: 2 :
INVERSE TRIGONOMETRIC FUNCTIONS

STUDY NOTES

- **Inverse of trigonometric functions :**

Functions	Domain	Range	Principal Value
$y = \sin^{-1}x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \cos^{-1}x$	$[-1, 1]$	$[0, \pi]$	$0 \leq y \leq \pi$
$y = \tan^{-1}x$	R	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$	$\frac{-\pi}{2} < y < \frac{\pi}{2}$
$y = \text{cosec}^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \sec^{-1}x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	$0 \leq y \leq \pi$
$y = \cot^{-1}x$	R	$(0, \pi)$	$0 < y < \pi$

- **Properties of Inverse Trigonometric functions :**

(I) (i) $\sin^{-1}\frac{1}{x} = \text{cosec}^{-1}x,\ x \geq 1$ or $x \leq -1$

(ii) $\cos^{-1}\frac{1}{x} = \sec^{-1}x,\ x \geq 1$ or $x \leq -1$

(iii) $\tan^{-1}\frac{1}{x} = \cot^{-1}x,\ x > 0$

(II) (i) $\sin^{-1}(-x) = -\sin^{-1}x,\ x \in [-1, 1]$

(ii) $\tan^{-1}(-x) = -\tan^{-1}x,\ x \in R$

(iii) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1}x,\ |x| \geq 1$

(III) (i) $\cos^{-1}(-x) = \pi - \cos^{-1}x,\ x \in [-1, 1]$

(ii) $\sec^{-1}(-x) = \pi - \sec^{-1}x,\ |x| \geq 1$

(iii) $\cot^{-1}(x) = \pi - \cot^{-1}x,\ x \in R$

(IV) (i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2},\ x \in [-1, 1]$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2},\ x \in R$

(iii) $\text{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2},\ |x| \geq 1$

(V) (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy},\ x\,y < 1$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy},\ x\,y > -1$

(iii) $\tan^{-1}x + \tan^{-1}y = \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right),\ x\,y > 1,\ x, y > 0$

(VI) (i) $2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2},\ |x| \leq 1$

(ii) $2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2},\ x \geq 0$

(iii) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2},\ -1 < x < 1$

QUESTION BANK

MULTIPLE CHOICE QUESTIONS

1. $\cot^{-1}[(\cos\alpha)^{1/2}] + \tan^{-1}[(\cos\alpha)^{1/2}] = x$, then $\sin x$ is equal to :

(a) 1 (b) $\cot^2\left(\frac{\alpha}{2}\right)$ (c) $\tan\alpha$ (d) $\cot\left(\frac{\alpha}{2}\right)$

2. The trigonometric equation $\sin^{-1}x = 2\sin^{-1}a$, has a solution for :

(a) $\frac{1}{2} \le |a| \le \frac{1}{\sqrt{2}}$ (b) all real values of 'a' (c) $|a| < \frac{1}{2}$ (d) $|a| \ge \frac{1}{\sqrt{2}}$

3. If $\cos^{-1}x - \cos^{-1}\frac{y}{2} = \alpha$, then $4x^2 - 4xy\cos\alpha + y^2$ is equal to :

(a) $2\sin 2\alpha$ (b) $4\sin^2\alpha$ (c) 4 (d) $-4\sin^2\alpha$

4. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then value of x is :

(a) 1 (b) 4 (c) 3 (d) 5

5. If x, y, z are in AP and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$, are also in AP, then :

(a) $2x = 3y = 6z$ (b) $6x = 4y = 3z$ (c) $6x = 3y = 2z$ (d) $x = y = z$

6. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is :

(a) $\frac{6}{17}$ (b) $\frac{3}{17}$ (c) $\frac{4}{17}$ (d) $\frac{5}{17}$

7. Let $\tan^{-1}y = \tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $|x| < \frac{1}{\sqrt{3}}$. then value of y is :

(a) $\frac{3x + x^3}{1 - 3x^2}$ (b) $\frac{3x - x^3}{1 + 3x^2}$ (c) $\frac{3x + x^3}{1 + 3x^2}$ (d) $\frac{3x - x^3}{1 - 3x^2}$

8. If $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$, then the area represented by the locus of point (x, y) if $|x| \le 1$, $|y| \le 1$ is :

(a) π (b) $\frac{4-\pi}{4}$ (c) $\pi - 2$ (d) $\frac{2-\pi}{2}$

9. Which of the following is equal $\cot^{-1}(\cot 4)$?

(a) π (b) 4 (c) $4 - \pi$ (d) $\pi - 4$

10. The value of $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$ is :

(a) $n\pi$ (b) $n\pi + \frac{\pi}{4}$ (c) $n\pi + \frac{\pi}{2}$ (d) $n\pi - \frac{\pi}{4}$

11. If $\tan^{-1}y = 4\tan^{-1}x$ ($|x| < \tan\frac{\pi}{8}$), y as an algebraic function of x. Then which of the following is the root of the equation of $x^4 - 6x^2 + 1 = 0$.

(a) $\tan\frac{\pi}{8}$ (b) $\tan\frac{\pi}{2}$ (c) $\tan\pi$ (d) $\tan\frac{\pi}{4}$

12. The solution set of $[\sin^{-1}x] > [\cos^{-1}x]$ is :

(a) $x \in \left[\sin\frac{\pi}{2}, \frac{\pi}{2}\right]$ (b) $x \in [\sin 1, 1]$ (c) $x \in [1, 0]$ (d) $x \in [\cos 1, 1]$

13. The value of : $\cos\left[\cos^{-1}\left(-\frac{1}{3}\right)-\sin^{-1}\left(\frac{1}{3}\right)\right]$ is :

(a) 0 (b) 1 (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

14. The value of x, if x, $\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right)=\frac{\pi}{2}$.

(a) 1 (b) 0 (c) 13 (d) 12

15. The value of x, if $\tan\left[\sec^{-1}\frac{1}{x}\right] = \sin\,[\tan^{-1}2)$

(a) $\frac{5}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{\sqrt{5}}{3}$ (d) $\frac{2}{3}$

