Chapter - 5 CONTINUITY AND DIFFERENTIABILITY

STUDY NOTES

- **Continuity of a function :**

  A function $f(x)$ is said to be continuous at $x = a$ if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

  i.e., LHL = RHL = Value of function at 'a', i.e., $\lim_{x \to a} f(x) = f(a)$

  If $f(x)$ is not continuous at $x = a$, we say that $f(x)$ is discontinuous at $x = a$.

- $f(x)$ is not continuous at $x = a$ in any of following case :

  (i) $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exists but are not equal.

  (ii) $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exists and are equal but not equal to $f(a)$.

  (iii) $f(a)$ is not defined.

  (iv) At least one of the limits does not exist.

- **Continuity of a function in an interval.**

  (a) Continuity in an open interval :

  A function is said to be continuous in an open interval (a, b) if it is continuous at each point of (a, b).

  (b) Continuity in closed Interval

  A function is said to be continuous in a closed interval $[a, b]$ if :

  (i) $f(x)$ is continuous from right at $x = a$, i.e., $\lim_{x \to a^+} f(x) = f(a)$

  (ii) $f(x)$ is continuous from left at $x = b$, i.e., $\lim_{x \to b^-} f(x) = f(b)$

  (iii) $f(x)$ is continuous at each point of interval (a, b)

- **Geometrical meaning of Continuity**

  (i) The function 'f' will be continuous at $x = a$ if there is no break in the graph of the function $y = f(x)$ at the point $(a, f(a))$.

  (ii) The function $f(x)$ will be continuous in the closed interval $[a, b]$ if the graph of $y = f(x)$ is an unbroken line from the point $(a, f(a))$ to $(b, f(b))$.

- **Continuity of Composite Function**

  If the function $u = f(x)$ is continuous at the point $x = a$, and the function $y = g(u)$ is continuous at the point $u = f(a)$, then the composite function $y = (gof)\ x = g(f(x))$ is continuous at point $x = a$.

- **Differentiability**

  Let $y = f(x)$ is continuous in (a, b). Then the derivative or differential of $f(x)$ at $x \in (a, b)$ denoted by $\frac{dy}{dx}$ or $f'(x)$ and is defined as

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Right and Derivative

Right hand derivative $f(x)$ at $x = a$ is denoted by $Rf'(a)$ or $f'(a^+)$ and defined as

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}, \ h > 0$$

Left and Derivative

Left hand derivative of $f(x)$ at $x = a$ is denoted by $Lf'(a)$ or $f'(a^-)$ and is defined as

$$Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}, h > 0$$

Standard Results on Differentiation :

(a) $\frac{d}{dx}(c) = 0$, c is constant

(b) $\frac{d}{dx}(x^n) = nx^{n-1}$

(c) $\frac{d}{dx}(\sin x) = \cos x$

(d) $\frac{d}{dx}(\cos x) = -\sin x$

(e) $\frac{d}{dx}(\tan x) = \sec^2 x$

(f) $\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

(g) $\frac{d}{dx}(\text{cosec } x) = -\text{cosec} x \ cot x$

(h) $\frac{d}{dx}(sec \ x) = sec \ x \ tan x$

(i) $\frac{d}{dx}(a^x) = a^2 \log_e^a$

(j) $\frac{d}{dx}(e^x) = e^x$

(k) $\frac{d}{dx}(\log x) = \frac{1}{x} \ (x > 0)$

(l) $\frac{d}{dx}|x| = \frac{x}{|x|}$ or $\frac{|x|}{x}$

(m) $\frac{d}{dx}\sqrt{x} = \frac{2}{\sqrt{x}}$

(n) $\frac{d}{dx}x^x = x^x(1 + \log x)$

(o) $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

(p) $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

(q) $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

(r) $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

(s) $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{|x|\sqrt{x^2-1}} \ |x|>|$

(t) $\frac{d}{dx}(\text{cosec}^{-1}x) = -\frac{1}{|x|\sqrt{x^2-1}} \ |x|>|$

- **Chain rule :** If y is a function of z and 'z' is a function of x, then

$$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$$

- **Algebraic functions of Differentiation**

(a) $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}f(x)$

(b) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

(c) $\frac{d}{dx}[f(x).g(x)] = g(x)\frac{d}{dx}f(x) + f(x)\frac{d}{dx}g(x)$

(d) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(e) $\frac{d}{dx}[f(x)]^{g(x)}] = [f(x)^{g(x)}\frac{d}{dx}g(x).\log f(x)$

(f) $\frac{d}{dx}\frac{1}{f(x)} = -\frac{1}{f(x)^2} \cdot \frac{d}{dx}f(x).\ f(x) \neq 0$

- **Some Important Results**

(a) $a^2 + x^2 \longrightarrow x = a \tan\theta$ (b) $a^2 - x^2 \longrightarrow x = a \sin\theta$ or $a\cos\theta$

(c) $x^2 - a^2 \longrightarrow x = a \sec\theta$ or $x = a \ \text{cosec}\theta$

(d) $\frac{a+x}{a-x} \longrightarrow x = a \tan\theta$

(e) $\sqrt{\frac{a+x}{a-x}} \longrightarrow x = a\cos\theta$ (f) $\frac{2x}{1+x^2} \longrightarrow x = \tan\theta$

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. The points of discontinuity of $y = g(u) = \frac{1}{u^2 + u - 2}$ is ; where $u = \frac{1}{x-1}$

(a) Only $\frac{1}{2}$ (b) only $-\frac{1}{2}$ (c) $\frac{1}{2}, -\frac{1}{2}$ (d) $1, \frac{1}{2}, 2$

2. If $f(x) = \frac{1}{(x-1)(x-2)}$ and $g(x) = \frac{1}{x^2}$, then points of discontinuity of $f(g(x))$ are

(a) $\left\{-1, 0, 1, \frac{1}{\sqrt{2}}\right\}$ (b) $\left\{-\frac{1}{\sqrt{2}}, -1, 0, 1, \frac{1}{\sqrt{2}}\right\}$ (c) $\{0, 1\}$ (d) $\left\{0, 1, \frac{1}{\sqrt{2}}\right\}$

3. If $f(x) = (1 + x)(2 + x^2)^{1/2} (3 + |x^3|)^{1/3}$, then $f'(-1)$ is:

(a) $\sqrt{3}\ 2^{1/3}$ (b) $\sqrt{2}\ 3^{1/3}$ (c) 0 (d) does not exists

4. Let $f : R \to R$ be a differentiable function at $x = 0$ satisfying $f(0) = 0$ and $f'(0) = 1$, then the value of $\lim_{x \to 0} \frac{1}{x} \sum_{n=1}^{\infty} (-1)^n f\left(\frac{x}{n}\right) =$

(a) 0 (b) –In2 (c) 1 (d) e

5. If $f(x)$ is continuous in [0, 2] and $f(0) = f(2)$ then the equation $f(x) = f(x + 1)$ has

(a) non real root in [0, 2] (b) at least one real root in [0, 1]

(c) at least one real root in [0, –2] (d) at least one real root in [1, 2]

6. Let $f(x) = \max(\cos x, x, 2x - 1)$, where $x \geq 0$, then the number of points of non-differentiability of $f(x)$ is :

(a) only 1 (b) two (c) three (d) four

7. If the function $g(x) = \begin{cases} k\sqrt{x+1}. & 0 \leq x \leq 3 \\ mx + 2, & 3 < x \leq 5 \end{cases}$

is differentiable, then the value of $k + m$ *is* :

(a) $\frac{16}{5}$ (b) $\frac{10}{3}$ (c) 4 (d) 2

8. If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to:

(a) $\frac{1}{2}$ (b) 1 (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

9. If a function $f(x)$ is differentiable at $x = a$ then $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ is :

(a) $2af(a) - a^2 f'(a)$ (b) $2af(a) + a^2 f'(a)$ (c) $-a^2 f'(a)$ (d) $af(a) - a^2 f'(a)$

10. Let $f : (-1, 1) \Rightarrow R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$.

Let $g(x) = (f(2f(x))$

Then $g'(0) =$

(a) –4 (b) 0 (c) –2 (d) 4

11. Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then $y'(1)$ equals

(a) –1 (b) log 2 (c) 1 (d) – log 2

12. Let $f(x) = \begin{cases} (x-1)\sin\left(\dfrac{1}{x-1}\right), & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$

Then which one of the following is true?

