also, $|\vec{c}| = 100$, i.e., $x^2 + y^2 + z^2 = (100)^2 \Rightarrow 3x^2 = (100)^2 \Rightarrow x = \pm\dfrac{100}{\sqrt{3}}$

$\therefore\ \vec{c} = \pm\dfrac{100}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

24. (c) Here, $|\vec{a} \times \vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 144$

$\Rightarrow (|\vec{a}|\,|\vec{b}|\sin\theta)^2 + (|\vec{a}|\,|\vec{b}|^2\cos\theta)^2 = 144 \Rightarrow |\vec{a}|^2\,|\vec{b}|^2\sin^2\theta + |\vec{a}|^2\,|\vec{b}|^2\cos^2\theta = 144$

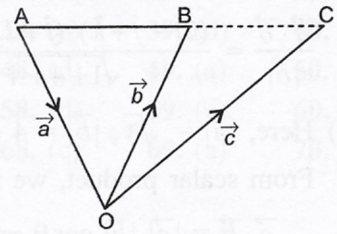
$\Rightarrow |\vec{a}|^2\,|\vec{b}|^2 = 144 \Rightarrow 4\cdot|\vec{b}| = 12 \Rightarrow |\vec{b}| = 3.$

25. (b) The number of vectors of unit length perpendicular or to vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$.

$\therefore\ \vec{c} = \pm(\vec{a} \times \vec{b})$

So, there will be two vectors of unit length perpendicular to vectors $\vec{a}$ and $\vec{b}$.

26. (b) We have, BC = 1.5 BA



$\Rightarrow \dfrac{BC}{BA} = 1.5 = \dfrac{3}{2} \Rightarrow \dfrac{\vec{c}-\vec{b}}{\vec{a}-\vec{b}} = \dfrac{3}{2}$

$\Rightarrow 2\vec{c} - 2\vec{b} = 3\vec{a} - 3\vec{b} \Rightarrow 2\vec{c} = 3\vec{a} - 3\vec{b} + 2\vec{b}$

$\Rightarrow 2\vec{c} = 3\vec{a} - \vec{b} \qquad \therefore\ \vec{c} = \dfrac{3\vec{a}-\vec{b}}{2}$

27. (a) D.c's, $l = m = n,\ l^2 + m^2 + n^2 = 1 \Rightarrow l^2 + l^2 + l^2 = 1 \Rightarrow l = \pm\dfrac{1}{\sqrt{3}}$

$\therefore\ \vec{r} = \pm\dfrac{1}{\sqrt{3}}\,\hat{i} \pm \dfrac{1}{\sqrt{3}}\,\hat{j} \pm \dfrac{1}{\sqrt{3}}\,\hat{k} \Rightarrow \vec{r} = \pm\dfrac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

$\Rightarrow \vec{r} = (\hat{r})\,|\vec{r}| = \pm\dfrac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})(2\sqrt{3}) \Rightarrow \vec{r} = \pm 2(\hat{i} + \hat{j} + \hat{k})$

29. (c) Unit vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1^2+(-2)^2+2^2}} = \dfrac{\hat{i}-2\hat{j}+2\hat{k}}{3}$

$\therefore$ Vector of magnitude $9 = \dfrac{9(\hat{i}-2\hat{j}+2\hat{k})}{3} = 3(\hat{i}-2\hat{j}-2\hat{k})$

30. (d) Since $\vec{a}$ and $\vec{b}$ are orthogonal $\vec{a}\cdot\vec{b} = 0$

$\Rightarrow (2\hat{i}+\lambda\hat{j}+\hat{k})\cdot(\hat{i}+2\hat{j}+3\hat{k}) = 0 \Rightarrow \lambda = \dfrac{-5}{2}$

31. (d) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k},\ \vec{a}^2 = a_1^2 + a_2^2 + a_3^2$

$$\vec{a} \times \hat{i} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix} = a_3\hat{j} - a_2\hat{k}$$