16. If $-1 < x < 0$, then which of the following could be equal to $\sin^{-1}x$?

(a) $\pi-\cos^{1}\left(\sqrt{1-x^2}\right)$ (b) $-\cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$ (c) $\tan^{-1}\frac{x}{\sqrt{1-x^2}}$ (d) $\operatorname{cosec}^{-1}x$

17. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$ is :

(a) zero (b) two (c) one (d) infinite

18. If $\sin^{-1}x > \cos^{-1}x$, then the set of exhaustive values of x is :

(a) $[-1, 1]$ (b) $\left(\frac{1}{\sqrt{2}},1\right]$ (c) $\left[-1,\frac{1}{\sqrt{2}}\right)$ (d) $\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$

19. Which of the following solution set of the equation $\sin^{-1}6x+\sin^{-1}6\sqrt{3}x=\frac{-\pi}{2}$?

(a) $\frac{1}{12}$ (b) $\frac{-1}{12},\frac{1}{12}$ (c) ϕ (d) $\frac{-1}{12}$

20. If $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then '$x$' equals :

(a) -1 (b) 0 (c) 1 (d) none of these

21. The solution set of $(\cot^{-1}x)^2 - (5\cot^{-1}x) + 6 > 0$ is :

(a) $(\cot 3, \cot 2)$ (b) $(\cot 2, \infty)$
(c) $(-\infty, \cot 3) \cup (\cot 2, \infty)$ (d) None of these

22. The value(s) of x satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ are/is.

(a) $n\pi-1$ (b) $n\pi+1$ (c) $n\pi$ (d) all of these

23. If $\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=\tan^{-1}(-7)$, the solution is :

(a) unique solution (b) no solution (c) exactly two solutions (d) exactly three solutions

24. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then the value of $x^2 + y^2 + z^2 + 2xyz$ is :

(a) 0 (b) 1 (c) 2 (d) π

25. The value of $\sin^{-1}\sin 12 + \cos^{-1}\cos 12$ is :

(a) 0 (b) 1 (c) π (d) $n\pi$

26. The sum of $\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{2}{9}\right)+\tan^{-1}\left(\frac{4}{33}\right)+ ... + \tan^{-1}\left(\frac{2^{n-1}}{1+2^{n-1}}\right)$ 1

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) none of these

27. If $\frac{1}{2} < |x| < 1$, then which of the following is not real?

(a) $\sin^{-1}x$ (b) $\sec^{-1}x$ (c) $\tan^{-1}x$ (d) $\cos^{-1}x$

28. Which of the given relation is true for permissible values of x ?

(I) $\tan|\tan^{-1}x| = |x|$ (II) $\cot|\cot^{-1}x| = |x|$

(III) $\tan^{-1}|\tan x| = |x|$ (IV) $\tan|\sin^{-1}x| = |x|$

(a) Both (I) and (II) (b) Both (II and IV) (c) Both (1) and III (d) All-I, II, III, IV

29. If $x + \frac{1}{x} = 2$, then the principal value of $\sin^{-1}x$ is :

(a) $\frac{\pi}{4}$ (b) π (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

30. Sum to infinite terms of the series $\cot^{-1}3 + \cot^{-1}7 + \cot^{-1}13 + ...$ is :

(a) $\frac{\pi}{2}$ (b) $\tan^{-1}2$ (c) $\cot^{-1}1$ (d) $\frac{\pi}{4}$

31. If $\cot^{-1}\left(\frac{n}{\pi}\right) > \frac{\pi}{6}$, $n \in N$, then maximum value of n is :

(a) 1 (b) 5 (c) 9 (d) 8

32. If a, b, c are distinct non-zero real numbers having same sign, then the value of

$\cot^{-1}\left(\frac{ab+1}{a-b}\right)+\cot^{-1}\left(\frac{bc+1}{b-c}\right)+\cot^{-1}\left(\frac{ca+1}{c-a}\right)$ is :

(a) π (b) 2π (c) $\frac{\pi}{2}$ (d) both (a) and (b)

33. Which of the following is equal to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ when $x \geq 0$?

(a) $-2\tan^{-1}x$ (b) $2\tan x$ (c) $2\tan^{-1}x$ (d) $\tan^{-1}x$

34. The value of $\tan\left[\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{2}{3}\right]$ is :

(a) $\frac{6}{17}$ (b) $\frac{17}{6}$ (c) $\frac{7}{16}$ (d) $\frac{7}{6}$

35. Equation $2\sin^{-1}x + 3\sin^{-1}y = \frac{5\pi}{2}$ and $y = kx - 5$ hold simultaneously when 'k' equals :

(a) 2 (b) 6 (c) 4 (d) none of these

36. If a, b, c be positive real number and $\theta = \tan^{-1}\sqrt{\frac{ak}{cb}}+\tan^{-1}\sqrt{\frac{bk}{ca}}+\tan^{-1}\sqrt{\frac{ck}{ab}}$, where $k = a + b + c$ then θ is equal to :

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

37. Arithmetic mean of the non-zero solution of the equation : $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$

(a) $\frac{7}{6}$ (b) $\frac{7}{9}$ (c) $\frac{2}{3}$ (d) $\frac{11}{6}$

38. Two angles of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$. The measure of third angle is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{3}$

39. The equation $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}x - \cos^{-1}x$ has :

(a) no solution (b) infinite solution (c) unique solution (d) none of these

40. If $\tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right] = \alpha, \left(\alpha \in \left[0, \frac{\pi}{4}\right]\right)$ then x^2 is equal to :

(a) $\cos 2\alpha$ (b) $\sin 2\alpha$ (c) $\tan 2\alpha$ (d) $\cot 2\alpha$

41. If $\frac{1}{2} \le x \le 1$, the find the value of $\cos^{-1}x + \cos^{-1}\left(\frac{x}{2}+\frac{\sqrt{3-3x^2}}{2}\right)$.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $2\cos^{-1}x - \frac{\pi}{3}$

42. Which of the following is equal to $\tan^{-1}\left\{\tan^{-1}\left(\frac{d}{1+a_1a_2}\right)+\tan^{-1}\left(\frac{d}{1+a_2a_3}\right)+...\left(\frac{d}{1+a_{n-1}a_n}\right)\right\}$

where a_1, a_2, a_3 ... are in A.P, 'd' is the common difference.