(a) f is neither differentiable at $x = 0$ nor at $x = 1$
(b) f is differentiable at $x = 0$ and $x = 1$
(c) f is differentiable at $x = 0$ but not at $x = 1$
(d) f is differentiable at $x = 1$ but not $x = 0$

13. The set of points where $f(x) = \dfrac{x}{1+|x|}$ is differentiable is

(a) $(-\infty, 0) \cup (0, \infty)$
(b) $(-\infty, -1) \cup (-1, \infty)$
(c) $(-\infty, \infty)$
(d) $(0, \infty)$

14. If $x^m y^n = (x + y)^{m+n}$, then $\dfrac{dy}{dx}$ is:

(a) $\dfrac{y}{x}$
(b) xy
(c) $\dfrac{x+y}{xy}$
(d) $\dfrac{x}{y}$

15. Let $f(x)$ is differentiable at $x = 1$ and $\lim\limits_{h\to 0}\dfrac{1}{h}f(1+h) = 5$, then $f'(1)$ equals

(a) 3
(b) 4
(c) 5
(d) 6

16. Let $f(x) = \dfrac{1-\tan x}{4x-\pi}$, $x \neq \dfrac{\pi}{4}$, $x \in \left[0, \dfrac{\pi}{2}\right]$.

If $f(x)$ is continuous in $\left[0, \dfrac{\pi}{2}\right]$ then $f\left(\dfrac{\pi}{4}\right)$ is:

(a) 1
(b) $\dfrac{1}{2}$
(c) $-\dfrac{1}{2}$
(d) -1

17. If f is a real valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals :

(a) -1
(b) 0
(c) 2
(d) 1

18. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$, for $x \in [1, 6]$ then :

(a) $f(6) \geq 8$
(b) $f(6) < 8$
(c) $f(6) < 5$
(d) $f(6) = 5$

19. If $x = e^{y+e^{y+\ldots+\infty}}$, $x > 0$, then $\dfrac{dy}{dx}$ is :

(a) $\dfrac{x}{1+x}$
(b) $\dfrac{1}{x}$
(c) $\dfrac{1-x}{x}$
(d) $\dfrac{x+1}{x}$

20. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$; then $f(x)$ is :

(a) continuous as well as differentiable for all x.
(b) continuous for all x but not differentiable at $x = 0$.
(c) Neither differentiable nor continuous at $x = 0$
(d) discontinuous everywhere

21. If $f(x) = \dfrac{2-\sqrt{x+4}}{\sin 2x}$, $(x \neq 0)$ is continuous function at $x = 0$, then $f(0)$ is equal to:

(a) $\dfrac{1}{4}$
(b) $-\dfrac{1}{4}$
(c) $\dfrac{1}{8}$
(d) $-\dfrac{1}{8}$

22. The value of $f(2)$, if the given function $f(x) = \dfrac{x(x-2)}{x^2-4}$, $x \neq 2$. is continuous at $x = 2$ is :

(a) 0
(b) $\dfrac{1}{2}$
(c) 1
(d) 2

23. If $f(x) = \begin{cases} 3x - 4 & 0 \le x \le 2 \\ 2x + \lambda & 2 < x \le 3 \end{cases}$ is continuous at $x = 2$, then what is the value of λ?

(a) 1 (b) –1 (c) 2 (d) –2

24. The value of $f(0)$ if the function $f(x) = \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x} (x \neq 0)$ is continuous at each point of its domain is :

(a) 2 (b) $\dfrac{1}{3}$ (c) $\dfrac{2}{3}$ (d) $-\dfrac{1}{3}$

25. The value of b is, if the function

$f(x) = \begin{cases} 5x - 4 & \text{if } 0 < x \le 1 \\ 4x^2 + 3bx & \text{if } 1 < x < 2 \end{cases}$ is continuous at every point of its domain

(a) –1 (b) 0 (c) 1 (d) 2

26. The value of k if $f(x) = \begin{cases} 2x + 1 & x > 1 \\ k & x = 1 \\ 5x - 2 & x < 1 \end{cases}$ is continuous at $x = 1$ is :

(a) 1 (b) 2 (c) 3 (d) 4

27. Which of the following the value of λ if the function $f(x) = \begin{cases} \left[\dfrac{1 - \sin x}{\pi - 2x}\right] & x \neq \dfrac{\pi}{2} \\ \lambda & x = \dfrac{\pi}{2} \end{cases}$ Continuous at $x = \dfrac{\pi}{2}$?

(a) –1 (b) 1 (c) 0 (d) 2

28. Which of the following is correct if the function?

$f(x) = \begin{cases} ax + 1 & x < \dfrac{\pi}{2} \\ \sin x + b & x > \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \dfrac{\pi}{2}$?

(a) $a = 1, b = 0$ (b) $a = \dfrac{n\pi}{2} + 1$ (c) $b = a\left(-\dfrac{\pi}{2}\right)$ (d) $b = \dfrac{9\pi}{2}$

29. Consider the function $f(x) = \begin{cases} ax^2 + b & x < -1 \\ bx^2 + ax + 4 & x \ge -1 \end{cases}$ is continuous everywhere, then, the values of a and b are:

(a) 3, 2 (b) 2, 3 (c) 1, 2 (d) 0, 2

30. $f(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ is

(a) continuous at $x = \dfrac{\pi}{2}$ (b) continuous at $x = 0$

(c) discontinuous at $x = 1$ (d) discontinuous at every point

31. What is the value of $f(0)$, if $f(x) = (x + 1)^{\cot x}$ is continuous at $x = 0$?

(a) 1 (b) e (c) $\dfrac{1}{e}$ (d) e^2

32. A function is defined as $f(x) = x^a \cos\left(\dfrac{1}{x}\right)$, $x \neq 0$, $f(0) = 0$. Which of the following must be true if the given function is continuous at $x = 0$?

(a) $a = 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \ge 0$

33. $f(x)$ is a real valued function, defined as $f(x) = \sin(|x|)$, which one of the following is correct?

(a) f is not differentiable only at 0. (b) f is differentiable at 0 only.

(c) f is differentiable everywhere. (d) f is non-differentiable at many points.

34. $f(x)$ is a real valued function such that $f(x) = -x$ if $x \geq 0$ and $f(x) = -x^2$ if $(x) < 0$. Then which of the following is correct?

(a) $f(x)$ is continuous at every $x \in R$.

(b) $f(x)$ is continuous at $x = 0$ only..

(c) $f(x)$ is discontinuous at $x = 0$ only.

(d) $f(x)$ is discontinuous at every $x \in R$.

35. Which of the following statement is correct for the function $f(x) = |x| + x^2$?

(a) $f(x)$ is not continuous at $x = 0$.

(b) $f(x)$ is differentiable at $x = 0$.

(c) $f(x)$ is continuous but not differentiable at $x = 0$.

(d) $f(x)$ is discontinuous at $x = 0$.

36. F (x) is defined as the product of two real functions $f_1(x) = x, x \in R$ and $f_2(x) = \begin{cases} \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

as $F(x) = \begin{cases} f_1(x).f_2(x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Then which of the following statement is correct?

(a) F (x) is continuous on R.

(b) $f_1(x)$ and $f_2(x)$ are continuous on R.

(c) F (x) is not continuous on R.

(d) $f_1(x)$ and $f_2(x)$ are differentiable on R.