$(\vec{a} \times \hat{i})^2 = (a_3\hat{j} - a_2\hat{k})(a_3\hat{j} - a_2\hat{k}) = a_3^2 + a_2^2$

Similarly, $(\vec{a} \times \hat{j})^2 = a_1^2 + a_3^2$ and $(\vec{a} \times \hat{k})^2 = a_1^2 + a_2^2$

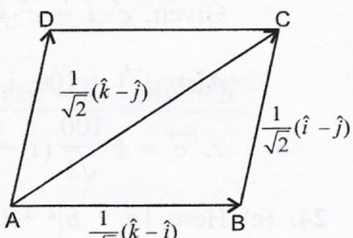
$\therefore\ (\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2 = 2\left(a_1^2 + a_2^2 + a_3^2\right) = 2\vec{a}^2.$

33. (a) By definition of scalar triple product $\vec{a}\cdot(\vec{b}\times\vec{c})$ can be written as $[\vec{a}\ \vec{b}\ \vec{c}]$

$$\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{a}\times\vec{c})}{\vec{c}\cdot(\vec{a}\times\vec{b})} = \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]} + \frac{[\vec{b}\ \vec{a}\ \vec{c}]}{[\vec{c}\ \vec{a}\ \vec{b}]} = \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} - \frac{[\vec{a}\ \vec{b}\ \vec{c}]}{[\vec{a}\ \vec{b}\ \vec{c}]} = 1 - 1 = 0$$

$\because [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}]$ but $[\vec{b}\ \vec{a}\ \vec{c}] = -[\vec{a}\ \vec{b}\ \vec{c}]$

34. (d) The position vector of points D, E, F are respectively.



$\dfrac{\hat{i}+\hat{j}}{2} + \hat{k},\ \hat{i} + \dfrac{\hat{k}+\hat{j}}{2}$ and $\dfrac{\hat{i}+\hat{k}}{2} + \hat{j}$

So, position vector of centre of $\Delta DEF = \dfrac{1}{3}\left[\dfrac{\hat{i}+\hat{j}}{2} + \hat{k} + \hat{i}\,\dfrac{\hat{k}+\hat{j}}{2} + \dfrac{\hat{i}+\hat{k}}{2} + \hat{j}\right] = \dfrac{2}{3}[\hat{i} + \hat{j} + \hat{k}]$

36. (a) According to question $a = -\hat{i} + \hat{j} + \hat{k}$ and $b = \hat{i} - \hat{j} + \hat{k}$

Then, $a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}[1 + 1] - \hat{j}\,[-1 - 1] + \hat{k}[1 - 1] = 2(\hat{i} + \hat{j})$ and $|a \times b| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Required unit vector $= \pm \dfrac{2(\hat{i} + \hat{j})}{2\sqrt{2}} = \pm \dfrac{\hat{i} + \hat{j}}{\sqrt{2}}$

37. (c) $a.b = a \cdot c \Rightarrow a \cdot (b - c) = 0$

$\Rightarrow a = 0$ or $b - c = 0$ or $a \perp (b - c)$

$\Rightarrow a = 0$ or $b = c$ or $a \perp (b - c)$ ...(i)

Also $a \times b = a \times c \Rightarrow a \times (b - c) = 0$

$\Rightarrow a = 0$ or $b - c = 0$ or $a \parallel (b - c)$

$\Rightarrow a = 0$ or $b = c$ or $a \parallel (b - c)$ ...(ii)

Observing to (i) and (ii) we find that $a = 0$ or $b = c$

38. (c) Unit vector perpendicular to both the given vectors is,

$$\frac{(6\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 6\hat{j} - 2\hat{k})}{|(6\hat{i} + 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 6\hat{j} - 2\hat{k})|} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

40. (a) $\vec{r} = \vec{a} + \vec{b} + \vec{c} = 4\hat{i} - \hat{j} - 3\hat{i} + 2\hat{j} - \hat{k} = \hat{i} + \hat{j} - \hat{k}$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{1^2 + 1^2 + (-1)^2}} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$$

41. (b) $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

$\Rightarrow \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

Squaring both the sides, we get

$\vec{A}^2 + \vec{B}^2 + 2\vec{A}\vec{B}\cos\theta = \vec{A}^2 + \vec{B}^2 - 2\vec{A}\vec{B}\cos\theta$

Or $4\vec{A}\vec{B}\cos\theta = 0$ or $\cos\theta = 0$ (since the scalar or dot product is zero).