(a) $\frac{(n-1)d}{a_1+a_n}$ (b) $\frac{nd}{1+a_1a_n}$ (c) $\frac{(n-1)d}{1+a_1a_n}$ (d) $\frac{a_n-a_1}{a_n+a_1}$

43. $\operatorname{cosec}^{-1}\sqrt{5}+\operatorname{cosec}^{-1}\sqrt{65}+\operatorname{cosec}^{-1}\sqrt{325}$ + ... to ∞ is :

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{4}$ (d) $\frac{\pi}{4}$

44. The sum to infinite terms of the series $\cot^{-1}\left(2^2+\frac{1}{2}\right)+\cot^{-1}\left(2^3+\frac{1}{2^2}\right)+\cot^{-1}\left(2^4+\frac{1}{2^3}\right)+ ...$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\cot^{-1}2$ (d) $\frac{\pi}{4}$

45. If $\theta = \cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18$, then the value of $\cot\theta$ is :

(a) 3 (b) $\frac{1}{3}$ (c) $\sqrt{3}$ (d) $3\sqrt{3}$

46. If $\tan\left[2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right] = \frac{-\lambda}{17}$, then the value of λ is :

(a) $\frac{7}{17}$ (b) $\frac{-7}{17}$ (c) $\frac{17}{7}$ (d) $\frac{9}{17}$

47. The number of solutions of the equation $\cos^{-1}\frac{x^2-1}{x^2+1}+\sin^{-1}\frac{2x}{x^2+1}+\tan^{-1}\frac{2x}{x^2-1} = \frac{2\pi}{3}$ is :

(a) only 1 (b) 2 (c) no solution (d) 3

48. $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}\left\{\left(\frac{1}{3}\right)\tan\theta\right\}$ if :

(a) $\tan\theta = -1$ (b) $\tan\theta = 1$ (c) $\tan\theta = 8$ (d) $\tan\theta = 2$

49. $\cos^{-1}\left[\cos(2\cot^{-1}(\sqrt{2}-1))\right]$ is equal to :

(a) $\sqrt{2}-1$ (b) $\frac{\pi}{4}$ (c) $1-\sqrt{2}$ (d) $\frac{3\pi}{4}$

50. If $f(x) = e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)}$, then which of the following is correct?

(a) $f\left(\frac{8\pi}{9}\right) = e^{\frac{5\pi}{18}}$ (b) $f\left(\frac{8\pi}{9}\right) = e^{\frac{3\pi}{18}}$ (c) $f\left(\frac{8\pi}{9}\right) = e^{\frac{13\pi}{18}}$ (d) $f\left(\frac{8\pi}{9}\right) = e^{\frac{11\pi}{12}}$

51. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then the value of $\alpha\beta + \beta\gamma + \gamma\alpha$ is :

(a) 3 (b) –3 (c) 0 (d) –2

52. The value of $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right)$ is :

(a) $\frac{\pi}{4}$ (b) π (c) 0 (d) ∞

53. The principal value of $\cos^{-1}x$ lies in the interval:

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$ (c) $[0, \pi]$ (d) $\left[0, \frac{\pi}{2}\right]$

54. Let $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 0$, for $0 \leq x, y, z, \leq 1$, what is the value of $x^{999} + y^{1000} + z^{1001}$?

(a) π (b) 0 (c) 3 (d) 1

55. What is the slope of tangent of $y = \cos^{-1}(\cos x)$ at $x = \frac{\pi}{4}$?

(a) 1 (b) –1 (c) 0 (d) 2

56. The value of $\tan\left\{2\tan^{-1}\left(\frac{1}{2}\right)\right\}$ is :

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) 1

57. The value of $\sin^{-1}x + \sin^{-1}\frac{1}{x} + \cos^{-1}x + \cos^{-1}\frac{1}{x}$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) 0

58. The value of $\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ is :

(a) $\pi - \frac{x}{2}$ (b) $\frac{x}{2}$ (c) $-\frac{x}{2}$ (d) $2\pi - \frac{x}{2}$

59. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then the value of $x^2 + y^2 + z^2 + 2x^2y^2 + 2y^2z^2 + 2z^2x^2$ is :

(a) 1 (b) 0 (c) 3 (d) 9

60. If $\sin^{-1}x + \sin^{-1}y = \pi$, then which of the following is equal to $\cos(\sin^{-1}x + \sin^{-1}y)$?

(a) 1 (b) 0 (c) –1 (d) $\frac{1}{2}$

61. If $\tan^{-1}\left(\frac{n}{x}\right) + \tan^{-1}\left(\frac{m}{x}\right) = \frac{\pi}{2}$, then which of the following is correct?