37. $f : R \to R$ be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$.

Then which of the following statement is correct?

(a) $f(c) = \frac{1}{3}$, for some $c \in R$.

(b) $0 < f(x) \leq \dfrac{1}{2\sqrt{2}}$ for all $x \in R$.

(c) $f(x)$ is differentiable at $x = 0$

(d) All of these

38. The function $f(x) = 1 + |\sin x|$ is :

(a) continuous no where

(b) continuous every where.

(c) differentiable at $x = 0$ only.

(d) not differentiable at infinite number of points.

39. For the function $f(x) = \begin{cases} |x-3| & x \geq 0 \\ \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}, & x < 1 \end{cases}$; which one of the following is incorrect?

(a) continuous at $x = 1$

(b) differentiable at $x = 1$

(c) continuous at $x = 3$

(d) differentiable at $x = 3$

40. If $f(x) = \min\{1, x^2, x^3\}$, then

(a) $f(x)$ is continuous $\forall\ x > R$

(b) $f'(x) > 0, \forall\ x > 1$

(c) $f(x)$ is differentiable but continuous $\forall\ x \in R$

(d) $f(x)$ is not differentiable for two values of x.

41. Let $f : R \to R$ be a function such that $f(x + y) = f(x) + f(y)\ \forall\ x, y \in R$. If $f(x)$ is differentiable at $x = 0$, then :

(a) $f(x)$ is differentiable only in a finite interval containing zero.

(b) $f'(x)$ is continuous $\forall\ x > R$

(c) $f(x)$ is constant $\forall\ x > R$

(d) $f(x)$ is differentiable except at finitely many points.

42. If $f(x) = |x - 3|$, then

(a) $\lim\limits_{x \to 3^-} f(x) \neq 0$

(b) $\lim\limits_{x \to 3^-} f(x) \neq 0$

(c) $\lim\limits_{x \to 3^+} f(x) \neq \lim\limits_{x \to 3^-} f(x)$

(d) $f(x)$ is continuous at $x = 3$

43. If the function $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{when } x \neq \dfrac{\pi}{2} \\ 3, & \text{when } x = \dfrac{\pi}{2} \end{cases}$; continuous at $x = \frac{\pi}{2}$, then $k =$

(a) 3 (b) 6 (c) 12 (d) 10

44. The function $f(x) = \dfrac{\log(1+ax) - \log(1-bx)}{x}$ is not defined at $x = 0$, so that it is continuous at $x = 0$ is:

(a) $a - b$ (b) $a + b$ (c) $\log a + \log b$ (d) $\log a - \log b$

45. Let $f(x) = \begin{cases} \dfrac{x^3 + x^2 - 16x + 20}{(x-2)^2} & \text{if } x \neq 2 \\ m & \text{if } x = 2 \end{cases}$. If $f(x)$ is continuous for all x, then $k =$.

(a) 7 (b) -7 (c) ± 7 (d) 6

46. If the function $f(x) = (x + 1)^{1/x}$ is continuous at $x = 0$, then $f(0)$ must be.

(a) 0 (b) $\dfrac{1}{e}$ (c) e (d) 1

47. The point at which the function $f(x) = \dfrac{x+1}{x^2 + x - 12}$ is discontinuous is :

(a) 3, 4 (b) 3, -4 (c) $-1, -3, 4$ (d) $-1, 3, 4$

48. If $f(x) = \begin{cases} \dfrac{\sin x}{x} + \cos x, & \text{when } x \neq 0 \\ 2, & \text{when } x = 0 \end{cases}$, then

(a) $\lim_{x \to 0^+} f(x) \neq 2$

(b) $\lim_{x \to 0^-} f(x) = 0$

(c) $f(x)$ is continuous at $x = 0$

(d) None of these

49. At which points the function $f(x) = \dfrac{x}{[x]}$, where $[\cdot]$ is the greatest integer function is discontinuous.

(a) only positive integer

(b) all positive and negative and (0, 1)

(c) all rational numbers

(d) none of these

50. For the function $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$. Which one is a true statement?

(a) $f(x)$ is continuous at $x = 0$

(b) $f(x)$ is discontinuous at $x = 0$ when $a \neq \pm 1$

(c) $f(x)$ is continuous at $x = a$

(d) None of these

51. If $f(x) = \begin{cases} \dfrac{\sin 2x}{5x}, & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ is continuous at $x = 0$. Then the value of x will be:

(a) 1 (b) $\dfrac{2}{5}$ (c) $-\dfrac{2}{5}$ (d) -1

52. If $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{x^2} & \text{when } x < 0 \\ a & \text{when } x = 0 \\ \dfrac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & \text{when } x > 0 \end{cases}$ is continuous at $x = 0$, then the value of a will be:

(a) 4 (b) 8 (c) 4 (d) -8

53. If $f(x) = \begin{cases} \dfrac{x^4 - 16}{x - 2}, & \text{when } x \neq 2 \\ 16, & \text{when } x = 2 \end{cases}$ Then,

(a) $f(x)$ is continuous at $x = 2$ (b) $f(x)$ is discontinuous at $x = 2$

(c) $\lim_{x \to 2} f(x) = 16$ (d) $\lim_{x \to 2^-} f(x) = 16$

54. If $f(x) = \begin{cases} \dfrac{x^2 + 3x - 10}{x^2 + 2x - 15}, & \text{when } x \neq -5 \\ a, & \text{when } x = -5 \end{cases}$; continuous at $x = -5$, then what is the value of a?

(a) $\frac{3}{2}$ (b) $\frac{7}{8}$ (c) $\frac{8}{7}$ (d) $\frac{2}{3}$

55. The value of m, if $f(x) = \begin{cases} x + m & x < 3 \\ 4 & x = 3 \\ 3x - 5 & x > 3 \end{cases}$ is continuous at $x = 3$.

(a) 1 (b) 4 (c) 3 (d) –1

56. If $f(x) = \begin{cases} \dfrac{\sin[x]}{[x]+1} & \text{for } x > 0 \\ \dfrac{\cos\frac{\pi}{2}[x]}{[x]} & \text{for } x < 0 \\ k & \text{at } x = 0 \end{cases}$, where $[x]$ denotes greatest integer function. The value of k, if the given function is continuous at $x = 0$ is :

(a) 0 (b) 1 (c) –1 (d) not defined

57. If $f(x) = \dfrac{1 - \cos 4x}{8x^2}$, where $x \neq 0$ and $f(x) = k$ where $x = 0$ is a continuous function at $x = 0$, then the value of k will be:

(a) 0 (b) 1 (c) –1 (d) ± 1

58. If $f(x) = \begin{cases} \dfrac{xe^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then which of the following is true?

(a) f is continuous and differentiable at every point (b) f is continuous at every point but is not differentiable

(c) f is differentiable at every point (d) f is differentiable only at the origin

59. If $f(x) = |x - 3|$, then f is.

(a) discontinuous at $x = 2$ (b) not differentiable at $x = 2$

(c) differentiable at $x = 3$ (d) continuous but not differentiable at $x = 3$

60. Let $h(x) = \min\{x, x^2\}$, for every real number of x. Then

(a) h is continuous for all x. (b) h is differentiable for all x.

(c) $h'(x) = 1$ for all $x > 1$. (d) h is not differentiable at two values of x.