Therefore, angle between $\vec{A}$ to $\vec{B}$ is 90°.

42. (d) If given vectors are coplanar, then there exists two scalar quantities x and y such that

$2\hat{i} - \hat{j} + \hat{k} = x(\hat{i} + 2\hat{j} - 3\hat{k}) + y(3\hat{i} + a\hat{j} + 5\hat{k})$

Comparing coefficient of $\hat{i}, \hat{j}$ and $\hat{k}$ on both sides

We get, $x + 3y = 2$ ...(i)

$2x + ay = -1$ ...(ii)

and $-3x + 5y = 1$ ...(ii)

Solving (i) and (iii) equations, we get $x = \dfrac{1}{2}, y = \dfrac{1}{2}$

Since, the vectors are coplanar, therefore, these values of x and y will satisfy the equation $2x + ay = -1$

$\therefore \left(\dfrac{1}{2}\right) + a\left(\dfrac{1}{2}\right) = -1 \Rightarrow a = -4$

43. (a) $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 676$

$(|\vec{a}| \cdot |\vec{b}| \sin\theta\hat{n})^2 + (|\vec{a}| \cdot |\vec{b}| \cos\theta)^2 = 676$

$\Rightarrow a^2b^2 \sin^2\theta + a^2b^2 \cos^2\theta = 676\ [(\hat{n})^2 = 1] \Rightarrow a^2b^2(\sin^2\theta + \cos^2\theta) = 676 \Rightarrow a^2 = \dfrac{676}{b^2} = \dfrac{676}{4}$

$\Rightarrow |\vec{a}| = \sqrt{\dfrac{676}{4}} \Rightarrow |\vec{a}| = \dfrac{26}{2} \Rightarrow |\vec{a}| = 13$

44. (d) If P and Q be the points represented by the coordinates (3, 1, 14) and (–2, –1, –6) respectively then, $\overrightarrow{PQ}$ = p.v. of Q – p.v. of P

$= (-2\hat{i} - \hat{j} - 6\hat{k}) - (3\hat{i} + \hat{j} + 14\hat{k}) = -5\hat{i} - 2\hat{j} - 20\hat{k}$ and $\overrightarrow{OQ} = -\overrightarrow{PQ} = 5\hat{i} + 2\hat{j} + 20\hat{k}$

45. (d) The given expression

$= \{\{a \times c + b \times a + b \times c\} \times (b \times c)\} \cdot (b + c) = \{(a \times c) \times (b \times c) + (b \times a) \times (b \times c)\} \cdot (b + c)$

$= [(a \cdot (b \times c)\ c - (c \cdot (b \times c))a + (b \cdot (b \times c))a - (a(b \times c)b)](b + c)$

$= [(a \cdot (b \times c))(c - b) \cdot (b + c)] = (a \cdot (b \times c)) = [|c|^2 - |b|^2] = 0\ [|b| = |c| = 1]$

46. (d) The position vector of points D, E, F are respectively.

$\frac{\hat{i} + \hat{j}}{2} + \hat{k},\ \hat{i} + \frac{\hat{k} + \hat{j}}{2}$ and $\frac{\hat{i} + \hat{k}}{2} + \hat{j}$

So, position vector of centre of ΔDEF

$= \frac{1}{3}\left[\frac{\hat{i} + \hat{j}}{2} + \hat{k} + \hat{i} + \frac{\hat{k} + \hat{j}}{2} + \frac{\hat{i} + \hat{k}}{2} + \hat{j}\right] = \frac{2}{3}[\hat{i} + \hat{j} + \hat{k}]$