(a) $x = \frac{n}{m}$ (b) $x = \frac{m}{n}$ (b) $x = nm$ (d) $x^2 = nm$

62. The value of $\tan^{-1}\left[\frac{\sqrt{1-\cos x}+\sqrt{1+\cos x}}{\sqrt{1-\cos x}-\sqrt{1+\cos x}}\right]$ is :

(a) $\frac{x}{2} - \frac{\pi}{4}$ (b) $\frac{\pi}{4} - \frac{x}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{x}{2} + \frac{\pi}{4}$

63. If $\sin^2 x + \sin^2 y = 1$, then the value of $\tan(x + y)$ is :

(a) 0 (b) not defined (c) π (d) 1

64. What is the value of $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$, for $x > 0$?

(a) $\frac{x^2+2}{x^2+1}$ (b) $\sqrt{\frac{x^2+2}{x^2+1}}$ (c) $\frac{x^2+1}{x^2-1}$ (d) $\sqrt{\frac{x^2+1}{x^2+2}}$

65. Which of the given value satisfy the equation $\sin^{-1}x + \sin^{-1}(1 - x) = \cos^{-1}x$ is :

(a) (0, 1) (b) $\left(0, \frac{1}{2}\right)$ (c) (–1, 1) (d) $\left(-1, \frac{-1}{2}\right)$

66. $\sin^{-1} + \sin^{-1} 1\frac{4}{5} = \sin^{-1}x$, then the value of x is :

(a) $\frac{2}{5}$ (b) $\frac{3}{2}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

67. The value of $\sum_{n=0}^{\infty} \cot^{-1}\left(\frac{1}{1+n+n^2}\right)$ is :

(a) $\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{-\pi}{2}$

68. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$ and $\cos^{-1}x - \cos^{-1}y = 0$. Then the value of $x^2 + y^2$ is :

(a) $\frac{1}{2}$ (c) $\frac{1}{4}$ (c) 1 (d) $\frac{1}{8}$

69. Which of the following is incorrect ?

(a) $\cos\left(\sin^{-1}\frac{8}{17}\right) = \frac{15}{17}$ (b) $\cos\left(\operatorname{cosec}^{-1}\frac{13}{12}\right) = \frac{5}{12}$ (c) $\cos\left(\sec^{-1}\frac{5}{3}\right) = \frac{3}{5}$ (d) $\cos\left(\tan^{-1}\frac{3}{4}\right) = \frac{4}{5}$

70. Given, $\tan^{-1}\frac{yz}{xr} + \tan^{-1}\frac{zx}{yr} + \tan^{-1}\frac{xy}{zr} = \frac{\pi}{2}$ then $x^2 + y^2 + z^2$ is equal to :

(a) 0 (b) 1 (c) r^2 (d) xyz

71. Solve for x : $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$ for $0 < x < 1$.

(a) $\frac{\sqrt{3}+1}{2}$ (b) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (c) $\frac{2\sqrt{3}}{\sqrt{3}+1}$ (d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

72. Solve the equation : $\tan^{-1}\sqrt{x^2+2x} + \sin^{-1}\sqrt{x^2+2x+1} = \frac{\pi}{2}$.

(a) 0, –1 (b) 0, –2 (c) –1, –2 (d) –1, 1

73. The equation $\sin^{-1}x - \cos^{-1}x = \sin^{-1}\left(\frac{1}{2}\right)$ has

(a) no solution (b) unique solution (c) infinite solution (d) cannot determine

74. The value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) π

75. If $\tan\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] = 0$. Then the value of x is :

(a) 0, 1, –1 (b) 0, 1, 2 (c) 0, 2, –2 (d) 1, –2, –1

76. The value of $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)\right]$ is :

(a) $\frac{3-\sqrt{2}}{7}$ (b) $\frac{3-\sqrt{2}}{\sqrt{2}}$ (c) $\frac{2\sqrt{3}}{5}$ (d) $\frac{\sqrt{2}-3}{5}$

77. If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then the value of x^2 is :

(a) 25 (b) $\frac{2}{\sqrt{5}}$ (c) $\frac{1}{5}$ (d) $\frac{1}{\sqrt{5}}$

78. If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$, what is the value of $\sin^{-1}x + \sin^{-1}y$?

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) 0 (d) π

79. If $\tan^{-1}\frac{3x-1}{3x+2} + \tan^{-1}\frac{3x+1}{3x+2} = \frac{\pi}{4}$. Then the value of x is :

(a) $\pm\sqrt{\frac{18}{5}}$ (b) $\pm\sqrt{\frac{5}{18}}$ (c) $\pm\frac{2}{3}$ (d) $\pm\sqrt{\frac{2}{3}}$

80. If $\cot^{-1}x + \cot^{-1}y = \frac{\pi}{2}$, then which of the following is correct ?

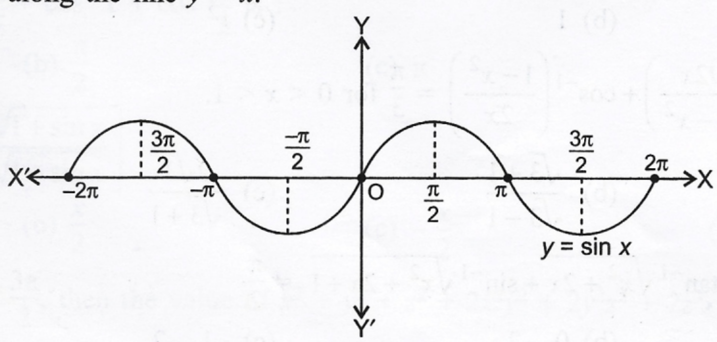
(a) $xy = 0$ (b) $xy = 1$ (c) $x + y = 0$ (d) $x + y = 1$

81. If $\cos^{-1}x + \cos^{-1}y = \pi$, then the value of $xy + yz$ is :

(a) 3 (b) 2 (c) 1 (d) 0

INPUT TEXT BASED MCQ's

82. It can be shown that the graph of an inverse function can be obtained from the corresponding graph of original function as a mirror image along the line $y = x$.



Answer the following questions :

(i) What is the range of arc sin in function for $x = \sin y$?

(a) $\frac{-\pi}{2} < y < \frac{\pi}{2}$ (b) $0 < y < p$ (c) $0 \leq y \leq \pi$ (d) $\frac{-\pi}{2} < y < 0$

(ii) The value of $\cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{7}{3}$ (c) π (d) 2π

(iii) The value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ is :

(a) 1 (b) −1 (c) 0 (d) $\frac{1}{2}$

(iv) The principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

(v) What is the domain of the given function?