61. The function $f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 < x \leq 2 \end{cases}$ is :

(a) continuous at all x, $0 \leq x \leq 2$ and differentiable at all x, except $x = 1$ in the interval $[0, 2]$

(b) continuous and differentiable at all x in $[0, 2]$

(c) not continuous at any point in [0, 2]

(d) not differentiable at any point [0, 2]

62. The function $f(x) = |x|$ at $x = 0$ is

(a) continuous but non-differentiable (b) discontinuous and differentiable

(c) discontinuous and non-differentiable (d) continuous and differentiable

63. Consider $f(x) = \begin{cases} \dfrac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

(a) $f(x)$ is discontinuous everywhere (b) $f(x)$ is continuous but not differentiable

(c) $f'(x)$ exists in (–1, 1) (d) $f'(x)$ exists in (–2, 2)

64. The function $y = |\sin x|$ is continuous for any x but it is not differentiate at

(a) $x = 0$ only (b) $x = \pi$ only

(c) $x = k\pi$ (k is an integer) only (d) $x = 0$ and $x = k\pi$ (k is integer)

65. If the left hand derivative of $f(x) = [x] \sin(\pi x)$ at $x = k$, is an integer and $[x]$ = greatest integer $\leq x$ is :

(a) $(-1)^k (k-1)\pi$ (b) $(-1)^{k-1} (k-1)\pi$ (c) $(-1)^k - k\pi$ (d) $(-1)^{k-1} k\pi$

66. If $f(x) = \dfrac{x}{1+|x|}$ for $x \in R$, then $= f'(0) =$

(a) 0 (b) 1 (c) 2 (d) 3

67. $f(x) = ||x| - 1||$ is not differentiable at :

(a) 0 (b) 1 (c) ±1, 0 (d) ±1

68. Let $f(x)$ is differentiable at $x = 1$ and $\lim\limits_{h \to 0} \dfrac{1}{h} f(1+h) = 5$, then $f'(1)$ equals :

(a) 5 (b) 6 (c) 3 (d) 4

69. If $f(x)$ is differentiable function such that $f : R \to R$ and $f\left(\dfrac{1}{n}\right) = 0 \ \forall \ n \geq 1, n \in 1$ then

(a) $f(x) = 0 \ \forall \ x \in (0, 1)$ (b) $f(0) = 0 = f'(0)$

(c) $f(0) = 0$ but $f'(0)$ may or may not be 0 (d) $|f(x)| \leq 1 \ \forall \ x \in (0, 1)$

70. If $f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2x - 1 & 1 < x \end{cases}$; then

(a) f is discontinuous at $x = 1$ (b) f is differentiable at $x = 1$

(c) f is continuous but not differentiable at $x = 1$ (d) none of these

71. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2) is :

(a) 1 (b) 2 (c) 3 (d) 4

72. If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then

(a) $f(x)$ is continuous but not differentiable at $x = 0$ (b) $f(x)$ is differentiable at $x = 0$

(c) $f(x)$ is not differentiable at $x = 0$ (d) none of these

73. Function $y = \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$ is not differentiable for

(a) $|x| < 1$ (b) $x = 1, -1$ (c) $|x| > 1$ (d) none of these

74. The set of all those points, where the function $f(x) = \dfrac{x}{1+|x|}$ is differentiable is :

(a) $(-\infty, \infty)$ (b) $[0, \infty]$ (c) $(-\infty, 0) \cup (0, \infty)$ (d) $(0, \infty)$

75. The function $f(x) = (x^2 - 1)\,|x^2 - 3x + 2| + \cos(|x|)$ is not differentiable at :

(a) -1 (b) 1 (c) 0 (d) 2

76. If $f(x) = x^2 - 2x + 4$ and $\dfrac{f(5) - f(1)}{5 - 1}$, then value of c will be :

(a) 0 (b) 1 (c) 2 (d) 3

77. Let $f(x) = \begin{cases} 1 & \forall x < 0 \\ 1 + \sin x & \forall\ 0 \le x \le \dfrac{\pi}{2} \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?

(a) 1 (b) -1 (c) ∞ (d) does not exist

78. The function which is continuous for all real values of $\cos x$ and differentiable at $x = 0$ is:

(a) $|x|$ (b) $\log x$ (c) $\sin x$ (d) $x^{1/2}$

79. If $f(x)$ is a differentiable function such that $f : \mathrm{R} \to \mathrm{R}$ and $f\left(\dfrac{1}{n}\right) = 0\ \forall\ n \ge 1,\ n \in \mathrm{I}$ then

(a) $f(x) = 0 \quad \forall\ x \in (0, 1)$ (b) $f(0) = 0 = f'(0)$

(c) $f(0) = 0$ but $f'(0)$ may or may not be 0 (d) $|f(x)| \le \mathrm{I},\ \forall\ x \in (0, 1)$

80. Let $f(x) = \begin{cases} \sin x & \text{for } x \ge 0 \\ 1 - \cos x & \text{for } x \le 0 \end{cases}$ and $g(x) = e^x$ then $(g \circ f)'(0)$ is :

(a) 1 (b) 0 (c) -1 (d) 2

81. Let $f(x) = \begin{cases} x + 1, & \text{when } x < 2 \\ 2x - 1, & \text{when } x \ge 2 \end{cases}$, then $f'(2)$

(a) 0 (b) 2 (c) 1 (d) does not exists

82. A function $f(x) = \begin{cases} 1 + x, & x \le 2 \\ 5 - x, & x > 2 \end{cases}$ is

(a) not continuous at $x = 2$ (b) differentiable at $x = 2$

(c) continuous but not differentiable at $x = 2$ (d) none of these

83. If $y = \log\sqrt{\tan\theta}$, then the value of $\dfrac{dy}{d\theta}$ at $\theta = \dfrac{\pi}{4}$ is :

(a) 0 (b) 1 (c) -1 (d) 2

84. If $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots}}}$ then $\dfrac{dy}{dx}$ is :

(a) $\dfrac{\sin x}{2y - 1}$ (b) $\dfrac{y^2}{\cos x - x}$ (c) $\dfrac{2y - 1}{\cos x}$ (d) $\dfrac{\cos x}{2y - 1}$

85. The value of $\dfrac{dy}{dx}$ at $y = 1$ for the function $\sqrt{x} + \sqrt{y} = 2$ is :

(a) 5 (b) 2 (c) 4 (d) -1

86. If $y = \dfrac{x + 1}{x - 1}$, then the value of $\dfrac{dy}{dx}$ is :

(a) $\dfrac{-2}{x - 1}$ (b) $\dfrac{-2}{(x - 1)^2}$ (c) $\dfrac{2}{(x - 1)^2}$ (d) $\dfrac{2}{x - 1}$

87. If $x = t^2$ and $y = t^3$, then the value of $\frac{d^2y}{dx^2}$ is :

(a) 1 (b) $\frac{3}{2t}$ (c) $\frac{3}{4t}$ (d) $\frac{3}{2}$

88. The value of differentiation of e^{x^3} *w.r.t.* x is :

(a) e^{x^3} (b) $3x^2 \cdot 2e^{x^3}$ (c) $3x^3e^{x^3}$ (d) none of these

89. If $y = \sqrt{x+\sqrt{x+\sqrt{x+\ldots}}}$, then the value of $\frac{dy}{dx}$ is :

(a) $\frac{1}{2-y}$ (b) $\frac{1}{2y-1}$ (c) $\frac{1}{2y+1}$ (d) $\frac{1}{2+y}$

90. The value of $\frac{dy}{dx}$ if $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!}$.

(a) $y - \frac{x^n}{n!}$ (b) $y + \frac{x^n}{n!}$ (c) $\frac{x^n}{n!}$ (d) $y - 1 - \frac{x^n}{n!}$

91. If $f(x) = mx + 1, x \leq \frac{\pi}{2}$ and $f(x) = \sin x + n, x > \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$, then which of the following is correct?