47. (c) Vectors $\vec{a} + 2\vec{b} + 3\vec{c}$, $\lambda\vec{b} + 4\vec{c}$, and $(2\lambda - 1)\vec{c}$ are coplanar if $\begin{vmatrix} 1 & 2 & 3 \\ 0 & \lambda & 4 \\ 0 & 0 & 2\lambda - 1 \end{vmatrix} = 0$

$\Rightarrow \lambda(2\lambda - 1) = 0 \Rightarrow \lambda = 0$ or $\frac{1}{2}$

∴ Forces are noncoplanar for all λ, except $\lambda = 0, \frac{1}{2}$

49. (a) $\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{3 + 4 + 3}{\sqrt{14}\sqrt{14}} = \frac{5}{7}$

50. (d) We have, $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} - \hat{k}$

We know that $\vec{A} \cdot \vec{B} = AB\cos\theta$ or $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$

Now, $A = \sqrt{1^2 + 1^2 + (-2)^2} = \sqrt{6}$, and $B = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$

$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} - 2\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -1 + 2 + 3 = 3$

$\Rightarrow \cos\theta = \frac{3}{\sqrt{6} \times \sqrt{6}} = \frac{3}{6} = \frac{1}{2}$ or $\theta = 60°$

51. (a) Let $\vec{A} = (\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{B} = (2\hat{i} + \hat{j} - 3\hat{k})$

Now, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{vmatrix}$ and $|\vec{A} \times \vec{B}| = \sqrt{25 + 25 + 25} = 5\sqrt{3}$

We know a unit vector along $\vec{a}$ is given as $\frac{\vec{a}}{|\vec{a}|}$.

∴ Unit vector along $|\vec{A} \times \vec{B}| = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \frac{5(\hat{i} + \hat{j} + \hat{k})}{5\sqrt{3}} = \left(\frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}\right)$

Thus, required vector of magnitude 5 is $\frac{5\sqrt{3}}{3}(\hat{i} + \hat{j} + \hat{k})$

52. (d) We have, $\hat{i} \times (\hat{j} \times \hat{k}) + \hat{j} \times (\hat{k} \times \hat{i}) + \hat{k} \times (\hat{i} \times \hat{j}) = \hat{i} \times \hat{i} + \hat{j} \times \hat{j} + \hat{k} \times \hat{k} = 0 + 0 + 0 = 0$

54. (a) Any vector r in the plane of b and c is $r = b + tc$

or $r = (1 + t)\hat{i} + (2 + t)\hat{j} - (1 + 2t)\hat{k}$ ...(i)

Projection of r on a is $\sqrt{\frac{2}{3}} \Rightarrow \frac{r\cdot a}{|a|} = \sqrt{\frac{2}{3}}$ or $\frac{2(1+t)-(2+t)-(1+2t)}{\sqrt{6}} = \pm\sqrt{\frac{2}{3}}$

$\therefore -t-1 = \pm 2 \Rightarrow t = -3, 1$

Putting in (i) we get $r = -2\hat{i} - \hat{j} + 5\hat{k}$ or $r = 2\hat{i} + 3\hat{j} - 3\hat{k}$

55. (c) We have, $r = a_1 + \lambda b_1$, $r = a_2 + \mu b_2$

Where $a_1 = 3\hat{i} - 2\hat{j} - 2\hat{k}$, $b_1 = \hat{i}$

$a_2 = \hat{i} - \hat{j} + 2\hat{k}$, $b_2 = \hat{j}$

$|b_1 \times b_2| = |\hat{i} \times \hat{j}| = (\hat{k}) = 1$

Now, $[(a_2 - a_1)b_1 \times b_2] = (a_2 - a_1)\cdot(b_1 \times b_2) = (-2\hat{i} + \hat{j} + 4\hat{k})(\hat{k}) = 4$

$\therefore$ Shortest distance $= \frac{(a_2-a_1)(b_1\times b_2)}{|b_1 \times b_2|} = \frac{4}{1} = 4.$