(a) $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$ (b) $\frac{-\pi}{2} < x < \frac{\pi}{2}$ (c) $-1 \le x \le 1$ (d) $-1 < x = 1$

83. The value of an inverse trigonometric function which lies in the range of principal branch is called the principal value of that inverse trigonometric functions.

Answer the following questions :

(i) If $x + \frac{1}{x} = 2$, the principal value of $\sin^{-1}x$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$

(ii) What is the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$?

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(iii) The value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{-\pi}{4}$ (d) $\frac{\pi}{6}$

(iv) If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\left(\frac{1-b^2}{1+b^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then the value of x is :

(a) $\frac{a}{b}$ (b) ab (c) $\frac{b}{a}$ (d) $\frac{a-b}{1+ab}$

(v) The principal value of $\sin^{-1}\left\{\sin\left(\frac{5\pi}{6}\right)\right\}$ is :

(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) $\frac{\pi}{3}$

ANSWERS

1. (a)	**2.** (a)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (b)	**9.** (c)	**10.** (b)
11. (a)	**12.** (b)	**13.** (a)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (b)	**19.** (d)	**20.** (a)
21. (c)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (b)	**28.** (d)	**29.** (c)	**30.** (d)
31. (b)	**32.** (d)	**33.** (c)	**34.** (b)	**35.** (b)	**36.** (b)	**37.** (b)	**38.** (c)	**39.** (c)	**40.** (b)
41. (b)	**42.** (c)	**43.** (d)	**44.** (c)	**45.** (b)	**46.** (b)	**47.** (a)	**48.** (b)	**49.** (d)	**50.** (c)
51. (a)	**52.** (a)	**53.** (c)	**54.** (b)	**55.** (a)	**56.** (b)	**57.** (c)	**58.** (a)	**59.** (d)	**60.** (c)
61. (d)	**62.** (a)	**63.** (b)	**64.** (d)	**65.** (b)	**66.** (c)	**67.** (c)	**68.** (c)	**69.** (b)	**70.** (c)
71. (d)	**72.** (b)	**73.** (b)	**74.** (b)	**75.** (a)	**76.** (a)	**77.** (c)	**78.** (b)	**79.** (b)	**80.** (b)
81. (a)									

82. (i) (a) (ii) (c) (iii) (a) (iv) (c) (v) (b)

83. (i) (b) (ii) (d) (iii) (d) (iv) (d) (v) (a)

Hints to Some Selected Questions

1. (a) Let $(\cos \alpha)^{1/2} = P$

Then, $\cot^{-1} P + \tan^{-1} P = x \Rightarrow \frac{\pi}{2} = x\left[\tan P = \cot\left(\frac{\pi}{2} - P\right)\right]$

$\therefore \sin \frac{\pi}{2} = 1$

2. (a) $\frac{-\pi}{4} \leq \frac{\sin^{-1}x}{2} \leq \frac{\pi}{4} \Rightarrow \frac{-\pi}{4} \leq \sin^{-1}a \leq \frac{\pi}{4} \Rightarrow \frac{1}{2} \leq |a| \leq \frac{1}{\sqrt{2}}.$

5. (d) $2y = x + z$ (x, y, z are in A.P) $\Rightarrow 2\tan^{-1} y = \tan^{-1}x + \tan^{-1}z \Rightarrow x = y = z$

6. (a) $\cot\left(\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{2}{3}\right)\right) = \cot\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)\right) = \cot\left(\tan^{-1}\frac{17}{6}\right) = \frac{6}{17}$

7. (d) $\tan^{-1}\left[\frac{x + \frac{2x}{1-x^2}}{1 - x.\frac{2x}{1-x^2}}\right] = \tan^{-1}\left[\frac{x - x^3 + 2x}{1-x^2-2x^2}\right] = \tan^{-1}\left[\frac{3x - x^3}{1-3x^2}\right]$

10. (b) $2\tan^{-1}(\cos x) = \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right)$

Now, $\tan^{-1}[2\cot x \operatorname{cosec} x] = \tan^{-1}(2\operatorname{cosec} x) \Rightarrow \cot x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}.$

11. (a) $\tan^{-1} y = 4\tan x = 2\tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{\frac{4x}{(1-x^2)}}{1-\frac{4x^2}{(1-x^2)^2}} = \tan^{-1}\frac{4x(1-x^2)}{x^4-6x^2+1}$

$\Rightarrow y = \frac{4x(1-x^2)}{x^4-6x^2+1}$

If $x = \tan\frac{\pi}{8} \Rightarrow \tan^{-1}y = 4\tan^{-1}x = \frac{\pi}{2} \Rightarrow y = \infty \Rightarrow x^4 - 6x^2 + 1 = 0$

13. (a) $\cos\left[\cos^{-1}\left(-\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{3}\right)\right] = \cos\left[\pi - \left\{\cos^{-1}\left(\frac{1}{3}\right) + \sin^{-1}\left(\frac{1}{3}\right)\right\}\right] = \cos\left(\pi - \frac{\pi}{2}\right) = 0$

14. (c) $\cos^{-1}\left(\frac{\sqrt{x^2-25}}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2} \Rightarrow \cos^{-1}\left(\frac{\sqrt{x^2-25}}{x}\right) = \cos^{-1}\left(\frac{12}{x}\right)$

$\Rightarrow \frac{\sqrt{x^2-25}}{x} = \frac{12}{x} \Rightarrow x^2 - 25 = 144 \Rightarrow x^2 = 169 \Rightarrow x = \pm 13.$

16. (c) $\sin^{-1}x = \alpha \Rightarrow \sin\alpha = x \Rightarrow \tan\alpha = \frac{x}{\sqrt{1-x^2}} \Rightarrow \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$