(a) $m = 1, n = 0$ (b) $m = \frac{n\pi}{2} + 1$ (c) $n = \frac{m\pi}{2}$ (d) $m = n = \frac{\pi}{2}$

92. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then the value of $(1 - x^2) \frac{d^2y}{dx^2} - 3x.\frac{dy}{dx} - y$ is :

(a) 0 (b) 1 (c) –1 (d) 2

93. If $f(x + y) = f(x). f(y)\ \forall\ x.y$ and $f(5) = 2, f'(0) = 3$, then $f'(5)$ is:

(a) 0 (b) 1 (c) 6 (d) 2

94. If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx} = ?$

(a) $\frac{-x}{y}$ (b) $\frac{x}{y}$ (c) $\frac{-y}{x}$ (d) $\frac{y}{x}$

95. If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then $\frac{dy}{dx} =$

(a) $-\sqrt[3]{\frac{x}{y}}$ (b) $-\sqrt[3]{\frac{y}{x}}$ (c) $\sqrt[3]{\frac{y}{x}}$ (d) $\sqrt[3]{\frac{x}{y}}$

96. If $y = 2^x\, 3^{2x+1}$, then $\frac{dy}{dx} =$

(a) log 2.log 3 (b) $(\log 18^2)y^2$ (c) (log 18) (d) y (log 18)

97. If $y = \tan^{-1}\left(\frac{\sqrt{x}-x}{1+x^{3/2}}\right)$, then $y'(1)$ is equal to:

(a) 0 (b) $\frac{\sqrt{x}-x}{1+x^{3/2}}$ (c) –1 (d) $-\frac{1}{4}$

98. If $y = (1 + x)(1 + x^2)(1 + x^4) + \ldots + (1 + x^{2n})$, then the value of $\frac{dy}{dx}$ at $x = 0$ is :

(a) 0 (b) –1 (c) 1 (d) 2

99. If $\sin y + e^{-x \cos y} = e$, then $\frac{dy}{dx}$ at $(1, \pi)$ is equal to :

(a) $\sin y$ (b) $- x \cos y$ (c) e (d) $\sin y - x \cos y$

INPUT TEXT BASED MCQ's

100. If α, β (where α < β) are the points of discontinuity of the of the function $g(x) = f(f(f(x)))$, where $f(x) = \frac{1}{1-x}$ and $P(a, a^2)$ is any point on xy-plane.

Answer the following questions :

(i) The point of discontinuity of $g(x)$ is :

(a) $x = 0, -1$ (b) $x = 1$ only (c) $x = 0$ only (d) $x = 0, x = 1$

(ii) The domain of $f(g(x))$ is :

(a) $x \in R$ (b) $x \in R - \{0, 1\}$ (c) $x \in R - \{1\}$ (d) $x \in R - \{0, 1, -1\}$

(iii) If point $P(a, a^2)$ lies on the same side as that of (α, β) with respect to line $x + 2y - 3 = 0$, then

(a) $a \in \left(-\frac{3}{2}, 1\right)$ (b) $a \in R$ (c) $a \in \left(\frac{-3}{2}, 0\right)$ (d) $a \in (0, 1)$

(iv) Consider the function $f : R \to R$ such that $f(x) = x$, if $x \geq 0$ and $f(x) = -x^2$, if $x < 0$, then which of the following is correct?

(a) $f(x)$ is continuous everywhere $x \in R$ (b) $f(x)$ is continuous at $x = 0$ only

(c) f(x) is discontinuous at $x = 0$ only (d) f(x) is discontinuous at every $x \in R$

(v) The function $f(x) = |x| + \frac{|x|}{x}$ is :

(a) Continuous at the origin

(b) discontinuous at origin because $|x|$ is discontinuous there

(c) discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there.

(d) discontinuous at the origin because $|x|$ and $\frac{|x|}{x}$ are discontinuous there

101. Let $f(x)$ be a real valued function, then

- Left hand derivative (LHD) : $Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$
- Right hand derivative (RHD) : $Lf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Also a function $f(x)$ is said to be differentiable at $x = a$ if its LHD and RHD at $x = a$ exits and are equal.

For the function $f(x) = \begin{cases} ax^2 + b & , \ x < -1 \\ bx^2 + ax + 4 & , \ x \geq -1 \end{cases}$

Answer the following questions :

(i) The value of a is :

(a) 1 (b) 2 (c) 4 (d) 5

(ii) Find the value of b.

(a) 2 (b) 4 (c) 3 (d) 5

(iii) $f(x)$ is continuous at

(a) $x = 1$ (b) $x = 4$ (c) $x = 3$ (d) everywhere

(iv) Find the value $f'(2)$:

(a) 6 (b) 10 (c) 8 (d) 5

(v) $f'(x)$ is

(a) continuous at $x = 1$ only (b) continuous at $x = 2$ only

(c) continuous everywhere (d) discontinuous at $x = 1$

ANSWERS

1. (d)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (d)
11. (a)	**12.** (a)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (a)	**19.** (c)	**20.** (b)
21. (d)	**22.** (b)	**23.** (d)	**24.** (b)	**25.** (a)	**26.** (c)	**27.** (c)	**28.** (d)	**29.** (b)	**30.** (b)
31. (b)	**32.** (b)	**33.** (a)	**34.** (b)	**35.** (a)	**36.** (a)	**37.** (a)	**38.** (b)	**39.** (a)	**40.** (d)
41. (c)	**42.** (d)	**43.** (b)	**44.** (b)	**45.** (a)	**46.** (c)	**47.** (b)	**48.** (c)	**49.** (b)	**50.** (b)
51. (b)	**52.** (b)	**53.** (b)	**54.** (b)	**55.** (a)	**56.** (a)	**57.** (b)	**58.** (a)	**59.** (d)	**60.** (d)
61. (a)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (a)	**66.** (b)	**67.** (b)	**68.** (a)	**69.** (b)	**70.** (c)
71. (b)	**72.** (d)	**73.** (d)	**74.** (c)	**75.** (d)	**76.** (d)	**77.** (d)	**78.** (c)	**79.** (b)	**80.** (b)
81. (d)	**82.** (c)	**83.** (b)	**84.** (d)	**85.** (d)	**86.** (b)	**87.** (c)	**88.** (d)	**89.** (b)	**90.** (a)
91. (c)	**92.** (a)	**93.** (c)	**94.** (c)	**95.** (b)	**96.** (d)	**97.** (d)	**98.** (c)	**99.** (c)	

100. (i) (c) (ii) (d) (iii) (d) (iv) (a) (v) (c)

101. (i) (b) (ii) (c) (iii) (d) (iv) (d) (v) (c)

Hints to Some Selected Questions

1. (d) $u = f(x) = \dfrac{1}{x-1}$ is discontinuous at $x = 1$

$$y = g(u) = \frac{1}{u^2+u-2} = \frac{1}{(u+2)(u-1)}$$ is discontinuous at $u = -2$ and $u = 1$.

$$u = -2, \Rightarrow \frac{1}{x-1} = u \Rightarrow x = \frac{1}{2}$$

when $u = 1 \Rightarrow \dfrac{1}{x-1} = 1 \Rightarrow x = 2$

$\therefore$ Function is discontinuous at three points $x = \dfrac{1}{2}, 1, 2$

2. (b) $$f(g(x)) = \frac{1}{\left(\frac{1}{x^2}-1\right)\left(\frac{1}{x^2}-2\right)} = \frac{x^4}{(1-x^2)(1-2x^2)}$$

$f(g(x))$ is discontinuous at $x = \pm 1$, $x = \pm\dfrac{1}{\sqrt{2}}$ and $x = 0$.