57. (a) $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & -3 \\ 2 & 1 & -1 \end{vmatrix} = 5\hat{i} - 5\hat{j} + 5\hat{k}$

Now, $(\vec{A} \times \vec{B}) \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -5 & 5 \\ 1 & 3 & -2 \end{vmatrix} = 5(-\hat{i} + 3\hat{j} + 4\hat{k})$

58. (d) $a\cdot[(b + c) \times (a + b + c)] = a\cdot(b \times a + b \times b + b \times c) + a\cdot(c \times a + c \times b + c \times c)$

$= [aba] + [abb] + [abc] + [aca] + [acb] + [acc]$

$= 0 + 0 + [abc] + 0 - [abc] + 0 = 0$

59. (c) Let $a = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow \hat{i} \times (a\times \hat{i}) + \hat{j} \times (a \times \hat{j}) + \hat{k} \times (a\times\hat{k}) = (\hat{i}\cdot\hat{i})\, a - \hat{i}(a\cdot\hat{i}) + (\hat{j}\cdot\hat{j})a - \hat{j}(a\cdot\hat{j}) + (\hat{k}\cdot\hat{k})a - \hat{k}(a\cdot\hat{k})$

$= 3a - a = 2a.$

60. (a) $[a - b \quad b - c \quad c - a] = \{(a - b) \times (b - c)\}\cdot(c - a)$

$= (a \times b - a \times c - b \times b + b \times c)\,(c - a)$

$= (a \times ab + ca \times a + b \times c)\,(c - a)$

$= (a \times b)\cdot c - (a \times b)\cdot a + (c \times a)\cdot c - (c \times a)\cdot a$

$= (a \times b)\cdot c - (a \times b)\cdot a + (c \times a)\cdot c - (c \times a)\cdot a + (b \times c)\cdot c - (b \times c)\cdot a$

$= [abc] - [aba] + [cac] - [caa] + [bcc] - [bca] = 0$

61. (c) $a \times (b \times c) = \frac{b+c}{\sqrt{2}} \Rightarrow (a\cdot c)b - (a\cdot b)c = \frac{b+c}{\sqrt{2}}$

$\Rightarrow \left[(a\cdot c) - \frac{1}{\sqrt{2}}\right]b - \left[(a\cdot b) + \frac{1}{\sqrt{2}}\right]c = 0$

$\Rightarrow a\cdot c = \frac{1}{\sqrt{2}},\ a\cdot b = \frac{-1}{\sqrt{2}} \Rightarrow |a||c| \cos\theta = \frac{1}{\sqrt{2}},\ |a||c| \cos\phi = \frac{-1}{\sqrt{2}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}},\ \cos\phi = \frac{-1}{\sqrt{2}} \quad \theta = \frac{\pi}{4},\ \phi = \frac{3\pi}{4}.$

62. (a) We have, $P(\vec{a} + 2\vec{b} + \vec{c})$, $Q(2\vec{a} + 3\vec{b})$ and $R(\vec{b} + \vec{c})$ are collinear.

$\therefore \overrightarrow{PQ} = \lambda\overrightarrow{QR}$ for some scalar λ.

$\Rightarrow \vec{a} + \vec{b} - \vec{c} = \lambda(-2\vec{a} - 2\vec{b} + t\vec{c}) \Rightarrow (2\lambda + 1)\vec{a} + a(1 + 2\lambda)\vec{b} - (t\lambda + 1)\vec{c} = 0$

$\Rightarrow 2\lambda + 1 = 0,\ 2\lambda + 1 = 0,\ t\lambda + 1 = 0 \Rightarrow t = 2$

63. (a) Let θ be the angle between $\vec{a}$ and $\vec{b}$.

We have, $|\vec{a} - \vec{b}| = 1 \Rightarrow |\vec{a} - \vec{b}|^2 = 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}\cdot\vec{b}) = 1 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \cos\theta = 1 \Rightarrow 1 + 1 - 2\cos\theta = 1$

$\Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \cos\theta = \dfrac{\pi}{3}$.