18. (b) $\sin^{-1}x > \cos^{-1}x \Rightarrow 2\sin^{-1}x > \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}x > \frac{\pi}{4}$

$\therefore \frac{\pi}{4} < \sin^{-1}x \leq \frac{\pi}{2} \Rightarrow \frac{1}{\sqrt{2}} < x \leq 1$

19. (d) $\sin^{-1}6x + \sin^{-1}6\sqrt{3}x = -\frac{\pi}{2} \Rightarrow \sin^{-1}6\sqrt{3}x = \frac{\pi}{2} - \sin^{-1}6x$

$6\sqrt{3}x = -\cos\sin^{-1}6x = -\sqrt{1-36x^2} \Rightarrow 108x^2 = 1 - 36x^2$

$\Rightarrow x = \pm\frac{1}{12}$, since $x < 0 \quad \therefore x = \frac{-1}{12}$

20. (a) $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8} \Rightarrow (\tan^{-1}x + \cot^{-1}x)^2 - 2\tan^{-1}x\left(\frac{\pi}{2} - \tan^{-1}x\right) = \frac{5\pi^2}{8}$

$\Rightarrow \frac{\pi^2}{4} - 2\frac{\pi}{2}\tan^{-1}x + 2(\tan^{-1}x)^2 = \frac{5\pi^2}{8} \Rightarrow 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 0$

$\Rightarrow \tan^{-1}x = \frac{-\pi}{4}, \frac{3\pi}{4} \Rightarrow x = -1.$

23. (b) Take tan on both side

$\Rightarrow \frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1} \times \frac{x-1}{x}} = -7 \Rightarrow x = 2 \Rightarrow \tan^{-1}3 + \tan^{-1}\frac{1}{2} = \tan^{-1}(-7)$

$\Rightarrow$ Therefore, the given equation has no solution

24. (b) $\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z \Rightarrow \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right] = \text{cost}^{-1}(-z)$

$\Rightarrow xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

$\Rightarrow x^2y^2 + z^2 + 2xyz = 1 - x^2 - y^2 + x^2y^2 \Rightarrow x^2 + y^2 + z^2 + 2xyz = 1$

31. (b) $\frac{n}{\pi} < \cot\frac{\pi}{6} \Rightarrow n < \pi \times \sqrt{3} \Rightarrow n < \sqrt{3}\pi \Rightarrow n < 5.16$

$\therefore$ Maximum value of $n = 5$.

32. (d) $\tan^{-1}\left(\frac{a-b}{ab+1}\right) + \tan^{-1}\left(\frac{b-c}{bc+1}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right)$

$\tan^{-1}a - \tan^{-1}b + \tan^{-1}b - \tan^{-1}c + \tan^{-1}c + \tan^{-1}a = 0$ or π or 2π

33. (c) Put $x = \tan\theta$

$\Rightarrow \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1}x$ when $x \geq 0$

34. (b) $\tan\left[\tan^{-1}\left\{\frac{\sqrt{1-\frac{16}{25}}}{\frac{4}{5}}\right\} + \tan^{-1}\frac{2}{3}\right] = \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right) = \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}.$

35. (b) $\frac{-2\pi}{2} \leq 2\sin^{-1}x \leq \frac{2\pi}{2}, \frac{-3\pi}{2} \leq 3\sin^{-1}y \leq \frac{3\pi}{2}$

$\Rightarrow 2\sin^{-1}x = \frac{2\pi}{2}, \Rightarrow \sin^{-1}x = \frac{\pi}{2} \Rightarrow 3\sin^{-1}y = \frac{3\pi}{2} \Rightarrow \sin^{-1}y = \frac{\pi}{2}$

$\Rightarrow y = kx - 5$; so $x = 1$ and $y = 1$ $\quad\therefore 1 = k - 5 \Rightarrow k = 6$.

36. (b) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

$$= \tan^{-1}\left[\frac{\sqrt{\frac{ak}{cb}} + \sqrt{\frac{bk}{ca}} + \sqrt{\frac{ck}{ab}} - \sqrt{\frac{ak}{cb}\cdot\frac{bk}{ca}\cdot\frac{ck}{ab}}}{1 - \sqrt{\frac{ak}{cb}\cdot\frac{bk}{ca}} - \sqrt{\frac{bk}{ca}\cdot\frac{ck}{ab}} - \sqrt{\frac{ck}{ab}\cdot\frac{ak}{cb}}}\right] = \tan^{-1}\left[\frac{\sqrt{k}\left(\frac{a+b+c}{\sqrt{abc}}\right) - \sqrt{a+b+c}\left(\frac{a+b+c}{\sqrt{abc}}\right)}{1-(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}\right] = \tan^{-1}0$$

$\Rightarrow \theta = \tan^{-1}0 \Rightarrow \theta = \pi$

37. (b) $\tan^{-1}\left(\dfrac{\dfrac{1}{2x+1}+\dfrac{1}{4x+1}}{1-\dfrac{1}{2x+1}\cdot\dfrac{1}{4x+1}}\right)=\tan^{-1}\left(\dfrac{2}{x^2}\right) \Rightarrow \dfrac{4x+1+2x+1}{8x^2+6x+1-1}=\dfrac{2}{2x^2}$ or $\dfrac{6x+2}{8x^2+6x}=\dfrac{2}{x^2}$

$\Rightarrow \dfrac{6x+2}{8x+6}=\dfrac{2}{x} \Rightarrow \dfrac{3x+1}{4x+3}=\dfrac{1}{x} \Rightarrow 3x^2+x=8x+6 \Rightarrow 3x^2-7x+6=0$

$\Rightarrow x = 0, -\dfrac{2}{3}, 3$

$\therefore$ Mean $= \dfrac{0-\dfrac{2}{3}+3}{3}=\dfrac{7}{9}$

39. (c) $\sin^{-1}x+\cos^{-1}x=\dfrac{\pi}{2}$

$\sin^{-1}x-\cos^{-1}x=\dfrac{\pi}{6}$ [Given]