3. (a) $f'(x) = (2+x^2)^{1/2}(3+|x|^3)^{1/3} + (1+x)[g(x)]f'(-1) = \sqrt{3}\,2^{1/3}$

5. (b) Let $g(x) = f(x) - f(x+1)$

$g(0) = f(0) - f(1)$

$g(1) = f(1) - f(2)$

$g(0) + g(1) = 0$, ($g(0)$ and $g(1)$ are of opposite side)

$f(x) = f(x+1)$ at least one root in $[0, 1]$

8. (d) $y = \sec(\tan^{-1}x)$

$$\frac{dy}{dx} = \sec(\tan^{-1}x)\tan(\tan^{-1}x)\,\frac{1}{1+x^2}$$

$$\left.\frac{dy}{dx}\right|_{x=1} = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}.$$

9. (a) $$\lim_{x\to a} \frac{x^2 f(a) - a^2 f(x)}{x-a} = \lim_{x\to a} 2xf(a) - a^2 f'(x) = 2af(a) - a^2 f'(a)$$

10. (d) $g'(x) = 2(f(2f(x)+2))\left[\frac{d}{dx}\left(f\left(2f(x)+2\right)\right)\right] = 2f(2f(x)+2) - f'(2f(x)+2)\cdot(2f'(x))$

$g'(0) = 2f(2f(0)+2)\cdot f'(2f(0)+2)\cdot(2f'(0)) = 4f(0)\cdot f'(0)(2f(0) = 4(-1)(-1) = 4$

13. (c) $f'(x) = \begin{cases} \dfrac{x}{1-x} & x < 0 \\ \dfrac{x}{1+x} & x \geq 0 \end{cases}$

$f'(x) = \begin{cases} \dfrac{1}{(1-x)^2} & x < 0 \\ \dfrac{1}{(1+x)^2} & x \geq 0 \end{cases}$

$\therefore f'(x)$ exists everywhere.

15. (c) $f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$\therefore$ If a function is differentiable so it is continuous,

$\lim_{x\to 0} \frac{f\left(1+h\right)}{h} = 5$ and hence $f(1) = 0$

Hence, $f'(1) = \lim_{h\to 0} \frac{f(1+h)}{h} = 5$.

16. (b) $f(x) = \frac{1-\tan x}{4x-\pi} = \lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{4x-\pi} = \frac{1}{2}$

17. (b) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$|f'(x)| = \lim_{h\to 0}\left|\frac{f(x+h)-f(x)}{h}\right| \leq \lim_{h\to 0}\left|\frac{(h)^2}{h}\right|$

$\therefore |f'(x)| \leq 0 \Rightarrow f'(1) = 0 \Rightarrow f'(x)$ is constant as $f(0) = 0 \Rightarrow f(1) = 0$

18. (a) as $f(1) = -2$ and $f'(x) \geq 2,\ \forall\ x \in [1, 6]$

Apply lagrange's mean value theorem, $\frac{f(6)-f(1)}{5} = f'(c) \geq 2$

$f(6) \geq 10 + f(1)$

$f(6) \geq 10 - 2 = 8$

19. (c) $x = e^{y+x}$

$\log x = y + x \Rightarrow \log x - x = y \Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$

20. (b) $f'(0) \Rightarrow f'(0-h) = 1$

$f'(0+h) = 0$

L.H.D. $\neq$ R.H.D.

21. (d) If $f(x)$ is continuous at $x = 0$

$f(0) = \lim_{x\to 0} \frac{2-\sqrt{x+4}}{\sin 2x}$ L' Hospital rule,

$f(0) = \lim_{x\to 0} \frac{\left(-\frac{1}{2\sqrt{x+4}}\right)}{2\cos 2x} = \frac{-\frac{1}{2\sqrt{4}}}{2\cos 0} = -\frac{1}{8}$

22. (b) $f(x) = \dfrac{x(x-2)}{x^2-4} = \dfrac{x}{x+2}$. Continuous at $x = 2$

$$\lim_{x\to 2} \frac{x}{x+2} = f(2) = f(2) = \frac{1}{2}$$

23. (d) $f(x)$ is continuous at $x = 2$

$$\lim_{x\to 2} f(x) = f(2) \Rightarrow \lim_{x\to 2}(2x+\lambda) = 6-4 \Rightarrow \lim_{h\to 0} 2(2-h)+\lambda = 2 \Rightarrow 4+\lambda = 2 \Rightarrow \lambda = -2$$

24. (b) Apply L'Hospital rule

$$f(0) = \lim_{x\to 0} \frac{\left(2 - \dfrac{-1}{\sqrt{1-x^2}}\right)}{\left(2 + \dfrac{1}{1+x^2}\right)} = \frac{2 - \dfrac{1}{\sqrt{1}}}{2 + \dfrac{1}{1}} = \frac{2-1}{2+1} = \frac{1}{3}$$

25. (a) Apply L'Hospital rule

$$\lim_{x\to 5} f(x) = \lim_{x\to 1-} f(x) = f(1)$$

$$\Rightarrow 5 \times 1 - 4 = 4 \times 1 + 3 \times b \times 1 \Rightarrow b = -1$$

26. (c) $\lim\limits_{x\to 5} f(x) - \lim\limits_{x\to 1^+} f(x) = f(1)$

$$\lim_{h\to 0} 5(1-h) - 2 = \lim_{h\to 0} 2(1+h)+1 = k \Rightarrow k = 3$$

27. (c) $f\left(\dfrac{\pi}{2}\right) = \lim\limits_{x\to\pi/2} f(x)$

$$\lambda = \lim_{x\to\pi/2} \frac{1-\sin x}{\pi - 2x}$$

Apply L'Hospital rule

$$\lambda = \lim_{x\to\pi/2} \frac{-\cos x}{-2} \Rightarrow \lambda = 0$$

30. (b) LHL $= \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} (-x)^2 \sin\left(-\dfrac{1}{x}\right) = 0$

RHL $= \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0} x^2 \sin\left(\dfrac{1}{x}\right) = 0$

$f(0) = \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = 0$

$\therefore f(x)$ is continuous at $x = 0$

31. (b) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} (x+1)^{\cot x}$

$$= \lim_{x\to 0} f(x) = \lim_{x\to 0}(x+1)^{\cot x} = e^{\lim\limits_{x\to 0} x\cot x} = e^{\lim\limits_{x\to 0}\frac{x}{\tan x}} = e^1 = e$$

33. (a) $f(x) = \begin{cases} \sin x & x \ge 0 \\ \sin(-x) & x < 0 \end{cases}$

$$f'(x) = \begin{cases} \cos x & x \ge 0 \\ -\cos x) & x < 0 \end{cases}$$

$\therefore f(x)$ is not differentiable at $x = 0$ and $f(x)$ is a periodic function.

34. (b) LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0} x^2 = 0$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} x^2 = 0$

$\therefore$ RHL = LHL

35. (a) $f(x) = \begin{cases} x^2 + x & x \geq 0 \\ x^2 - x & x < 0 \end{cases}$

LHL = $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} (-h)^2 + h = 0$

RHL = $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} (0+h) = \lim_{h\to 0} (h)^2 + h = 0$

$\therefore$ RHL = LHL = $f(0)$.

36. (a) $F(x) = \begin{cases} x.\sin\dfrac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

$F(0) = \lim_{x\to 0} x\sin\left(\dfrac{1}{x}\right) = 0$

$\therefore$ F(x) is continuous on R.

38. (b) $f(x) = 1 + |x|$, $g(x) = \sin x$ are continuous everywhere. Therefore *fog* is continuous everywhere.

$\Rightarrow 1 + |\sin x|$ is continuous everywhere.

40. (d) $f(x) = \begin{cases} 1, & x > 1 \\ x^3, & x < 1 \end{cases}$

$f(x)$ is continuous for $x \in$ R and not differentiable at $x = 1$.

41. (c) Since $f(0) = 0$

$f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \dfrac{f(h)}{h} = f'(0) = k(\text{say})$

$f(x) = kx + c \Rightarrow f(x) = kx$

42. (d) $f(3) = 0$

$\lim_{x\to 3^+} f(x) = \lim_{h\to 0} (3-h) = \lim_{h\to 0} |3-h-3| = 0$

$\lim_{x\to 3^-} f(x) = \lim_{h\to 0} (3+h) = \lim_{h\to 0} |3+h-3| = 0$

Hence, it is continuous at $x = 3$

43. (b) $f\left(\dfrac{\pi}{2}\right) = 3$ as $f(x)$ is continuous at $x = \dfrac{\pi}{2}$

$\lim_{x\to \pi/2} \left(\dfrac{k\cos x}{\pi - 2x}\right) = f\left(\dfrac{\pi}{2}\right) = \dfrac{k}{2} = 3 \Rightarrow k = 6$

44. (b) As limit of function is $a + b$ as $x \to 0$, therefore to be continuous at a function, its value must be $a + b$ at $x = 0 \Rightarrow f(0) = a + b$.