64. (b) Let θ be the angle between unit vector $\vec{a}$ and $\vec{b}$.

Then, $|\vec{a} - \vec{b}| = 1 \Rightarrow |\vec{a} - \vec{b}|^2 = 1$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 - 2(\vec{a}\cdot\vec{b}) = 1 \Rightarrow 1 + 1 - 2|\vec{a}||\vec{b}| \cos\theta = 1 \Rightarrow \cos\theta = \dfrac{1}{2} \Rightarrow \theta = \dfrac{\pi}{3}$

65. (c) Let θ be the angle between $\vec{a}$ and $\vec{b}$. Then,

$\tan\dfrac{\theta}{2} = \dfrac{|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|} \Rightarrow \tan\dfrac{\theta}{2} = \sqrt{3} \Rightarrow \theta = 120°$

$\therefore\ \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}| \cos\theta = \cos 120° = \dfrac{-1}{2}$

Now, $|3\vec{a}+2\vec{b}|^2 = 9|\vec{a}|^2 + 4|\vec{b}|^2 + 12(\vec{a}\cdot\vec{b}) = 9 + 4 + 12 \times \dfrac{-1}{2} = 7$

$\Rightarrow |3\vec{a}+2\vec{b}| = \sqrt{7}$.

67. (d) We have, $\vec{F} = 7\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{d} = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ Work done $= \vec{F}\cdot\vec{d} = (28 + 4 + 8)$ units $= 40$ units.

68. (c) We have, $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -3 \\ 4 & 3 & -1 \end{vmatrix} = 5(3\hat{i} - 2\hat{j} + 6\hat{k})$

$|\vec{a} \times \vec{b}| = 5\sqrt{9+4+36} = 35$

Hence, required unit vector $\hat{n}$ is

$\hat{n} = \dfrac{5}{35}(3\hat{i} - 2\hat{j} + 6\hat{k}) = \dfrac{1}{7}(3\hat{i} - 2\hat{j} + 6\hat{k})$.

70. (b) We have, $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} \Rightarrow \vec{a} \times \vec{b} = -(\vec{c} \times \vec{b})$

$\Rightarrow (\vec{a} + \vec{c}) \times \vec{b} = 0 \Rightarrow \vec{a} + \vec{c}$ is parallel to $\vec{b}$.

$\Rightarrow \vec{a} + \vec{c} = \lambda\vec{b}$ for some scalar $\lambda \Rightarrow \vec{c} \times (\vec{a} + \vec{c}) = \vec{c} \times \lambda\vec{b}$

$\Rightarrow \vec{c} \times \vec{a} + \vec{c} \times \vec{c} = \lambda(\vec{c} \times \vec{b}) \Rightarrow \vec{c} \times \vec{a} = -\lambda(\vec{b} \times \vec{c})$

$\Rightarrow (\vec{b} \times \vec{c}) = -\lambda(\vec{b} \times \vec{c}) \Rightarrow \lambda = -1$

$\therefore\ \vec{a} + \vec{c} = \lambda\vec{b} \Rightarrow \vec{a} + \vec{b} + \vec{c} = 0$

71. (c) Given vectors will be coplanar, if

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0 \Rightarrow \lambda^6 - 3\lambda^2 - 2 = 0$$

$\Rightarrow (1 + \lambda^2)^2(\lambda^2 - 2) = 0 \Rightarrow \lambda = \pm\sqrt{2}$

72. (c) We have, $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$

$\Rightarrow \vec{a} \times (\vec{a} \times \vec{b}) = \vec{a} \times (\hat{j} - \hat{k}) \Rightarrow (\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} = (\hat{i} + \hat{j} - \hat{k}) \times (\hat{j} - \hat{k})$

$\Rightarrow (\hat{i} + \hat{j} + \hat{k}) - 3\vec{b} = -2\hat{i} + \hat{j} + \hat{k} \Rightarrow 3\vec{b} = 3\hat{i} \Rightarrow \vec{b} = \hat{i}$.