$\Rightarrow 2\sin^{-1}x=\dfrac{2\pi}{3} \Rightarrow \sin^{-1}x=\dfrac{\pi}{3} \Rightarrow \cos^{-1}x=\dfrac{\pi}{6} \Rightarrow x=\dfrac{\sqrt{3}}{2}$ is the unique solution

40. (b) Let $x^2=\cos 2\theta$

$\tan^{-1}\left(\dfrac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right)=\alpha$

$\Rightarrow \tan^{-1}\left(\dfrac{1+\tan\theta}{1-\tan\theta}\right)=\alpha \Rightarrow \tan^{-1}\left(\tan\left(\dfrac{\pi}{4}+\theta\right)\right)=\alpha$

$\Rightarrow \dfrac{\pi}{4}+\theta=\alpha \Rightarrow \dfrac{\pi}{4}+\dfrac{1}{2}\cos^{-1}x^2=\alpha \Rightarrow x^2=\sin 2\alpha$

41. (b) Put $x=\cos\theta \Rightarrow \cos^{-1}(\cos\theta)+\cos^{-1}\left[\dfrac{1}{2}\cos\theta+\dfrac{\sqrt{3}}{2}\sin\theta\right]=\theta+\cos^{-1}\cos\left(\dfrac{\pi}{3}-\theta\right)=\dfrac{\pi}{3}$.

42. (c) $\tan^{-1}\left(\dfrac{d}{1+a_1a_2}\right)=\tan^{-1}\left(\dfrac{a_2-a_1}{1+a_1a_2}\right)=\tan^{-1}a_2-\tan^{-1}a_1$

Similarly,

$\tan^{-1}\left(\dfrac{d}{1+a_2a_3}\right)=\tan^{-1}a_3-\tan^{-1}a_2$ .... so on

$\tan\{\tan^{-1}a_2-\tan^{-1}a_1+\tan^{-1}a_3-\tan^{-1}a_2+....\}$

44. (c) $\sum_{n=1}^{\infty}\cot^{-1}\left(2^{n+1}+\dfrac{1}{2^n}\right)$

46. (b) $2\tan^{-1}\left(\dfrac{1}{5}\right)=\tan^{-1}\dfrac{5}{12} \Rightarrow \tan\left[\tan\dfrac{-5}{12}-\dfrac{\pi}{4}\right]=\dfrac{\dfrac{15}{12}-1}{1+\dfrac{5}{12}.1}=\dfrac{-7}{17}$

47. (a) Put $x=\tan\theta$

We get, $\cos^{-1}(\cos 2\theta)+\sin^{-1}(\sin 2\theta)+\tan^{-1}(\tan 2\theta)=\dfrac{2\pi}{3}$

$\Rightarrow 2\theta+2\theta+2\theta=\dfrac{2\pi}{3} \Rightarrow 6\theta=\dfrac{2\pi}{3} \Rightarrow \theta=\dfrac{\pi}{9} \Rightarrow x=\tan\dfrac{\pi}{9}$

48. (b) $\theta = \tan^{-1}\left[\dfrac{2\tan^2\theta - \frac{1}{3}\tan\theta}{1+(2\tan^2\theta)\left(\frac{1}{3}\tan\theta\right)}\right] \Rightarrow \tan\theta = \dfrac{2\tan^2\theta - \frac{1}{3}\tan\theta}{1+\frac{2}{3}\tan^3\theta}$

$\Rightarrow (\tan\theta - 1)^2(\tan\theta + 2) = 0 \Rightarrow \tan\theta = 1, \tan\theta = -2$

50. (c) $f(x) = e^{\cos^{-1}\sin\left(x+\frac{\pi}{3}\right)} \Rightarrow y = \cos^{-1}\sin\left(x - \frac{\pi}{3}\right)$

$\Rightarrow y = \frac{\pi}{2} - \sin^{-1}\left(\sin\left(\frac{8\pi}{9}+\frac{\pi}{3}\right)\right) = \frac{\pi}{2} - \sin^{-1}\left(\sin\left(-\frac{2\pi}{9}\right)\right) = \frac{\pi}{2} + \frac{2\pi}{9} = \frac{13\pi}{18}$

$\therefore f\left(\frac{8\pi}{9}\right) = e^{13\pi/18}$

51. (a) $\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi \Rightarrow \alpha = \beta = \gamma = -1 \Rightarrow \alpha\beta + \beta\gamma + \gamma\alpha = 1 + 1 + 1 = 3$

52. (a) $\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left[\dfrac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}\right] = \tan^{-1}\frac{3}{4}$

$\therefore \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}(1) = \frac{\pi}{4}$

53. (c) $y = \cos^{-1}x$ — Domain $[-1, 1]$

Range $[0, \pi]$

$\therefore$ Principal value of $\cos^{-1}x$ lies between $0 \le y \le \pi$

54. (b) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 0$, it means $\cos^{-1}x = 0, \cos^{-1}y = 0, \cos^{-1}z = 0$

$\therefore x = 0, y = 0, z = 0 \Rightarrow x^{999} + y^{1000} + z^{1001} = 0$

56. (b) $\tan\left\{2\tan^{-1}\left(\frac{1}{2}\right)\right\} = \tan\left\{\tan^{-1}\dfrac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right\} = \tan\left\{\tan^{-1}\dfrac{1}{\frac{3}{4}}\right\} = \tan\left\{\tan^{-1}\frac{4}{3}\right\} = \frac{4}{3}$

57. (c) $(\sin^{-1}x + \cos^{-1}x) + \left(\sin^{-1}\frac{1}{x} + \cos^{-1}\frac{1}{x}\right) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