46. (c) $\lim_{x\to 0} f(x) = f(0) = \lim_{x\to 0} (1+x)^{1/x} = e$

47. (b) $f(x) = \dfrac{x+1}{(x-3)(x+4)}$. Hence the points are (3, –4)

48. (c) $f(0^+) = f(0^-) = 2$ and $f(0) = 2$. Hence, $f(x)$ is continuous at $x = 0$

49. (b) (i) which $0 \le x < 1$

$\therefore f(x)$ doesn't exists as $[x] = 0$, here,

(ii) Also, $\lim\limits_{x \to 1^+} f(x)$ and $\lim\limits_{x \to 1^-} f(x)$ does not exist.

Hence, $f(x)$ is discontinuous at all integer and also (0, 1)

50. (b) $\lim\limits_{x \to 0} f(x) = \dfrac{\sin^2 ax}{(ax)^2} . a^2 = a^2$ and $f(0) = 1$

$\therefore f(x)$ is discontinuous at $x = 0$, when $a \ne 0$.

51. (b) $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{2 \sin 2x}{2x.5} = \dfrac{2}{5} = k$

52. (b) $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \left(\dfrac{2 \sin^2 2x}{(2x)^2} \right) 4 = 8$

$\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \sqrt{16 + \sqrt{x}} + 4 = 8$

Hence, $a = 8$

53. (b) $\lim\limits_{x \to 2} f(x) = \lim\limits_{x \to 2} (x + 2)(x^2 + 4) = 32$

$f(2) = 16$

$\therefore f(x)$ is discontinuous at $x = 2$

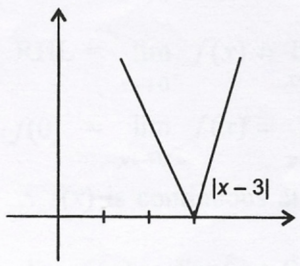
54. (b) $\lim\limits_{x \to -5} f(x) = \dfrac{(x - 2)(x + 5)}{(x + 5)(x - 3)} = \dfrac{-7}{-8} = \dfrac{7}{8}$

55. (a) $\lim\limits_{x \to 3^+} f(x) = f(3) = \lim\limits_{x \to 5} f(x)$

$\lim\limits_{x \to 3^-} = 4$ or $\lim\limits_{h \to 0} 3 - h + m = 4$

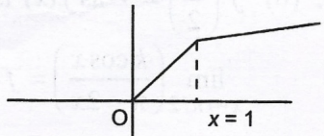
$3 + m = 4 \Rightarrow m = 1$

59. (d)



Can seen in graph it is continuous but tangent. is not defined at $x = 3$.

61. (a) $f(x) = \begin{cases} x & 0 \le x \le 1 \\ 1 & 1 < x \le 2 \end{cases}$



$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} (1 - h)$

$\lim\limits_{h \to 1^+} f(x) = \lim\limits_{h \to 0} (1 + h) = 1$

$\therefore$ Function is continuous in (0, 2) seen in the graph it is not differentiable

62. (a) $f(x)$ is continuous at $x = 0$

$f(x) = |x| = |0| = 0$

$f(0+h) = |0+h| = |h|$

$$\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{|h|}{h} = -1$$

$$\lim_{h\to 0^+} \frac{f(1+h)-f(0)}{h} = \lim_{x\to 0^+} \frac{|h|}{h} = 1$$

Not differentiable.

65. (a) $f(k-0) = \lim_{h\to 0} \dfrac{[k-h]\sin \pi(k-h) - [k]\sin \pi k}{-h} = \lim_{h\to 0} \dfrac{(-1)^{k-1}(k-1)\sin \pi h}{-h} = (-1)^k (k-1)\pi$

66. (b) Let $x < 0 \Rightarrow |x| = -x \Rightarrow f(x) = \dfrac{d}{dx}\left(\dfrac{x}{1-x}\right)$

$[f'(x)]_{x=0} = 1$ again $x > 0 \Rightarrow |x| = x$

$f(x) = \dfrac{d}{dx}\left(\dfrac{x}{1+x}\right) = \dfrac{1}{1+x)^2} \Rightarrow [f'(x)]_{x=0} = 1$

$f'(0) = 1$

68. (a) $f'(1) = \lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$

$\lim_{x\to 0} \dfrac{f(1+h)}{h} = 5$ and hence $f(1) = 0$

Hence, $f'(1) = \lim_{h\to 0} \dfrac{f(1+h)}{h} = 5$

69. (b) $f(1) = f\left(\dfrac{1}{2}\right) = f\left(\dfrac{1}{3}\right) = \ldots \lim_{x\to 0}\left(\dfrac{1}{n}\right) = 0$

Since there are infinitely many points in $x \in ((0, 1)$

where $f(x) = 0$ and $\lim_{n\to\infty} f\left(\dfrac{1}{n}\right) = 0 \Rightarrow f(0) = 0]$

There are infinitely many points in the neighbourhood of $x = 0$ such that $f(x)$ remains constant in the neighbourhood of $x = 0 \Rightarrow f'(0) = 0$

71. (b) Function $f(x) = |x - 0.5| + |x - 1| + \tan x$

$x = 0.5, 1, \dfrac{\pi}{2} \in (0, 2)$

72. (d) Since the function is defined for $x \geq 0$ i.e., not defined for $x < 0$.

Hence, the function neither continuous nor differentiable at $x = 0$.

74. (c) Let $h(x) = x,\ x \in (-\infty, \infty)$

$g(x) = 1 + |x|\ x \in (-\infty, \infty)$

Here h is differentiable in $(-\infty, \infty)$ but $|x|$ is not differentiable at $x = 0$.

Therefore, g is differentiable in $(-\infty, 0) \cup (0, \infty)$ and $g(x) \in 0,\ x \in (-\infty, \infty)$

$f(x) = \dfrac{h(x)}{g(x)} = \dfrac{x}{1+|x|}$.

It is differentiable in $(-\infty, 0) \cup (0, \infty)$ for $x = 0$.

75. (d) Since $|x|$ is not differentiable at $x = 0$,

$|x^2 - 3x + 2| = (x-1)(x-2)$

It is not differentiable at $x = 1$ and 2

$f(x) = (x^2 - 1)\ |x^2 - 3x + 2| \cos + (|x|)$ is not differentiable at $x = 2$.

77. (d) $f'(x) = \begin{cases} 0 & \forall x < 0 \\ \cos x & 0 \le x \le \dfrac{\pi}{2} \end{cases}$

$f'(0) = \begin{cases} 0 & x < 0 \\ \cos 0 = 1 \end{cases} \Rightarrow f'(0)$ does not exists.

78. (c) Since, $\dfrac{dy}{dx} = \cos x$ which is defined at $x = 0$ and no other differentiable coefficient is defined at $x = 0$

79. (b) $f(1) = f\left(\dfrac{1}{2}\right) = f\left(\dfrac{1}{3}\right) = \ldots \lim\limits_{h\to\infty} f\left(\dfrac{1}{n}\right) = 0$

$f(0) = 0$

$\Rightarrow f(x)$ remains constant in the neighbourhood of $x = 0 \Rightarrow f'(0) = 0$.