59. (d) $\sin^{-1}x = \frac{\pi}{2}, \sin^{-1}y = \frac{\pi}{2}, \sin^{-1}z = \frac{\pi}{2} \Rightarrow x = 1, y = 1, z = 1$

$= 1 + 1 + 1 + 2 \times 1 \times 1 + 2 \times 1 \times 1 + 2 \times 1 \times 1 = 3 + 2 + 2 + 2 = 9$

62. (a) $\tan^{-1}\left[\dfrac{\sqrt{1-\cos x}+\sqrt{1+\cos x}}{\sqrt{1-\cos x}-\sqrt{1+\cos x}} \times \dfrac{\sqrt{1-\cos x}+\sqrt{1+\cos x}}{\sqrt{1-\cos x}+\sqrt{1+\cos x}}\right]$

$= \tan^{-1}\left[\dfrac{1-\cos x+1+\cos x+2\sqrt{\sin^2 x}}{1-\cos x-1-\cos x}\right] = \tan^{-1}\left[\dfrac{2(1+\sin x)}{-2\cos x}\right] = \tan^{-1}\left[\dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{-\left[\cos\frac{x}{2}-\sin\frac{x}{2}\right]}\right]$

$= \tan^{-1}\left[-\left(\dfrac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}\right)\right] = \tan^{-1}\tan\left[-\left(\frac{\pi}{4}-\frac{x}{2}\right)\right] = \left[\tan^{-1}\left(\tan\left(\frac{x}{2}-\frac{\pi}{4}\right)\right)\right] = \frac{x}{2} - \frac{\pi}{4}.$

63. (b) $\sin^2 x = 1 - \sin^2 y \Rightarrow \sin^2 x = \cos^2 y \Rightarrow \sin x = \cos y$

similarly $\sin y = \cos x$

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \frac{\cos^2 y + \cos^2 x}{0} = \text{Not defined}$$

66. (c) $\sin^{-1}\left[1.\sqrt{1-\left(\frac{4}{5}\right)^2} + \frac{4}{5}\sqrt{1-1^2}\right]$

$$= \sin^{-1}x = \sin^{-1}\left[\frac{3}{5}+0\right] = \sin^{-1}x \Rightarrow \sin^{-1}\frac{3}{5} = \sin^{-1}x \Rightarrow x = \frac{3}{5}$$

69. (b) $\cos\left(\operatorname{cosec}^{-1}\frac{13}{12}\right) = \cos\left(\cos^{-1}\frac{5}{13}\right) = \frac{5}{13}.$

73. (b) $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ (we know)

$2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{\pi + 3\pi}{6} = \frac{2\pi}{3} \Rightarrow \sin^{-1}x = \frac{\pi}{3}$ and $\cos^{-1}x = \frac{\pi}{6}$

$\therefore \sqrt{\frac{3}{2}}$ is unique solution.

74. (b) $\tan^{-1}\left(\frac{x}{y}\right) - \left\{\tan^{-1}(1) - \tan^{-1}\left(\frac{y}{x}\right)\right\} = \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}1 + \cot^{-1}\left(\frac{x}{y}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

78. (b) $\left(\frac{\pi}{2} - \sin^{-1}x\right) + \left(\frac{\pi}{2} - \sin^{-1}y\right) = \frac{\pi}{2} \Rightarrow \pi - (\sin^{-1}x + \sin^{-1}y) = \frac{\pi}{2}$

$\Rightarrow \pi - \frac{\pi}{2} = \sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

79. (b) $\tan^{-1}\left[\frac{\frac{3x-1}{3x+2}+\frac{3x+1}{3x+2}}{1-\left(\frac{3x-1}{3x+2}\right)\left(\frac{3x+1}{3x+2}\right)}\right] = \frac{\pi}{4} \Rightarrow \frac{6x}{3x+2} \times \frac{(3x+2)^2}{12x+5} = 1$

$\Rightarrow \frac{6x(3x+2)}{12x+5} = 1 \Rightarrow 18x^2 = 5 \Rightarrow x = \pm\sqrt{\frac{5}{18}}$

82. (i) (a) The range of $x = \sin y$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. i.e., $\frac{-\pi}{2} < y < \frac{\pi}{2}$.

(ii) (c) $\left[\pi - \cos^{-1}\cos\left(\frac{1}{2}\right)\right] + 2\sin^{-1}\left(\frac{1}{2}\right)$

$$= \left[\pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right)\right] + 2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\left(\frac{1}{2}\right)$$

$$\left(\pi - \frac{\pi}{3}\right) + 2\times\frac{\pi}{6} = \pi$$

(iii) (a) $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right] = \sin\left[\frac{\pi}{3} + \sin^{-1}\left(\sin\frac{\pi}{6}\right)\right] = \sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1.$

(iv) (c) We know that $\sin^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3} = \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

(v) (b) The domain of the given function is –1 to 01.

83. (i) (b) $x + \frac{1}{x} = 2$

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 0 \Rightarrow \frac{x-1}{\sqrt{x}} = 0 \Rightarrow x = 1 \Rightarrow \sin^{-1}x = \frac{\pi}{2}.$$

(ii) (d) The range of $\sec^{-1} x$ is $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{\pi}{2}, \pi\right] = \sec^{-1}\left(\sec\frac{\pi}{6}\right) = \frac{\pi}{6} = \frac{\pi}{6}$.

(iii) (d) The range of $\operatorname{cosec}^{-1}x$ is $\left[\frac{-\pi}{2}, 0\right) \cup \left[\frac{\pi}{2}, \pi\right]$.

(iv) (d) $2\tan^{-1} a - 2\tan^{-1}b = 2\tan^{-1}x$

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) = x \Rightarrow x = \frac{a-b}{1+ab}$$

(v) (a) $\sin^{-1}\left(\sin\left(\pi - \frac{\pi}{6}\right)\right) = \sin^{-1}\left\{\sin\left(\frac{\pi}{6}\right)\right\} = \frac{\pi}{6}$.