80. (b) $(g \circ f)(x) = g[f(x)] = g[1 - \cos x] = e^{1-\cos x}$, for $x \le 0$

$(g \circ f)'(x) = e^{1-\cos x}.\sin x$ for $x \le 0$

$(g \circ f)'(0) = 0$

81. (d) $Rf'(2) = \lim\limits_{h\to 0} \dfrac{f(2+h) - f(2)}{h} = \lim\limits_{h\to 0} \dfrac{4 + 2h - 1 - 3}{h} = 2$ and $Lf'(2) = \lim\limits_{h\to\infty} \dfrac{f(2-h) - f(2)}{-h} = 1$

Thus, $f'(2)$ does not exist

83. (b) $y = \log\sqrt{\tan\theta} \Rightarrow y = \dfrac{1}{2}\log\tan\theta = \dfrac{dy}{d\theta} \Rightarrow \dfrac{1}{2\tan\theta}\sec^2\theta \Rightarrow \left(\dfrac{dy}{d\theta}\right)_{\theta=\pi/4} = \dfrac{1}{2}\times(\sqrt{2})^2 = 1$

84. (d) $y = \sqrt{\sin x + y} \Rightarrow y^2 = \sin x + y$

$2y\dfrac{dy}{dx} = \cos x + 1\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{\cos x}{2y - 1}$

85. (d) At $y = 1 \Rightarrow \sqrt{x} + 1 = 2 \Rightarrow \sqrt{x} = 1 \Rightarrow x = 1 \Rightarrow \dfrac{1}{2\sqrt{x}} + \dfrac{1}{2\sqrt{y}}\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -1$

86. (b) $\dfrac{dy}{dx} = \dfrac{(x-1)(1) - (x+1)(1)}{(x-1)^2} \Rightarrow \dfrac{dy}{dx} = \dfrac{x - 1 - x - 1}{(x-1)^2} = \dfrac{-2}{(x-1)^2}$

87. (c) $x = t^2$, $y = t^3$

$\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2 \Rightarrow \dfrac{dy}{dx} = \dfrac{3}{2}t \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{3}{2}\dfrac{dx}{dt} = \dfrac{3}{2}.\dfrac{1}{2t} = \dfrac{3}{4t}$

88. (d) $y = e^{x^3} \Rightarrow \dfrac{dy}{dx} = e^{x^3}(3x^2) = 3x^2 e^{x^3}$

89. (b) $y = \sqrt{x + y} \Rightarrow y^2 = x + y \Rightarrow 2y\dfrac{dy}{dx} = 1 + \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2y - 1}$

90. (a) $\dfrac{dy}{dx} = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \ldots + \dfrac{x^{n-1}}{n-1!} \Rightarrow \dfrac{dy}{dx} + \dfrac{x^n}{n!} = 1 + x + \dfrac{x^2}{21} + \ldots\dfrac{x^n}{n!} \Rightarrow \dfrac{dy}{dx} = y - \dfrac{x^n}{n!}$

91. (c) Since $f(x)$ is continuous at $x = \dfrac{\pi}{2}$

So, $\lim\limits_{x\to\frac{\pi}{2}^-} f(x) = \lim\limits_{x\to\frac{\pi}{2}^+} f(x) = f(x) \Rightarrow \dfrac{m\pi}{3} + 1 = \sin\dfrac{\pi}{2} + n \Rightarrow n = \dfrac{m\pi}{2}$

92. (a) $(1 - x^2)\dfrac{dy}{dx} - xy = 1$ (differentiable first time)

Again differentiate w.r.t. x, we get $(1 - x^2)\dfrac{d^2y}{dx^2} - 3x.\dfrac{dy}{dx} - y = 0$

93. (c) $f(x+y) = f(x) \times f(y) \Rightarrow f'(x+y) = f'(x)\cdot f(y)$

Put $x = 0$ and $y = x \Rightarrow f'(x) = f'(0)\cdot f(x) \Rightarrow f'(5) = 3f(5) = 3 \times 2 = 6$

94. (c) $\dfrac{dx}{dt} = \dfrac{(-2t)(1-t^2)-(1-t^2)(2t)}{(1+t^2)^2} = \dfrac{-2t+2t^3-2t+2t^3}{(1+t)^2} = \dfrac{-4t+4t^3}{(1+t^2)^2} = \dfrac{-4t(1-t^2)}{(1+t^2)^2}$

$\dfrac{dy}{dt} = \dfrac{2(1+t^2)-2t(2t)}{(1+t^2)^2} = \dfrac{2+2t^2-4t^2}{(1+t^2)^2} = \dfrac{2-2t^2}{(1+t^2)^2} = \dfrac{2(1-t^2)}{(1+t^2)^2} \Rightarrow \dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = \dfrac{-y}{x}.$

95. (b) $\dfrac{dy}{dx} = -3a\cos^2\theta\sin\theta,\ \dfrac{dy}{dx} = 3a\sin^2\theta\cos\theta \Rightarrow \dfrac{dy}{dx} = \dfrac{3a\sin^2\theta\cos\theta}{3a\cos^2\theta\sin\theta} = -\dfrac{\sin\theta}{\cos\theta} = -\sqrt[3]{\dfrac{y}{x}}$

97. (d) $y = \tan^{-1}\left(\dfrac{\sqrt{x}-x}{1+\sqrt{x}.x}\right) = \tan^{-1}\left(\sqrt{x}\right) - \tan^{-1}(x)$

Differentiate w.r.t x, we get $\Rightarrow y' = \dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}} - \dfrac{1}{1+x^2}$

$y'(1) = \dfrac{1}{2}\cdot\dfrac{1}{2} - \dfrac{1}{2} = -\dfrac{1}{4}$

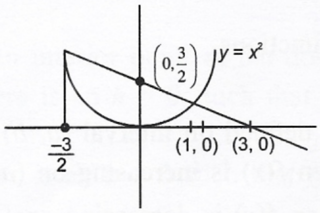
99. (c) $\cos y\dfrac{dy}{dx} + e^{-x\cos y}\left[x\sin y\dfrac{dy}{dx} - \cos y\right] = 0$

$\left(\dfrac{dy}{dx}\right)_{\text{at}(1,\pi)} = \dfrac{\cos y\, e^{-x\cos y}}{\cos y + x\sin y\, e^{-x\cos y}} = \dfrac{-1\times e}{-1+0\times e} = e$

100. (i) (c) $f(x) = \dfrac{1}{1-x} \Rightarrow f(f(x)) = \dfrac{x-1}{x} \Rightarrow x \neq 0, 1$

(ii) (d) $g(x) = f(f(f(x))) = x,\ x \in R - \{0, 1\} \Rightarrow f(g(x)) = \dfrac{1}{1-x} \Rightarrow x \neq 0, 1$

(iii) (d) From graph $a \in \left(\dfrac{-3}{2}, 1\right)$



(iv) (a) $f(x) = \begin{cases} x & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

LHL = $\lim\limits_{x\to 0^-} f(x) = -\lim\limits_{x\to 0} x^2 = 0$; RHL = $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0} x = 0$ and $f(0) = 0$ $\because$ it is continuous

(v) (c) since, $|x|$ is discontinuous at $x = 0$ and $\dfrac{|x|}{x}$ is discontinuous at $x = 0$

$\therefore f(x) = |x| + \dfrac{|x|}{x}$ is discontinuous $x = 0$.

101. (i) (b) $f(-1) = \lim\limits_{x\to 1^-} f(x) = -\lim\limits_{h\to 0} a(-1-h)^2 + b \Rightarrow b - a + 4 = a + b;\ a = 2$

(ii) (c) $f(-1) = \lim\limits_{x\to 1^-} f(x) = -\lim\limits_{h\to 0}(2a)(-1-h) - 2a \Rightarrow 2b(-1) + a = -2a$; $3a - 2b = 0 \rightarrow b = 3$

(iii) (d) since, $f'(x)$ is continuous everywhere, so $f(x)$ is also continuous everywhere.

(iv) (d) $f'(x) = 2ax \Rightarrow f'(x) = 4x \Rightarrow f'(x) = 4 \times 2 = 8.$

(v) (c) $f'(x)$ is continuous everywhere